Lecture 9 Evolution after the main sequence

As a star evolves, gravitational contraction makes it hotter and denser. Until now, our analysis was limited by working with a simplest possible version of the equation of state — that of an ideal gas composed of classical particles (Lecture 3). We now need to introduce some generalizations of this simple description.

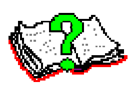
9.1. Relativistic and quantum effects in the equation of state.

Physically, the gas pressure is a measure of momentum exchange inside the gas. A general expression for the gas pressure, applicable for classical and relativistic particles, is

$$P = \frac{1}{3}\int_0^\infty vpn(p)\,dp,$$

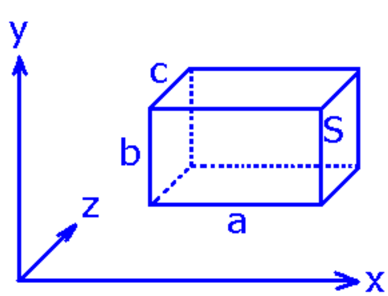
$$P = \frac{1}{3}\int_0^\infty vpn(p)\,dp, P = \frac{1}{3}\int_0^\infty vpn(p)\,dp, \qquad (9.1)$$

where $n(p)dp$ is the number density of particles with momentum between p and $p+dp$, and v is velocity. This equation can be derived in a way which is very similar to our derivation of the equation (3.11) in Lecture 3.

Exercise 9.1. Derive equation (9.1). 

Solution to Exercise 9.1.

Consider a gas of particles contained in a rectangular box



When a single particle hits the wall of the box labeled with area $S=bc$, it changes its momentum by an amount $2p_x$. The time interval between two consecutive hits is $2a/v_x$, and hence the average force on area S, produced by a single particle, is $v_x p_x a$, and the average pressure is $v_x p_x/(abc)=v_x p_x/V$, where V is box volume. Energy equipartition between three degrees of freedom gives the average value $<v_x p_x>=vp/3$, and hence the pressure produced by a single particle is $vp/3V$. If we have N particles with momentum p, the pressure is $vpN/3V=vpn/3$, where $n=N/V$ is number density of particles with momentum p. When we have many particles described by a continuous distribution in momentum p, we replace n by $n(p)dp$ and integrate to obtain

$$P = \frac{1}{3}\int_0^{\infty} vpn(p)\,dp.$$

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For an ideal gas of classical (i.e. non-relativistic) particles, the energy of a single particle is $E=mv^2/2=vp/2$, and hence the internal energy density of the gas is

$$u = \frac{1}{2}\int_0^{\infty} vpn(p)\,dp = \frac{3}{2}P.$$

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(nonrelativistic particles) (9.2)

In the limiting case of extremely relativistic (ultrarelativistic) particles, we have $E=cp$, where c is speed of light, and hence

$$u = \int_0^\infty cpn(p)\,dp = 3P.$$

$$u = \int_0^\infty cpn(p)\,dp = 3P. u = \int_0^\infty cpn(p)\,dp = 3P.$$

(ultrarelativistic particles) (9.3)

One example of purely relativistic particles is photons. With radiation energy density $u_R = aT^4$, where a is Stefan radiation constant (equation 7.14 of Lecture 7), we get immediately the *radiation pressure*, i.e. the pressure from the photons:

$$P_R = \frac{1}{3}aT^4.$$

$$P_R = \frac{1}{3}aT^4 P_R = \frac{1}{3}aT^4.$$

(9.4)

At low temperature and high density, quantum-mechanical effects must be taken into account in the description of the gas. According to Pauli's exclusion principle, at most two fermions (i.e., electrons or nucleons), with different spin, can occupy a given energy state. Each particular state occupies volume h^3 in the 6-dimentional space of spatial coordinates and momentum components, where h is Planck's constant. Thus, if p is the length of the 3-dimentional momentum vector, the number density of particles with momentum in the interval between p and $p+dp$ is

$$n(p)\,dp = \frac{2}{h^3}4\pi p^2 dp\,F(p),$$

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$$n(p)\,dp = \frac{2}{h^3}4\pi p^2 dp\,F(p),$$

(9.5)

where $F(p)$ is the occupation probability number for the Fermi gas.

Here we only consider a gas of electrons in the limit of zero temperature (limit of complete degeneracy). In this limit, all the quantum states ore occupied up to some maximum momentum p_F, known as *Fermi momentum*, but no states with higher p are occupied:

$$n(p)\,dp = \frac{2}{h^3} 4\pi p^2 dp, \qquad p \le p_F$$

$$n(p)\,dp = 0, \qquad p > p_F.$$

$$n(p)\,dp = \frac{2}{h^3} 4\pi p^2 dp, \qquad p \le p_F$$

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$$n(p)\,dp = \frac{2}{h^3} 4\pi p^2 dp, \qquad p \le p_F$$

$$n(p)\,dp = 0, \qquad p > p_F. \qquad (9.6)$$

Integrating over all possible momenta, the electron number density is

$$n_e = \int_0^\infty n_e(p)\,dp = \frac{8\pi p_F^3}{3h^3}.$$

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$$n_e = \int_0^\infty n_e(p)\,dp = \frac{8\pi p_F^3}{3h^3}. \qquad (9.7)$$

Using equation (9.6), the energy density of the degenerate electrons is

$$u_e^{(deg)} = \frac{1}{2m_e} \int_0^\infty p^2 n_e(p)\,dp = \frac{4\pi p_F^5}{5h^3 m_e},$$

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(nonrelativistic) (9.8)

$$u_e^{(deg)} = \int_0^\infty cp\,n_e(p)\,dp = \frac{2\pi c p_F^4}{h^3},$$

$$u_e^{(deg)} = \int_0^\infty cp\,n_e(p)\,dp = \frac{2\pi c p_F^4}{h^3}, u_e^{(deg)} = \int_0^\infty cp\,n_e(p)\,dp = \frac{2\pi c p_F^4}{h^3},$$

(ultra-relativistic) (9.9)

and the pressure of the degenerate electrons is

$$P_e^{(deg)} = \frac{2}{3}u_e^{(deg)} = \frac{8\pi p_F^5}{15h^3 m_e},$$

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(nonrelativistic) (9.10)

$$P_e^{(deg)} = \frac{1}{3}u_e^{(deg)} = \frac{2\pi c p_F^4}{3h^3}.$$

$$P_c^{(deg)} = \frac{1}{3}u_c^{(deg)} = \frac{2\pi c p_F^4}{3h^3}.$$

$$P_e^{(deg)} = \frac{1}{3}u_e^{(deg)} = \frac{2\pi c p_F^4}{3h^3}.$$

(ultra-relativistic) (9.11)

When chemical composition is described by the mass fractions of hydrogen X, helium Y and heavy elements $Z=1-X-Y$, the electron number density is approximately

$$n_e \simeq \frac{\rho}{m_H}X + \frac{\rho}{2m_H}Y + \frac{\rho}{2m_H}Z$$
$$= \frac{\rho}{2m_H}[2X + 1 - X] = \frac{\rho(1+X)}{2m_H}.$$

$$n_e \simeq \frac{\rho}{m_H}X + \frac{\rho}{2m_H}Y + \frac{\rho}{2m_H}Z$$
$$= \frac{\rho}{2m_H}[2X + 1 - X] = \frac{\rho(1+X)}{2m_H}.$$
$$n_e \simeq \frac{\rho}{m_H}X + \frac{\rho}{2m_H}Y + \frac{\rho}{2m_H}Z$$
$$= \frac{\rho}{2m_H}[2X + 1 - X] = \frac{\rho(1+X)}{2m_H}.$$

(9.12)

Using equation (9.7), we obtain

$$P_e^{(deg)} = \frac{h^2}{20 m_e m_H}\left(\frac{3}{\pi m_H}\right)^{\frac{2}{3}}\left(\frac{1+X}{2}\rho\right)^{\frac{5}{3}},$$

$$P_e^{(deg)} - \frac{h^2}{20m_e m_H}\left(\frac{3}{\pi m_H}\right)^{\frac{2}{3}}\left(\frac{1+X}{2}\rho\right)^{\frac{5}{3}},$$

$$P_e^{(deg)} = \frac{h^2}{20m_e m_H}\left(\frac{3}{\pi m_H}\right)^{\frac{2}{3}}\left(\frac{1+X}{2}\rho\right)^{\frac{5}{3}},$$

(nonrelativistic) (9.13)

$$P_e^{(deg)} = \frac{hc}{8m_H}\left(\frac{3}{\pi m_H}\right)^{\frac{1}{3}}\left(\frac{1+X}{2}\rho\right)^{\frac{4}{3}}.$$

$$P_e^{(deg)} = \frac{hc}{8m_H}\left(\frac{3}{\pi m_H}\right)^{\frac{1}{3}}\left(\frac{1+X}{2}\rho\right)^{\frac{4}{3}}.$$

$$P_e^{(deg)} = \frac{hc}{8m_H}\left(\frac{3}{\pi m_H}\right)^{\frac{1}{3}}\left(\frac{1+X}{2}\rho\right)^{\frac{4}{3}}.$$

(ultra-relativistic) (9.14)

Exercise 9.2. Verify equations (9.13, 9.14).

The equations (9.13, 9.14) represent a particularly simple form of the equation of state, since they are independent of the temperature. This is a consequence of applying the low-temperature limit (equation 9.6) in the derivations. One may wonder why the low-temperature limit is relevant for describing stellar interiors, which are very hot in our everyday standards. The answer is that it is essentially the high-density requirement, which can also be viewed as a low-temperature requirement.

The low-temperature limit is applicable when the Fermi momentum p_F is much bigger than the classical momentum of the electron provided by its thermal motion, which is $m_e v=(2m_e E)^{1/2}=(3m_e kT)^{1/2}$, i.e. when

$$kT \ll \frac{h^2 n_e^{2/3}}{m_e}.$$

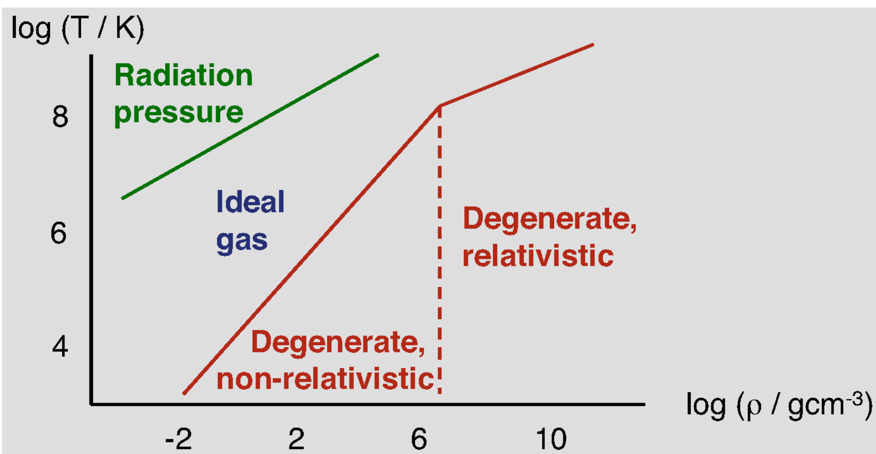
$$kT \ll \frac{h^2 n_e^{2/3}}{m_e}.$$

$$kT \ll \frac{h^2 n_e^{2/3}}{m_e}.$$

(9.15)

Thus, a quantum gas is a cold gas, but the standard of "coldness" is set by the density of the gas; a temperature of a billion degrees can be cold in a very dense gas.

The following diagram illustrates when the different pressures matter:



Different stars occupy different portions of the plane:

Solar-type stars — ideal gas throughout;
Massive stars — significant contribution of radiation pressure;
White dwarfs — nonrelativistic degeneracy pressure.

Relativistic degeneracy implies an unstable equation of state, and hence there is no stable stars in that part of the plane. Indeed, according to equation (4.18) of Lecture 4, the gravitational binding energy of the star is

$$\Omega = -3\int_V P\,dv,$$

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and for an extremely relativistic gas with internal energy density $u=3p$ (equation 9.3) we obtain

$$\Omega = -\int_V u\, dv = -U,$$

$$\Omega = -\int_V u\, dv = -U, \Omega = -\int_V u\, dv = -U,$$

where U is internal energy, and hence the total energy E is

$$E = \Omega + U = 0.$$

$$E = \Omega + U = 0.$$
$$E = \Omega + U = 0.$$

This result shows that an extremely relativistic system is marginally stable: it may expand or contract indefinitely without any change in the total energy. Hence a small change to the system may me sufficient to push it into instability. In addition to the ultra-relativistic degeneracy, another example of extremely relativistic systems is a star dominated by radiation pressure; such stars are also unstable.

9.2. Red giants.

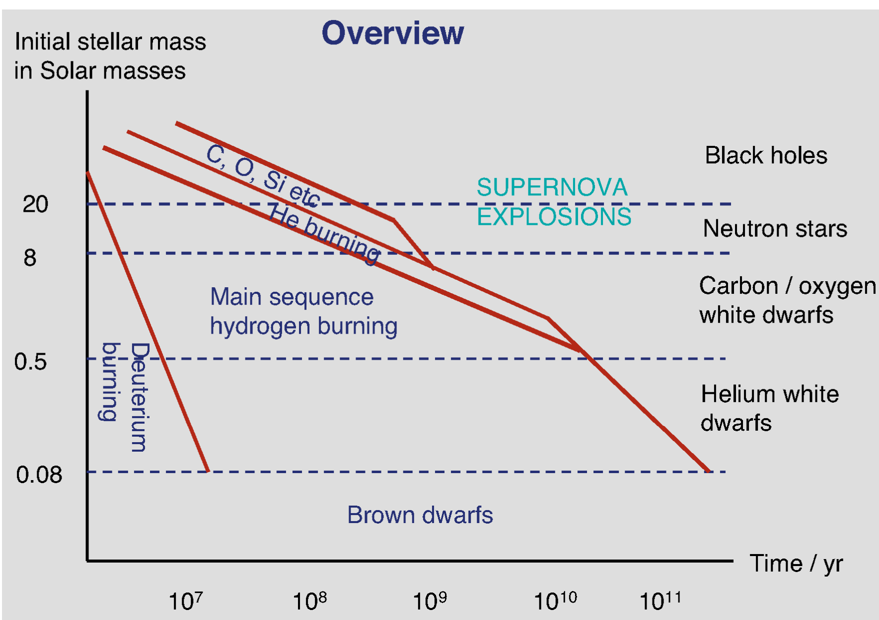
When hydrogen is exhausted near the centre, the star is left with a core consisting of helium and a small amount of heavy elements. Initially the temperature of the core is far below the 10^8 K required for helium ignition, and hence there is no nuclear energy generation in the core. Although there may still be some release of energy due to gravitational contraction, the luminosity in the core is generally very low, and the core is almost isothermal.

Surrounding the core is a region containing hydrogen where the temperature is still high enough for hydrogen burning to proceed. This region, which is known as a *hydrogen shell source*, provides the energy from which the luminosity of the star is derived. As the hydrogen is converted into helium in the shell source, the mass of the inert helium core increases. This leads to contraction of the core. As usual the contraction releases gravitational potential energy, part of which goes towards increasing the thermal energy of the core. As long as the core is not degenerate, the increase in thermal energy leads to an increase in temperature, up to the point where the temperature of the core is sufficiently high for helium burning to begin. Very roughly the process is then repeated: the star burns helium in the core (while still maintaining a hydrogen shell source) until helium is exhausted; the star then has a contracting core consisting of ^{12}C and ^{16}O, surrounded by a helium shell source and a hydrogen shell source; the contraction of the core may proceed up to the point where the temperature is high enough for carbon ignition; and so on.

This rough sketch ignores a large amount of fascinating detail. Particularly important is the response of the observable properties of the star to the changes in the core: when the region inside a burning shell contracts, the region outside the shell expands. This response is behind the dominant observational signature of the post-main-sequence evolution, which is rapid expansion of the envelope to form a *red giant* star.

This phenomenon is undoubtedly confirmed by computations, but, despite of many efforts, has no simple and fully accepted explanation in simple physical terms. One good plausibility argument is the following. Suppose the core contraction at the end of hydrogen burning occurs on a time scale shorter than the Kelvin-Helmholtz time scale of the whole star. From energy conservation, we have the sum of gravitational and internal energy $\Omega + U = const$ during the core-contraction phase (very small energy loss to the outer space). From the virial theorem, we have $\Omega + 2U = const$ (equation 4.19 of Lecture 4). But this is only possible when Ω and U are conserved separately. The contraction of the stellar core makes the gravitational binding energy Ω more negative; this change has to be compensated by the expansion of the envelope.

The subsequent evolution of the star depends crucially on its mass, as illustrated by the following diagram:



9.3. White dwarfs

For a star with contracting core consisting of ^{12}C and ^{16}O, surrounded by a helium and hydrogen shell source, one may now expect a repetition of the story, the core contraction leading to sufficiently high temperatures for carbon burning to set in. However, stars of mass smaller than about $8M_\odot$ never get that far. The carbon-oxygen core becomes degenerate, and the pressure of the degenerate electrons prevents the core from further contraction before the temperature reaches the values required for carbon ignition.

The subsequent evolution is complex, and not fully understood. Numerical computations indicate that a thermal instability develops in the helium shell source, causing thermal pulses where the star alternates between having a hydrogen and a helium shell source. At the same time the luminosity of the star increases greatly, as does its radius. Possibly as a result of the increase in radius and luminosity, the thermal pulses, or instabilities in the outer layers of the star, the star begins to loose mass at a fairly rapid rate. This process has been called a *"superwind"* (however, the fact that it has been given a name does not mean that the underlying physical mechanism is understood). The result appears to be that the star eventually looses essentially all the material outside the degenerate carbon-oxygen core. The core is initially extremely hot and quite luminous, despite its small size. It illuminates the material which has been lost, and which for a few thousand years forms a fairly-well defined shell around the star, and causes it to shine as a *planetary nebulae*.

Planetary nebulae

Subsequently the material is dispersed in the interstellar medium; the degenerate core continues to shine through loss of its thermal energy. It cools gradually, reaching an effective temperature of about 4000 K in about 10^{10} years. These objects are called *white dwarfs*. Their masses are typically between $0.5M_\odot$ and $1.4M_\odot$.

As a white dwarf cools, the pressure generated by the thermal motion of the ions will become less important, and eventually a pressure due to degenerate electrons will provide the bulk of the pressure needed to support the star.

We now assume for a while that the star is supported by the pressure of a gas of non-relativistic degenerate electrons (equation 9.13). The pressure and density profiles in such a star are described by a polytropic model ($P=K\rho^\gamma$, equation 5.1 of Lecture 5) with $\gamma=5/3$ and polytropic index $n=1/(\gamma-1)=3/2$ (equation 5.4). Using equation (5.14) of Lecture 5, we obtain immediately the mass-radius relation

$$R \propto M^{-1/3}, \qquad (9.16)$$

$R \propto M^{-1/3}$,$R \propto M^{-1/3}$,

which tells us that more massive the white dwarf is, smaller its radius. The constant of proportionality in this relation depends on the composition; for a star with $X=0$,

$$R \simeq \frac{R_\odot}{74}\left(\frac{M_\odot}{M}\right)^{1/3}.$$

$$R \simeq \frac{R_\odot}{74}\left(\frac{M_\odot}{M}\right)^{1/3} R \simeq \frac{R_\odot}{74}\left(\frac{M_\odot}{M}\right)^{1/3}.$$

(9.17)

In many white dwarfs the electron gas is relativistic in the central part of the star, while it is non-relativistic further out. Indeed, the degenerate electrons become relativistic when Fermi momentum p_F is large compared with $m_e c$, i.e. when the number density of the electrons (equation 9.7) is large compared with $(m_e c/h)^3$.

If we now consider an opposite limit when the entire white dwarf is filled with extremely relativistic degenerate electrons, the equation of state (9.13) shall be replaced with (9.14), which corresponds to a polytrope with $\gamma=4/3$, $n=3$. For polytropic index 3, the equation (5.14) predicts a unique value of mass M, which does not depend on radius R, being only governed by the constant K in the polytropic equation of state. This mass is known as Chandrasekhar mass M_{Ch}. The constant K in the equation of state depends upon the composition; for a white dwarf with $X=0$, we obtain

$$M_{Ch} = 1.44 M_\odot.$$

$$M_{Ch} = 1.44 M_\odot M_{Ch} = 1.44 M_\odot.$$

(9.18)

According to our discussion earlier in this Lecture, an extremely relativistic system is marginally stable. It follows that the Chandrasekhar mass is the maximum mass possible for a white dwarf star. When the mass reaches M_{Ch}, the star collapses and new physics must be sought to explain what happens next. For the moment, the only firm conclusion we draw is that a degenerate electron gas cannot support a star with mass larger than the Chandrasekhar mass.

9.4. Supernova explosion.

Stars with initial mass greater than about $8M_\odot$ are expected to evolve through all the stages of nuclear burning. The process begins with hydrojen burning at about 2×10^7 K and proceeds at successfully high temperatures through helium, carbon, neon, oxygen and silicon burning. Silicon burning at about 3×10^9 K leads to a star with a central core of iron surrounded by concentric shells containing silicon, oxygen, neon, carbon, helium and hydrogen. Because energy cannot be released by the thermonuclear fusion of iron (in normal circumstances the most stable form of nuclear matter consists of nuclei near ^{56}Fe in the periodic table), the central core contracts. Initially, this contraction can be controlled by the pressure of the dense gas of degenerate electrons in the core. But as silicon burning in the surrounding shell deposits more iron onto the central core, the degenerate electrons in the core become increasingly relativistic. When the core mass reaches the Chandrasekhar limit of about $1.4M_\odot$, the electrons become ultra-relativistic and they are no longer able to support the core. At this stage the stellar core is on the brink of a catastrophe. What follows is an uncontrolled collapse of the stellar core.

To understand the onset of the collapse, we note that when a body contracts under gravity, gravitational energy is converted into internal energy. If this leads to the activation of exothermic nuclear fusion, the internal kinetic energy increases, the pressure rises and the contraction is opposed. The opposite happens if an energy-absorbing process is activated: kinetic energy is absorbed, the effectiveness of the pressure is diminished and gravitational contraction turns into gravitational collapse.

There are two energy-absorbing processes which could drive the iron core of a star into an uncontrollable collapse. They are the photodisintegration of atomic nuclei and the capture of electrons via inverse beta decay. During photodisintegration, in reactions like

$$\gamma + {}^{56}\mathrm{Fe} \rightarrow 13\,{}^{4}\mathrm{He} + 4n,$$

$$\gamma + {}^{56}\mathrm{Fe} \rightarrow 13\,{}^{4}\mathrm{He} + 4n,$$

$$\gamma + {}^{56}\mathrm{Fe} \rightarrow 13\,{}^{4}\mathrm{He} + 4n,$$

kinetic energy is used to unbind atomic nuclei; and during inverse beta decay

$$e^- + p \rightarrow n + \nu_e,$$

$$e^- + p \rightarrow n + \nu_e, e^- + p \rightarrow n + \nu_e,$$

or in reactions like

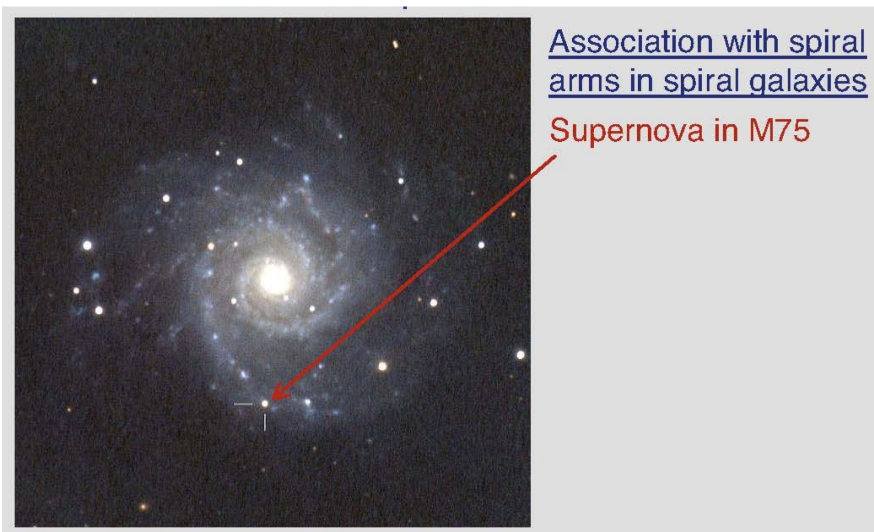
$$e^- + {}^{56}\mathrm{Fe} \rightarrow {}^{56}\mathrm{Mn} + \nu_e$$

$$e^- + {}^{56}\mathrm{Fe} \rightarrow {}^{56}\mathrm{Mn} + \nu_e$$

$$e^- + {}^{56}\mathrm{Fe} \rightarrow {}^{56}\mathrm{Mn} + \nu_e$$

(the conversion of protons to neutrons is often called *neutronization*) kinetic energy of degenerate electrons is converted into the kinetic energy of electron neutrinos which escape from the core. These energy-absorbing processes are so effective that the collapse of the stellar core is almost unopposed. Indeed, the core can collapse almost freely under gravity, on a free-fall time scale (equation 1.3 of Lecture 1) which is remarkably short, of the order of 1 millisecond.

The collapse is rapid and almost unopposed until a density comparable to the density of nuclear matter is reached. The nuclear forces (and neutron degeneracy) are expected to resist further compression and bring the collapse to a halt. The core is expected to rebound strongly and set up a shock wave that travels through the material that is falling towards the center. Theoretical calculations suggest that this shock wave may be able to reverse the inward fall of stellar material surrounding the core and produce an outward expulsion, a *supernova*.



Supernovae are very energetic explosions: the observed kinetic energy of the debris is typically 10^{44} J and the optical energy output, during the year following the explosion, is of the order of 10^{42} J.

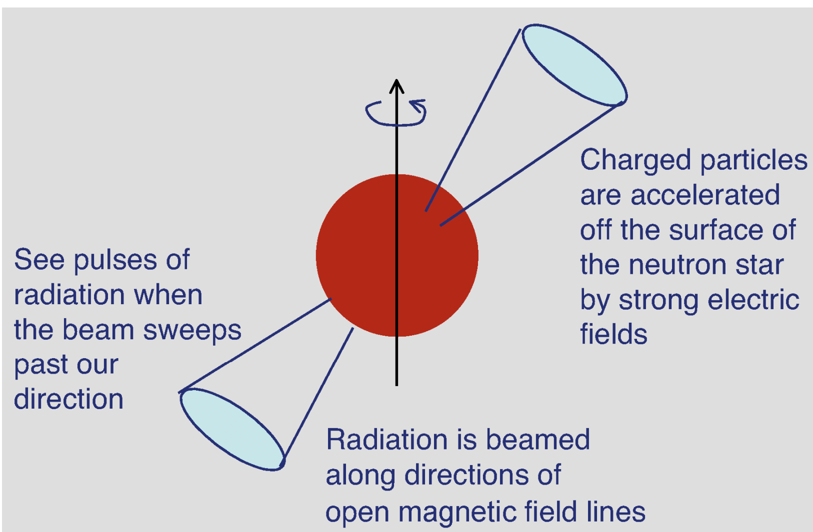
The mixture of products of thermonuclear reactions accumulated around the core is ejected into the interstellar medium and hence enriches is by heavy elements.

The collapse of the iron core of a massive star is the most likely cause of a so-called Type II supernova (most of Type I supernovae are thought to arise from a thermonuclear detonation of a carbon-oxygen white dwarf which can increase its mass by drawing mass from a nearby companion star). The collapse is expected to leave a core residue, either a *neutron star* or an overweight neutron star that collapses to form a *black hole*.

9.5. Neutron stars.

A neutron star is born as a hot residue of the collapsed core of a massive star. The typical internal temperature is initially between 10^{11} K and 10^{12} K. It rapidly cools by neutrino emission and is expected to reach a temperature of the order of 10^9 K in a day and 10^8 K in a 100 years. These are high temperatures according to terrestrial and solar standards, but they are low when compared to the standards set by the high densities in the matter inside a neutron star. The electrons, photons and above all the neutrons, which appear to be the dominant constituent of neutron stars, are degenerate and occupy the lowest possible states consistent with the Pauli's exclusion principle. The characteristic size of a neutron star is about 17 km, which is about 2000 times smaller than the typical size of a white dwarf given by equation (9.17).

Observationally, neutron stars have been detected in the form of the *pulsars*, which emit pulses at very regular intervals, with periods between a few milliseconds and a few seconds. These are most often observed in radio emission. The interpretation of the observations is that the pulses originate from a rotating neutron star, which is predominantly radiating in specific directions; a pulse is observed when the beam of radiation sweeps past the observer.



To a first approximation, neutrons play the same supporting role in a neutron star as electrons in a white dwarf. They can also fail to support in similar ways. Just as degenerate electrons are unable to support a white dwarf with a mass above a critical limit, the Chandrasekhar limit, degenerate neutrons are unable to support a neutron star with a mass above a certain value.

The physics underlying the Chandrasekhar limit is clear-cut. As the mass of the white dwarf approaches the limit, the central density increases and the degenerate electrons become increasingly relativistic. At the Chandrasekhar limit, the electrons are ultra-relativistic, the density approaches "infinity" and the star collapses. A similar phenomenon involving neutrons is expected in a neutron star, but there are a number of important differences. First, the interactions between neutrons are very important at the high densities found in a neutron star. Second, the gravitational fields are very strong and Einstein's theory, not Newton's, should be used to describe the equilibrium of a neutron star under gravity. However, these important differences do not alter the fundamental result that there is a maximum mass for a neutron star. Their main effect is to make the calculation of this maximum mass very difficult.

The first calculation of this kind was by Oppenheimer and Volkoff in 1939. They found that the maximum mass of a star composed of non-interacting neutrons is $0.7M_\odot$. Modern estimates range from approximately $1.5M_\odot$ to $3M_\odot$. The uncertainty in the value reflects the fact that the equations of state for extremely dense matter are not well-known.

9.6. Black holes.

In stars with initial mass bigger than about $20M_\odot$, the collapsing core is too massive to end its life as a neutron star. As the collapse proceeds, the gravitational field becomes stronger and stronger, and the internal pressure becomes larger and larger. But the source of the gravitational field in general relativity is the energy density and the pressure. Hence the increase in pressure accelerates the final stages of collapse. According to general relativity, the star enters a region of space-time called a black hole; it is more accurate to describe a black hole in terms of a distortion of the unified concept of space-time. In general relativity, gravity is not a force, but a distortion of the geometrical properties of space-time due to the presence of matter and radiation. The Sun only produces a slight "dent" in space-time, but a collapsed core of a massive star can produce a "hole". Nothing can escape from this hole because there are no outward paths in this distorted region of space-time; every path is towards the center of the hole. It is a hole of no return.

The most important property of a black hole is the existence of an *event horizon* at radius

$$R_S = \frac{2GM}{c^2}, \qquad (9.19)$$

$$R_S = \frac{2GM}{c^2} R_S = \frac{2GM}{c^2},$$

Known as *Schwarzchild radius*. For a collapsed mass equal to $10M_\odot$ the Schwarzchild radius is 30 km. The Schwarzchild radius marks the boundary of the one-way surface of the black hole. This surface is not made of anything. It encloses an unobservable region of space in which all motion is towards the center. No matter, radiation, or information can propagate outwards through this surface. A black hole is formed when the radius of a collapsing star reaches the Schwarzchild radius.

Any method for detecting a black hole depends on observing the effects of its intense gravitational field. Observations of some compact X-ray sources indicate the presence of intense gravitational fields due to compact objects which are too massive to be neutron stars. These objects, by default, are thought to be black holes.

Gravity is the driving force for stellar evolution. It leads to the formation of a star and to temperatures which make thermonuclear fusion possible. The energy released by fusion only serves to delay the gravitational contraction of the matter inside the star. The endpoint may be a white dwarf or a neutron star, stars in which cold matter resists the force of gravity. An alternative endpoint is a black hole in which gravity is completely triumphant. The outcome is neat and tidy - nothing is left of the collapsed matter apart from an intense gravitational field.