Lecture 2 Basic observational data

We cannot perform experiments on stars, or take surface samples from them. Thus we are restricted to studying them through the electromagnetic radiation they emit, or possibly through the effect of their gravitational field. A further restriction is that almost all stars can be observed only as points of light. The most obvious exception is the Sun, where very detailed observations are possible. However, the Sun is the only star, caught at a particular moment of its evolution. It has been possible to measure the diameter directly for some stars, and in few cases also to observe very large-scale features on the stellar surface, although the interpretation of these observations is somewhat questionable. In all other cases we have determinations only of the position of the star in the sky, and of the properties of the emitted radiation, integrated over the surface of the star. As we shall see, even these limited data allow us to learn a great deal about the stars, and hence to test computations of stellar evolution.

2.1. Stellar positions and distances

Stellar positions have been measured since antiquity. The most basic quantity is the *angular distance* between two stars, i.e. the angle between the lines-of-sight to the stars.

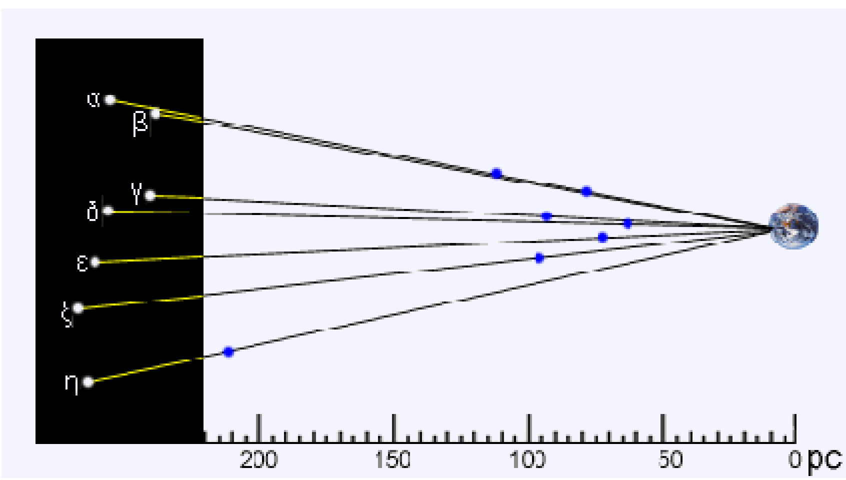

Figure 2.1. Stars in Ursa Major are located very far from each other, and do not constitute any connected system. Brightest star in a constellation is usually designated as α, then β and so on.

The angular distances are traditionally measured in degrees (°) or its subdivisions arcminutes (′) or acrseconds (″),

defined by

$$1^\circ = 60' = 3600''.$$

$$1^\circ = 60' = 3600'' 1^\circ = 60' = 3600''. \qquad (2.1)$$

Note also that 1 radian = 206265″. Under the best conditions an optical telescope on the surface of the Earth can separate two stars that are at distance of about 0.3″.

From the point of view of investigating stellar structure and evolution, the apparent positions of stars are in themselves of little interest. However, measurements of the change in apparent position as the Earth moves in its orbit around the Sun provide our only direct determinations of distances to stars other than the Sun.

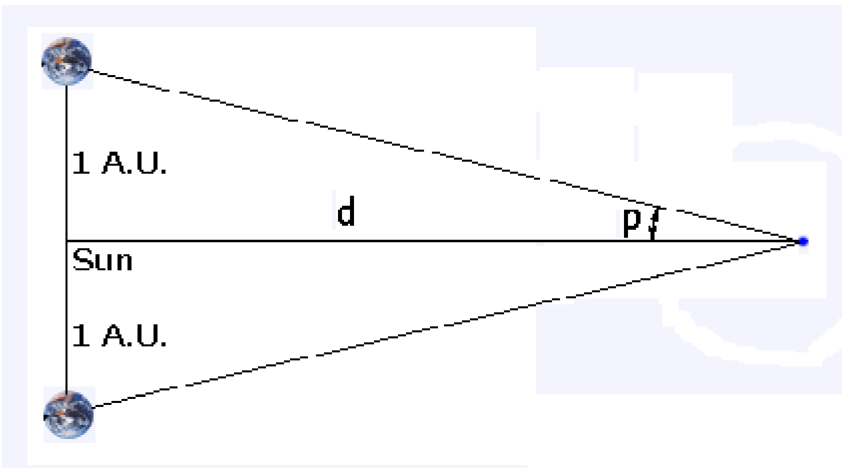

Figure 2.2. Parallax p of a star at a distance d from the Earth.

The change in direction to the star, as measured relative to very distant stars, as the Earth moves from a point at its orbit to the opposite point, is defined to be $2p$, where p is the *parallax* of the star. Hence p is the angle subtended by the radius of the Earth's orbit (1 astronomical unit, or A.U.) as seen from the star, so that

$$\tan p = \frac{1\ \text{A.U.}}{d},$$

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(2.2)

where d is the distance to the star, and $1\text{A.U.} = 1.496\times10^{11}$ m is the radius of the Earth orbit. Since p is a very small angle, equation (2.2) gives

$$d \approx \frac{1\ \text{A.U.}}{p\ (\text{radian})} = \frac{1\ \text{pc}}{p''},$$

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$$d \approx \frac{1\ \text{A.U.}}{p\ (\text{radian})} = \frac{1\ \text{pc}}{p''},$$

(2.3)

where p'' is the parallax measured in arcsec, and we have introduced the distance measure *parsec* (or pc), where 1 pc = 206265 A.U. = 3.086×10^{16} m.

The closest star other than the Sun has a parallax of 0.76″, and hence a distance of 1.32 pc. The best terrestrial observations yield parallaxes with a precision of about 0.01″. This allows determination of distances of a few thousand stars in the solar neighbourhood. Much better measurements where provided by the satellite HIPPARCOS, which was launched by the European Space Agency (ESA) in 1989.

2.2. Stellar brightness

In early star catalogues stars were classified according to their *magnitude*, the brightest stars having magnitude 0 and the faintest stars visible to the naked eye having magnitude 6. This scheme for describing the brightness of stars has essentially been maintained, but has been made precise.

What is measured on the Earth is the apparent luminosity l of a star, i.e. the energy from the star that passes through a unit area (orthogonal to the direction to the star) in unit time. Hence the unit for l is J m^{-2} s^{-1}. It was found that this precisely defined quantity could be related to the loosely defined magnitude scale by defining the *apparent magnitude* m_{ap} of a star as

$$m_{app} = -2.5 \log l + K_1,$$

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(2.4)

where K_1 is a constant which is determined by specifying the magnitude of a given star, say. The reason for the "-" in the definition of m_{ap} is evidently that the magnitude of stars, according to the old definition, increases as the stars get fainter. Since the magnitude is defined only to within a constant, a more convenient form of equation (2.4) is

$$m_{app\,1} - m_{app\,2} = -2.5 \log\left(\frac{l_1}{l_2}\right),$$

$$m_{app\,1} - m_{app\,2} = -2.5 \log\left(\frac{l_1}{l_2}\right), m_{app\,1} - m_{app\,2} = -2.5 \log\left(\frac{l_1}{l_2}\right),$$

(2.5)

where l_1 and l_2 are the apparent luminosities of two stars, and $m_{app\ 1}$ and $m_{app\ 2}$ are the corresponding magnitudes.

For the purpose of comparing with stellar evolution calculations, a much more interesting quantity is the *absolute luminosity* L , i.e. the total amount of energy radiated by the star per unit time. If we assume that the radiation is emitted isotropically (equally in all directions), and that there is no absorption between the star and us, then

$$l = \frac{L}{4\pi d^2},$$

$$l = \frac{L}{4\pi d^2}, l = \frac{L}{4\pi d^2},$$

(2.6)

where d is the distance to the star. Corresponding to the apparent magnitude m_{app}, we introduce the absolute magnitude M_{abs}, by

$$M_{abs} = -2.5 \log L + K_2,$$

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$$M_{abs} = -2.5 \log L + K_2,$$

(2.7)

where K_2 is another constant. From equations (2.4), (2.6) and (2.7) then follows a relation between m_{app} and M_{abs}. It is conventional to choose the constant K_2 such that this relation has the form

$$m_{app} = M_{abs} + 5 \log d - 5,$$

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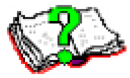
(2.8)

where d is measured in parsec. Thus M_{abs} corresponds to the apparent magnitude the star would have had if it had been at a distance of 10 pc. The Sun has an absolute magnitude of 4.62.

Exercise 2.1. Show that one can obtain a relation of the form (2.8) by suitable choice of K_2. If you need help, press 

Solution to Exercise 2.1.

We have

$$\begin{aligned} m_{app} &= -2.5 \log I + K_1 \\ &= -2.5 \log L + 2.5 \log\left(4\pi d^2\right) + K_1 \\ &= M_{abs} - K_2 + 2.5 \log(4\pi) + 5 \log d + K_1, \end{aligned}$$

$$\begin{aligned} m_{app} &= -2.5 \log I + K_1 \\ &= -2.5 \log L + 2.5 \log\left(4\pi d^2\right) + K_1 \\ &= M_{abs} - K_2 + 2.5 \log(4\pi) + 5 \log d + K_1, \end{aligned}$$

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which gives the required relation (2.8) when we choose

$$K_2 = K_1 + 2.5 \log(4\pi) + 5.$$

$$K_2 - K_1 + 2.5 \log(4\pi) + 5 K_2 = K_1 + 2.5 \log(4\pi) + 5.$$

It is obvious that the description of star’s brightness in terms of its magnitude is entirely conventional, and for the uninitiated somewhat awkward. However, it does reflect one important feature of observations of stellar brightness, namely that it is quite difficult to determine the amount of energy received from a given star, since it requires an absolute calibration of the measuring device. It is far easier to measure the ratio between the luminosities of two stars, and hence their magnitude difference. Once the zero-point of the magnitude scale has been established by arbitrarily assigning a given magnitude to a given star, one can then determine the magnitudes of other stars.

2.3. Colour indices and surface temperature

Different stars on the sky have different colours, which depend on their temperature; blue stars are hotter than red. We are thus interested in measuring not only the total amount of energy coming from the star in electromagnetic waves, but also the distribution of this energy with the wavelength.

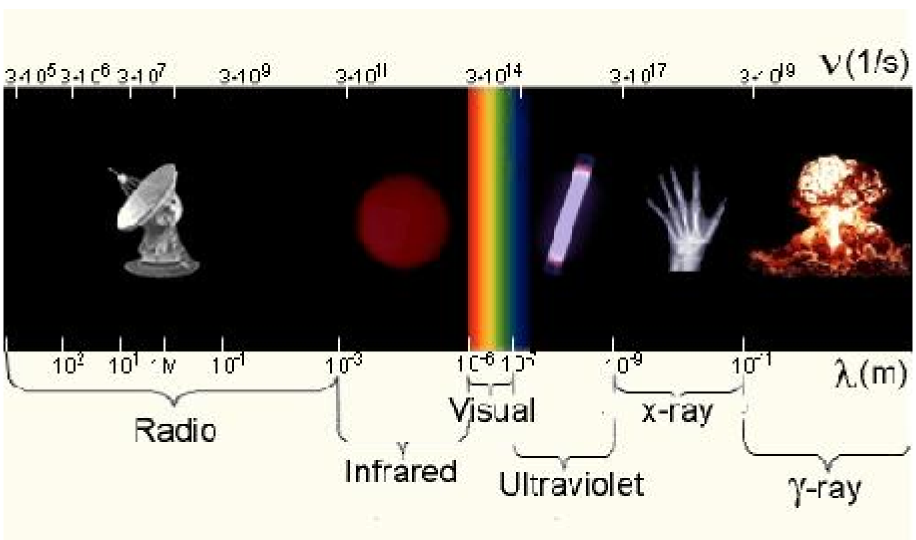

Figure 2.3. Scale of electromagnetic waves.

Some indication of the distribution of the stellar luminosity with wavelength can be obtained by observing the star through differently coloured filters. To enable comparisons between results obtained at different observatories, standard sets of filters are used. A commonly used system is the so-called UBV system. It uses three filters, which sensitivity ranges are roughly

Ultraviolet	(U)	300 – 400 nm
Blue	(B)	350 – 550 nm
Visual	(V)	480 – 650 nm

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In this system, U , B and V are used to denote the apparent magnitudes m_{app} measured with corresponding filters. Corresponding apparent luminosities are usually denoted as l_U, l_B, l_V and absolute magnitudes -- as M_U, M_B, M_V.

To characterize the distribution of energy with wavelength, one introduces the *colour indices* $U\text{-}B$ and $B\text{-}V$, so that, for example,

$$U - B = 2.5 \log\left(\frac{l_B}{l_U}\right) + K_U - K_B,$$

$$U - B = 2.5 \log\left(\frac{l_B}{l_U}\right) + K_U - K_B, U - B = 2.5 \log\left(\frac{l_B}{l_U}\right) + K_U - K_B,$$

(2.9)

where K_U and K_B are the constants in the definition of the U and B magnitudes. In the UBV system the constants are chosen such that $U - B = B - V = 0$ for a particular type of a star (the so-called A0 dwarf stars). For the Sun, the UBV apparent magnitudes and the colour indices are

$$U = -25.96,$$
$$B = -26.09, \quad U - B = 0.13 \qquad \text{(solar values)}$$
$$V = -26.74, \quad B - V = 0.65$$

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$$V = -26.74, \quad B - V = 0.65$$

Since the filters defining the UBV magnitudes let through light over fairly broad wavelength ranges, the magnitudes and colour indices can be measured even for a very faint stars. Furthermore, in the absence of interstellar absorption the colour indices are independent of the distance to the star, which is most often not known. Hence they can be used to characterize the intrinsic properties of a star.

Exercise 2.2. Show that the colour indices are independent on distance.

Solution to Exercise 2.2.

We have

$$l_B = \frac{L_B}{4\pi d^2}, \qquad l_U = \frac{L_U}{4\pi d^2},$$

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$$l_B = \frac{L_B}{4\pi d^2}, \qquad l_U = \frac{L_U}{4\pi d^2},$$

$$\frac{l_B}{l_U} = \frac{L_B}{L_U},$$

$$\frac{l_B}{l_U} = \frac{L_B}{L_U}, \frac{l_B}{l_U} = \frac{L_B}{L_U},$$

And hence $U - B$ is independent on distance d ; the same with other colour indices.

The colour index is predominantly determined by the surface temperature of the star. Hotter stars radiate more of the energy at short wavelengths; hence their U magnitude tends to be low (recall that the magnitude decreases as the luminosity increases), relative to their B magnitude, and so they have a low colour index $U - B$, relative to cooler stars; the same is true for the index $B - V$. To describe the relation between temperature and colour indices more precisely we assume, as a first rough approximation, that the star radiates as black body with a temperature T . Then the emission from the stellar surface is given by the Planck function, which specifies the *spectral density* of the apparent luminosity (i.e. the apparent luminosity per unit interval of the wavelength λ) as

$$l_\lambda = \frac{2\pi hc^2}{\lambda^5} \frac{1}{\exp\left(\frac{hc}{\lambda kT}\right) - 1} \left(\frac{R}{d}\right)^2;$$

$$l_\lambda = \frac{2\pi hc^2}{\lambda^5} \frac{1}{\exp\left(\frac{hc}{\lambda kT}\right) - 1} \left(\frac{R}{d}\right)^2 ; l_\lambda = \frac{2\pi hc^2}{\lambda^5} \frac{1}{\exp\left(\frac{hc}{\lambda kT}\right) - 1} \left(\frac{R}{d}\right)^2;$$

(2.10)

here h is Planck's constant, k is Boltzmann's constant, c is speed of light, and R is the radius of the star.

Equation (2.10) predicts that the wavelength λ_{max} at which stellar luminosity has its maximum value is inversely proportional to the surface temperature,

$$\lambda_{max} = \frac{2.8978 \times 10^6}{T},$$

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(2.11)

where λ is measured in nanometers (1nm=10^{-9}m) and T is in Kelvins (K).

Equation (10) also allows to evaluate total apparent luminosity of the star (the so-called *bolometric luminosity*).

Exercise 2.3. Show that total apparent luminosity of the star, integrated over wavelength, may be written as

$$I_{bol} = \int_0^\infty I_\lambda d\lambda = \sigma T^4 \left(\frac{R}{d}\right)^2,$$

(2.12)

where σ is some constant, which you are not expected to explicitly specify

(this constant σ is the so-called *Stefan-Boltzman constant*).

Solution to Exercise 2.3.

We introduce

$$x = \frac{hc}{\lambda kT}$$

as new independent variable, which gives

$$
\begin{aligned}
I_{bol} &= \int_0^{\infty} I_\lambda d\lambda \\
&= 2\pi hc^2 \left(\frac{kT}{hc}\right)^4 \left(\frac{R}{d}\right)^2 \int_0^{\infty} \frac{x^3}{e^x - 1} dx \\
&= \sigma T^4 \left(\frac{R}{d}\right)^2
\end{aligned}
$$

where

$$\sigma = 2\pi hc^2 \left(\frac{k}{hc}\right)^4 \int_0^{\infty} \frac{x^3}{e^x - 1} dx.$$

Even though real stars do not radiate as black bodies, it is convenient to describe their total energy emission by means of equation (2.12), by defining the *effective temperature* T_{eff} of the star such that

$$L = 4\pi\sigma T_{eff}^4 R^2, \tag{2.13}$$

and

$$I_{bol} = \sigma T_{eff}^{4} \left(\frac{R}{d}\right)^2 ,$$

(2.14)

are exactly satisfied.

Equation (2.10) can also be used to relate the colour indices with stellar temperature. With known spectral properties of the UBV filters, equation (2.10) allows to calculate theoretical values of the $U - B$ and $B - V$ colour indices for any “trial” temperature T. Matching the theoretical colour indices with their observational values allows one to obtain an estimate of the surface temperature of the star, the so-called *colour temperature*. Since stars do not radiate like black bodies, the colour temperature is in general different from the effective temperature, although they are normally quite similar.

2.4. Colour-magnitude diagrams

Given measurements of the brightness and surface temperature of a group of stars, as determined by their magnitudes and colour indices, it is natural to plot these quantities against each other, to look for systematic correlations. This was first done independently by E. Hertzsprung and H. N. Russell, and these diagrams are collectively known as *Hertzsprung-Russell*, or HR, diagrams; the term colour-magnitude diagrams is also commonly used. An example of such a plot, for stars that are near enough to make possible a determination of their distance, and hence their absolute magnitude and luminosity, is illustrated by Figure 2.4.

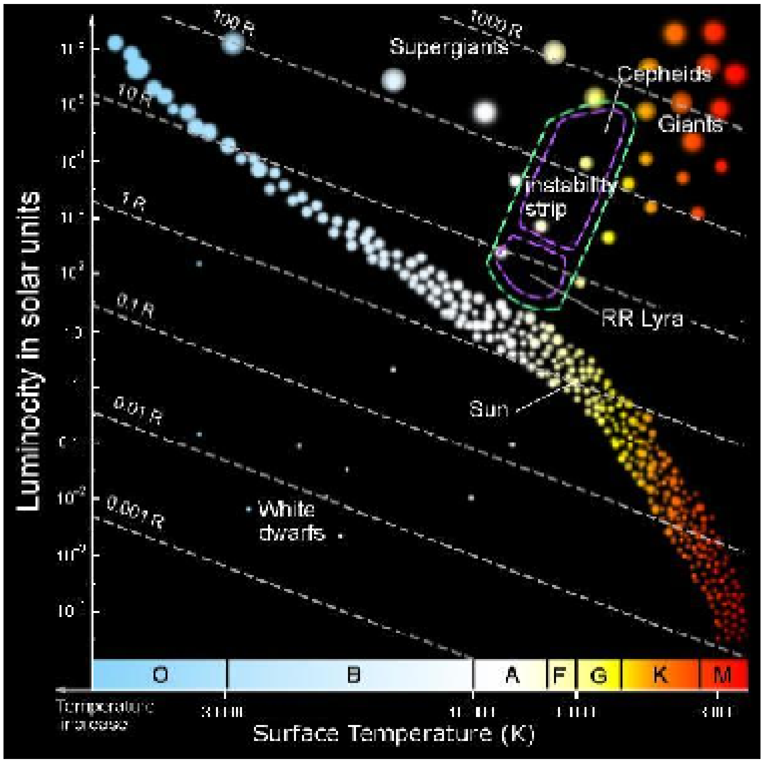

Figure 2.4. Colour-magnitude diagram.

It is obvious that the distribution of stars is far from random. Most of the stars are concentrated in the *main sequence*, a fairly narrow band of stars with steeply increasing luminosities with increasing temperature (it is an unfortunate historical accident that such diagrams are always plotted with the temperature increasing *to the left*). The *red giant* stars are located in the upper-right corner of the diagram. Below the main sequence, there is a small number of very faint and relatively hot stars, the white dwarfs. The appropriateness of this name follows immediately from equation (2.13): if hot stars are very faint, compared with main sequence stars of the same temperature, their radii must be very small. The understanding of distribution of stars in this diagram, and its relation to stellar evolution, is a major goal of these lectures.

Further exercises

Exercise 2.4. A star is at a distance of 1.2×10^{17} m, calculate its parallax. A second star has a parallax of 0.5, calculate its distance. This star appears 10 times as bright as the first star, what is the true ratio of brightness?

Solution to Exercise 2.4.

Parallax

$$p'' = \frac{1\,\mathrm{pc}}{d(\mathrm{pc})} = \frac{3.1\times10^{16}\,\mathrm{m}}{1.2\times10^{17}\,\mathrm{m}} = 0.258$$

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Distance

$$d(\mathrm{pc}) = \frac{1\,\mathrm{pc}}{p''} = 2\,\mathrm{pc} = 6.2\times10^{16}\,\mathrm{m}$$

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True ratio of brightness is

$$10\times\left(\frac{6.2}{12}\right)^2 = 2.67\,.$$

$$10\times\left(\frac{6.2}{12}\right)^2 = 2.67\; 10\times\left(\frac{6.2}{12}\right)^2 = 2.67\,.$$

Exercise 2.5. A star has apparent magnitude 7.62 and is at a distance of 100 parsecs from us.

a) What is its absolute magnitude?
b) What is its luminosity?
c) Its effective temperature is 5780K, what is its radius?

A second star has the same effective temperature but its radius is 100 times greater. Its apparent magnitude is 13.12. What is its distance from the observer in parsecs? What would its parallax be?

Solution to Exercise 2.5.

a)

$$
\begin{aligned}
M_{abs} &= m_{app} - 5 \log d(pc) + 5 \\
&= 7.62 - 10 + 5 = 2.62
\end{aligned}
$$

$$
\begin{aligned}
M_{abs} &= m_{app} - 5 \log d(pc) + 5 \\
&= 7.62 - 10 + 5 = 2.62
\end{aligned}
$$

$$
\begin{aligned}
M_{abs} &= m_{app} - 5 \log d(pc) + 5 \\
&= 7.62 - 10 + 5 = 2.62
\end{aligned}
$$

b) We have an equation

$$M_{abs} - M_{abs,\odot} = -2.5 \log \frac{L}{L_\odot} = 2.62 - 4.62 = -2.$$

$$M_{abs} - M_{abs,\odot} = -2.5 \log \frac{L}{L_\odot} = 2.62 - 4.62 = -2.00$$

$$M_{abs} - M_{abs,\odot} = -2.5 \log \frac{L}{L_\odot} = 2.62 - 4.62 = -2.00$$

from which

$$\log \frac{L}{L_\odot} = 0.8, \quad \frac{L}{L_\odot} = 10^{0.8} = 6.3,$$

$$\log \frac{L}{L_\odot} = 0.8, \quad \frac{L}{L_\odot} = 10^{0.8} = 6.3,$$

$$\log \frac{L}{L_\odot} = 0.8, \quad \frac{L}{L_\odot} = 10^{0.8} = 6.3,$$

$$L = 6.3 \times 3.86 \times 10^{26}\,\mathrm{J/s} = 2.43 \times 10^{27}\,\mathrm{J/s}$$

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$$L = 6.3 \times 3.86 \times 10^{26}\,\mathrm{J/s} = 2.43 \times 10^{27}\,\mathrm{J/s}$$

c)

$$R^2 = \frac{L}{4\pi\sigma T_{eff}^{\ 4}} = \frac{2.43 \times 10^{27}}{4\pi \times 5.67 \times 10^{-8} \times 5780^4}$$
$$= 3.06 \times 10^{18}(\mathrm{m}^2), \quad R = 1.75 \times 10^9\,\mathrm{m}$$

$$R^2 = \frac{L}{4\pi\sigma T_{eff}^{\ 4}} = \frac{2.43 \times 10^{27}}{4\pi \times 5.67 \times 10^{-8} \times 5780^4}$$
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$$= 3.06 \times 10^{18}(\mathrm{m}^2), \quad R = 1.75 \times 10^9\,\mathrm{m}$$

Second star:

$$L_2 = 100^2 L_1,$$

$$M_{abs,2} = M_{abs,1} - 2.5 \log \frac{L_2}{L_1} = 2.62 - 10 = -7.38$$

$$m_{app,2} = M_{abs,2} + 5 \log d_2 - 5 = 13.12,$$

$$5 \log d_2 = 13.12 + 5 + 7.38 = 25.5,$$

$$\log d_2 = 5.1, \quad d_2 = 10^{5.1} = 1.25 \times 10^5 (\text{pc}),$$

$$p = 8.0 \times 10^{-6} \text{arcsec}$$

Exercise 2.6. A stellar field was observed using a CCD detector twice, once in a red (R) filter and once through a blue (B) filter. The exposure time for the R filter was 20s and for the B filter 60s. One star was of Solar Type and known to have an apparent R magnitude of 12.1.

A second star was also measured in the same field. The total proton counts from each star through each filter were as follows:

	Solar-type star:	Other star:
R filter 20s:	23456	58919
B filter 60s:	20954	49405

Obtain the B and R magnitudes of the other star (B-R for the Sun is 1.17). Is this star redder or bluer than the Sun? 

Solution to Exercise 2.6.

$$m_{app} = -2.5 \log(\text{count}) + \text{const}$$

$$m_{app} = -2.5 \log(\text{count}) + \text{const} m_{app} = -2.5 \log(\text{count}) + \text{const}$$

In R filter:

$$m_{1R} - m_{2R} = -2.5 \log \frac{23456}{58919} = 1.000,$$

$$m_{2R} = m_{1R} - 1.0 = 12.1 - 1.0 = 11.1$$

$$m_{1R} - m_{2R} = -2.5 \log \frac{23456}{58919} = 1.000,$$

$$m_{2R} = m_{1R} - 1.0 = 12.1 - 1.0 = 11.1$$

$$m_{1R} - m_{2R} = -2.5 \log \frac{23456}{58919} = 1.000,$$

$$m_{2R} = m_{1R} - 1.0 = 12.1 - 1.0 = 11.1$$

In B filter:

$$m_{1B} - m_{2B} = -2.5 \log \frac{20954}{49405} = 0.932$$

$$m_{1B} - m_{2B} = -2.5 \log \frac{20954}{49405} = 0.932$$

$$m_{1B} - m_{2B} = -2.5 \log \frac{20954}{49405} = 0.932$$

For solar-type star,

$$B_1 - R_1 - 1.17$$

$$B_1 - R_1 = 1.17$$

$$B_1 - R_1 = 1.17$$

(same as for the Sun), hence

$$B_1 \equiv m_{1B} = 1.17 + 12.1 = 13.27$$

$$B_1 \equiv m_{1B} = 1.17 + 12.1 = 13.27$$

$$B_1 \equiv m_{1B} = 1.17 + 12.1 = 13.27$$

For other star,

$$B_2 \equiv m_{2B} = m_{1B} - 0.932 = 12.34,$$
$$B_2 - R_2 \equiv m_{2B} - m_{2R} = 12.34 - 11.1 = 1.24$$

$$B_2 \equiv m_{2B} = m_{1B} - 0.932 = 12.34,$$
$$B_2 - R_2 \equiv m_{2B} - m_{2R} = 12.34 - 11.1 = 1.24$$
$$B_2 \equiv m_{2B} = m_{1B} - 0.932 = 12.34,$$
$$B_2 - R_2 \equiv m_{2B} - m_{2R} = 12.34 - 11.1 = 1.24$$

For the second star, $B - R$ is bigger, hence it is redder.

Exercise 2.7. The stars Rigel and β Canis Majoris have the same effective temperature, 12 000K. Their absolute magnitudes are respectively -6.77 and -1.33.

a) Calculate their luminosities in solar units

b) Calculate the ratio of their radii.

Solution to Exercise 2.7.

a)

$$M_{abs} = 4.62 - 2.5 \log \frac{L}{L_\odot}$$

$$M_{abs} = 4.62 - 2.5 \log \frac{L}{L_\odot}$$

$$M_{abs} = 4.62 - 2.5 \log \frac{L}{L_\odot}$$

Reigel:

$$-6.77 = 4.62 - 2.5 \log \frac{L_1}{L_\odot},$$

$$\log \frac{L_1}{L_\odot} = \frac{11.39}{2.5} = 4.56, \quad \frac{L_1}{L_\odot} = 3.6 \times 10^4$$

$$-6.77 = 4.62 - 2.5 \log \frac{L_1}{L_\odot},$$

$$\log \frac{L_1}{L_\odot} = \frac{11.39}{2.5} = 4.56, \quad \frac{L_1}{L_\odot} = 3.6 \times 10^4$$

$$-6.77 = 4.62 - 2.5 \log \frac{L_1}{L_\odot},$$

$$\log \frac{L_1}{L_\odot} = \frac{11.39}{2.5} = 4.56, \quad \frac{L_1}{L_\odot} = 3.6 \times 10^4$$

β Canis Majoris:

$$-1.33 = 4.62 - 2.5 \log \frac{L_2}{L_\odot},$$

$$\log \frac{L_2}{L_\odot} = 2.38, \quad \frac{L_2}{L_\odot} = 240$$

$$-1.33 = 4.62 - 2.5 \log \frac{L_2}{L_\odot}, -1.33 = 4.62 - 2.5 \log \frac{L_2}{L_\odot},$$

$$\log \frac{L_2}{L_\odot} = 2.38, \quad \frac{L_2}{L_\odot} = 240 \log \frac{L_2}{L_\odot} = 2.38, \quad \frac{L_2}{L_\odot} = 240$$

b)

$$\frac{L_1}{L_2} = \left(\frac{R_1}{R_2}\right)^2,$$

$$\frac{3.6 \times 10^4}{240} = \left(\frac{R_1}{R_2}\right)^2, \quad \frac{R_1}{R_2} = 12.$$

$$\frac{L_1}{L_2} = \left(\frac{R_1}{R_2}\right)^2, \qquad \frac{L_1}{L_2} = \left(\frac{R_1}{R_2}\right)^2,$$

$$\frac{3.6 \times 10^4}{240} = \left(\frac{R_1}{R_2}\right)^2, \quad \frac{R_1}{R_2} = 12 \, \frac{3.6 \times 10^4}{240} = \left(\frac{R_1}{R_2}\right)^2, \quad \frac{R_1}{R_2} = 12.$$

Exercise 2.8. For yellow stars, the main sequence has a slope of 6 (i.e. log L=6log T + const). Show that their effective temperature is proportional to their radius.

Solution to Exercise 2.8.

We have

$$L \propto T^6$$

$L \propto T^6 L \propto T^6$

And also

$$L \propto R^2 T^4,$$

$L \propto R^2 T^4, L \propto R^2 T^4,$

hence

$$T^6 \propto R^2 T^4, \quad T^2 \propto R^2, \quad T \propto R\,.$$

$T^6 \propto R^2 T^4, \quad T^2 \propto R^2, \quad T \propto R\,.$
$T^6 \propto R^2 T^4, \quad T^2 \propto R^2, \quad T \propto R\,.$