A counterexample to Tang and Padubidri's claim about the bisection width of a diagonal mesh

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Abstract: A counterexample is presented to disprove Tang and Padubidri's claim about the bisection width of a diagonal mesh.

Index terms: Diagonal mesh, bisection width.

Tang and Padubidri [2] introduce a $k \times n$ *diagonal mesh* where k and n are odd integers greater than or equal to three. The node set consists of all ordered pairs (x, y) where $x \in \{-\frac{k-1}{2}, \cdots, \frac{k-1}{2}\}$ and $y \in \{-\frac{n-1}{2}, \cdots, \frac{n-1}{2}\}$. Further, each (x, y) has four neighbors:

- $(<x+1>_k, <y+1>_n)$

- $(<x+1>_k, <y-1>_n)$

- $(<x-1>_k, <y+1>_n)$

- $(<x-1>_k, <y-1>_n)$

where

$$<x>_n = \begin{cases} x & \text{if } |x| \leq \frac{n-1}{2} \\ x-n & \text{if } x > \frac{n-1}{2} \\ x+n & \text{if } x < -\frac{n-1}{2}. \end{cases}$$

Figure 1 presents the 5×5 diagonal mesh that consists of 25 nodes and 50 edges. It is essentially a reproduction of Figure 1(b) that appears in [2] (p. 816).

<u>Remark</u>: The graph has been presented in a way that minimizes edge crossings. In particular, nodes in the leftmost column (resp. top row) are to be identified with the corresponding nodes with the same label in the rightmost column (resp. bottom row).
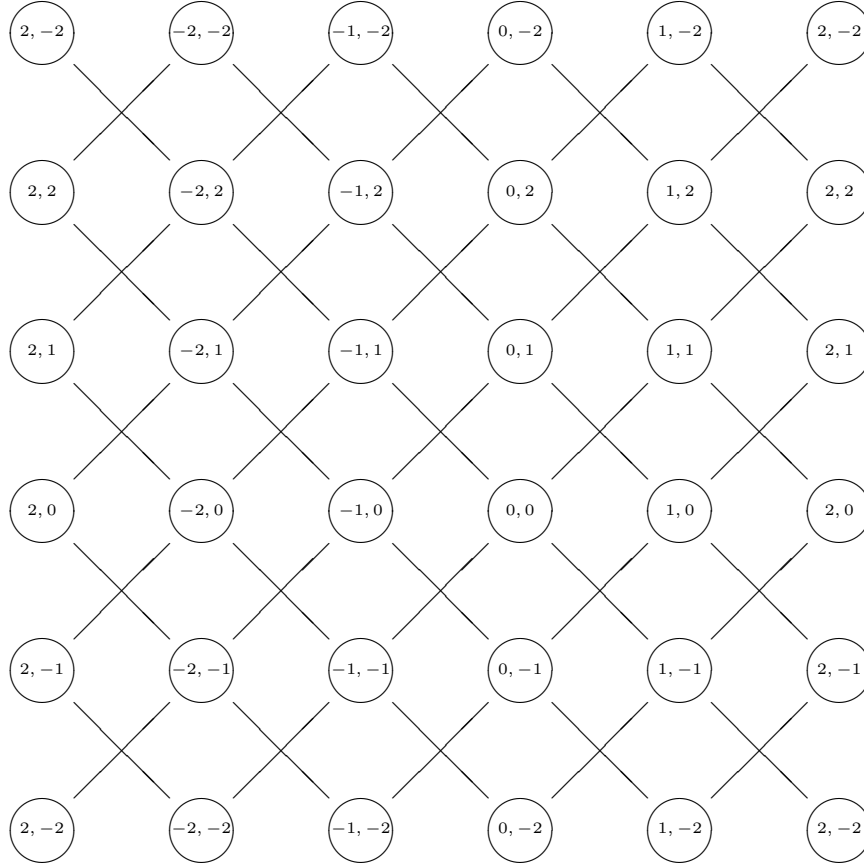
Figure 1: The 5×5 diagonal mesh

It is relevant to note that the $k \times n$ diagonal mesh is actually isomorphic to the direct product of two cycles of lengths k and n, respectively [1].

The *bisection width* of a connected graph is defined to be the least number of edges that need to be removed so that the resulting graph consists of two connected components having the same number (plus/minus one) of nodes. Tang and Padubidri "claim" ([2], p. 817) that the bisection width of a $k \times n$ diagonal mesh is equal to $4n$ where $k \geq n \geq 3$. The objective of this note is to disprove their claim. To that end, Figure 2 presents two node-disjoint connected subgraphs of the 5×5 diagonal mesh. The nodes of the first subgraph (resp. second subgraph) appear within "rectangles" (resp. "ovals"). While the first subgraph accounts for 13 nodes and 20 edges, the second subgraph accounts for 12 nodes and 18 edges. In other words, the two connected subgraphs (whose numbers of nodes differ by just one) are obtainable by removing only

12 edges of the original graph. Thus the bisection width of the 5×5 diagonal mesh is at most 12 (*not* 20 as suggested by the authors).

<u>Note</u>: The two subgraphs appearing in Figure 2 are to be viewed in the light of the earlier remark, viz., nodes in the leftmost column (resp. top row) are to be identified with the corresponding nodes with the same label in the rightmost column (resp. bottom row).
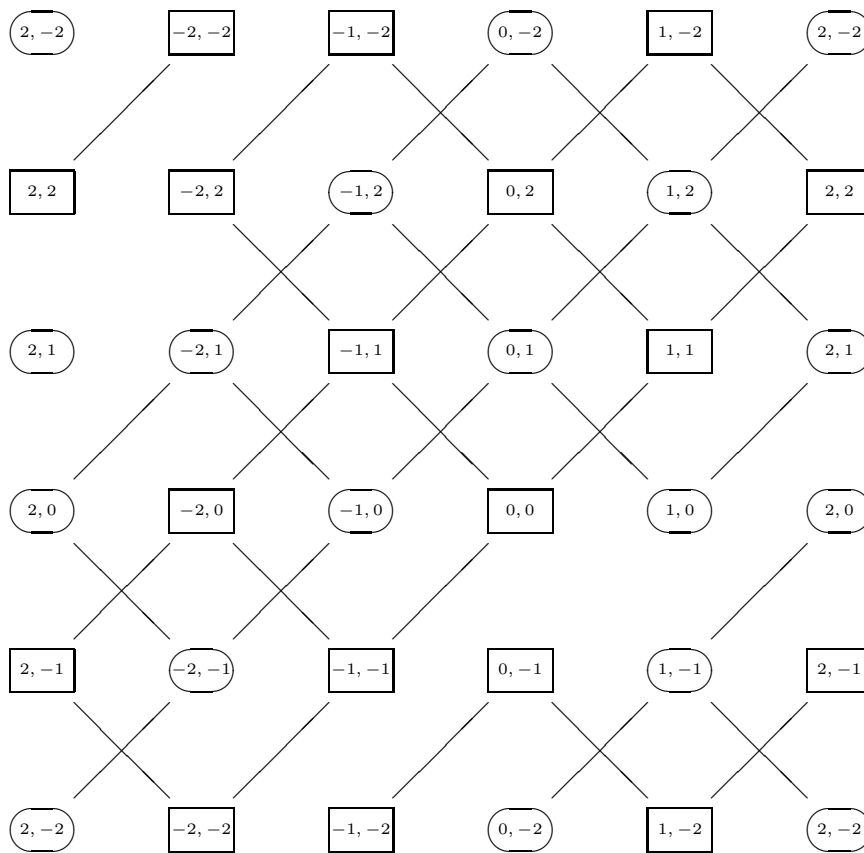
Figure 2: Two connected subgraphs of the 5×5 diagonal mesh

References

[1] W. Imrich and S. Klavžar, *Product Graphs: Structure and Recognition*, John Wiley & Sons, New York, 2000.

[2] K.W. Tang and S.A. Padubidri, "Diagonal and toroidal mesh networks," *IEEE Trans. Comput.*, vol. 43, no. 7, pp. 815-826, July 1994.