The Interpretations of Quantum Mechanics.

1 Introduction.

Quantum Mechanics is an extremely successful branch of science. It has enabled us to explain a wide range of phenomena for which classical mechanics fails to find any explanation at all - the structure of atoms and the details of atomic spectra, radioactivity, chemical bonding and the Ramsauer-Townsend effect. Elaborations of the fundamental theory have led to satisfactory explanations of nuclear structure and reactions, the electrical and thermal properties of solids, superconductivity, the creation and annihilation of elementary particles, the production of anti-matter, Bose-Einstein condensation, the stability of white dwarfs and neutron stars, and much else. It has also made possible major practical developments such as the electron microscope, the laser and the computer chip. Exceedingly delicate experiments have confirmed subtle quantum effects to an astonishing degree of accuracy. It has never been shown to contradict the results of 50 years of experimenting.

If all that is asked of a theory is that it should provide correct predictions of the results of experiments, quantum mechanics works perfectly, and, to paraphrase John Bell, "Ordinary quantum mechanics (as far as we know) is just fine for all practical purposes (FAPP)" .

However, the basic conceptual foundations of quantum mechanics, when closely examined, can lead, depending on their interpretation, to puzzling paradoxes and strange, counterintuitive, and to some physicists unacceptable, features. The problems in quantum theory are not with its technical aspects, but in its interpretation. Up until about 1984, the standard interpretation of quantum mechanics, due to Bohr, was generally accepted; though many physicists never really thought about the questions it raised, it remained a problem by those few who did. But in the past 20 years a renewed interest has arisen, stimulated in part by advances in technology which have enabled experimental studies which were hitherto only possible in the imagination (thought experiments) to be actually carried out. For example, experiments can be made with single photons. The results have thrown fresh light on the apparent paradoxes at the heart of quantum mechanics, without always having solved them; indeed, they have also revealed deeper and perhaps even more bizarre manifestations of the theory.

2 Wave-particle duality and indeterminacy

Experiments have shown that material particles such as electrons display wave-like properties like interference and diffraction, while electromagnetic waves can in turn exhibit particle-like behaviour, as in the photoelectric effect. Now particle and wave aspects combined in the same system seem to be mutually exclusive and incompatible; classically, a particle is an object with a precise position in space, whereas a wave is a disturbance that is spread out in space and time, and does not have a sharply defined location. The fact that there are two seemingly contradictory descriptions leads to some intriguing conclusions about nature.

If a beam of polarised light is incident upon a piece of polarising material with the planes of polarisation of light and polariser at 45°, the transmitted light will have half the intensity of the incident beam. Classically, polarisation of light is described by the orientation of the electric field vector, perpendicular to the direction of propagation of the light wave. Quantum mechanically, the polarisation states of photons are described as linear superpositions of two basis states (eigenstates) corresponding to horizontal and vertical polarisation respectively, horizontal and vertical referring to the direction of the axis of a polariser that transmits those photons with probability 1. According to the photon picture, 50% of photons will get through, 50% blocked.

This is because the state vector corresponding to say, horizontal polarisation is of the form

$$|\rightarrow>= \frac{1}{\sqrt{2}}(|\ 45^\circ > + \mid -45^\circ >)$$

But what if an experiment is performed such that only one photon passes through the polariser? A photon cannot be split up: any given photon must be either passed or blocked. Quantum mechanics explains this by saying that any given photon has a 50% probability of getting through but it is impossible to predict in advance whether a given one will be passed or blocked. Thus two identical experiments could produce different results; whereas, according to classical mechanics, identical results should always be obtained. If an experiment is repeated many times, with identical, identically prepared apparatus, then according to quantum mechanics only the statistical frequency (in the above case 50%) of obtaining a particular result can be predicted. We cannot ever know what the fate of a particular photon in an ensemble will be.

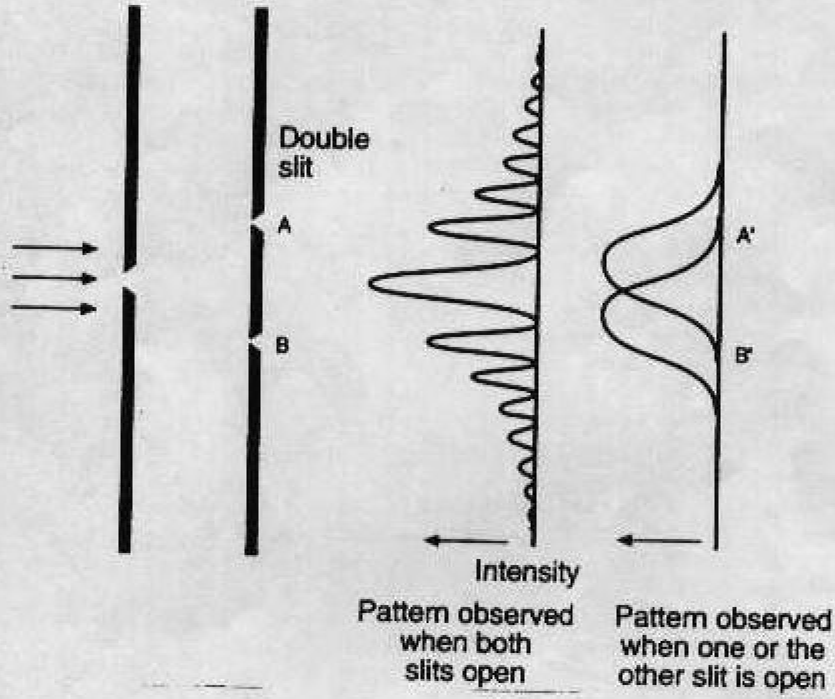
This example illustrates the probabilistic nature of quantum mechanics, and its lack of determinism. When a single measurement of an observable is made, the result of the measurement will be one of its eigenvalues; but it is absolutely impossible to predict which particular one will be found (unless the system is known to be already in an eigenstate). All that can be predicted is the probability of finding upon measurement any given eigenvalue. This implies that in the quantum world there is an absolute randomness that does not exist in classical physics, where randomness is only apparent.

Heisenberg's Uncertainty Principle puts the element of uncertainty inherent in the micro world on a quantitative basis. In classical mechanics, uncertainty and fuzziness exist but are caused by our incomplete knowledge or perception of systems because of, for example, their complexity. But in quantum mechanics, uncertainty is intrinsic to nature and not merely a result of a lack of precision in experimental technique or the difficulty of obtaining precise descriptions of complicated physical systems. According to the position-momentum uncertainty relation, the more precisely we try to locate a particle, the more we are forced to forgo information about its momentum: the act of pinning down its exact location introduces an increased indeterminacy in its momentum. The more precisely we try to measure its motion, the less the particle becomes localised in space. Heisenberg's well-known imaginary experiment of the "gamma-ray microscope" provides an example of this and illustrates how both the observer and the apparatus used are inextricably linked to and involved in the process of measurement.

The question then arises as to where the system ends and the measuring apparatus begins (the shifty-split of Bell) and indeed, what constitutes a "measurement". This problem, which is by no means a straightforward one, will be discussed later.

The Uncertainty Principle $\Delta x \Delta p \geq \hbar/2$ also illustrates the lack of determinism in quantum mechanics. According to classical mechanics, all variables can be specified simultaneously with arbitrary precision, so that given a precise specification of the initial state of a system, all future states may be predicted precisely; the system is completely determinate. (Even when Chaotic motion sets in, it is still in principle deterministic). In quantum mechanics the Uncertainty Principle renders both the precise specification of the initial state and the precise prediction of its subsequent behaviour impossible at all times.

2.1 The Double-Slit Experiment.



2.1 The Double-Slit Experiment.

Richard Feynman has been quoted as remarking that all the mysteries of quantum mechanics are encapsulated in the double-slit interference experiment. This well-known experiment was first carried out, with visible light, by Thomas Young in the late eighteemth century, hence the name "Young's Slits." When the experiment is performed with both slits open, the familiar interference pattern is observed, clearly illustrating the wave nature of light. The pattern seen with both slits open is not the same as would be obtained by closing each slit in turn and adding the results.

Modern experimental techniques have enabled the experiment to be carried out with a single photon at a time passing through the apparatus. If the photon is a true particle, it must pass through one or other of the slits. But in fact, it passes through both slits at the same time like a wave and interferes with itself. When it arrives at the recording screen however it produces a single dot. After many identical photons have passed through the apparatus, the pattern of dots on the screeen builds up into the standard interference pattern. Each photon appears to start out as a particle, travel like a wave, but arrive as a particle. If one slit is closed, the photons must travel through the slit that is open and a different pattern is seen. A photon places itself at a different place on the screen than it would have occupied if both slits had been open. The photon seems in a strange way to have some kind of "knowledge" about whether one or both slits are open, and selects a path accordingly. If the experiment is set up such that slit A is open (or closed) when the photon leaves the source, but is closed (opened) before it reaches slit B, it will choose the appropriate path. It seems as if the photons are in touch with the entire apparatus, of more of the world than their own immediate locality. This is an example of a fundamental property of quantum mechanics - non-locality.

What happens if we introduce detectors to try to determine the trajectories followed by the photons, to find out which slit a photon passes through? We find that each photon behaves as a particle, passing through one slit or the other. The act of observing each photon wave makes it collapse and behave like a particle as it passes through the slit. At the same time, the interference pattern disappears and the charateristic pattern produced by a particle at a single slit is seen on the screen. The photons thus behave differently depending on whether or not you are looking at them. The human observer is thus an integral part of the experiment; set it up to observe particles, and you find them; set it up to observe waves and you find interference. Variations on this experiment have been performed not just with single photons, but also with single electrons (an experiment chosen by readers of Physics World in September 2002 to be the most beautiful experiment in physics). The results obtained were the same. Interference patterns have also been observed in similar experiments using atoms as projectiles, illustrating the wave nature of whole atoms, and showing that like electrons and photons, they can also be in two places at once. Other experiments in which pairs of trapped atoms play the role of the slits (and hence behave like particles) and produce interference of light waves have also been successfully carried out. These experimental studies involving atoms provide an excellent example of wave-particle duality.

In the double slit experiment the experimenter could delay the choice about whether to observe interference or not by deciding to switch on or switch off the detectors after the photons have passed through the slits; the results would still be the same. This would mean that the way the light had behaved at the slits was determined only **after** it had passed through them. Such delayed-choice experiments have actually been carried out with beam-splitters (see below), and the results confirm quantum mechanics. I told you it was weird.

The behaviour of electrons or photons passing through a double-slit apparatus, and the role played by monitors in an attempt to detect them may be put on a quantitative basis as follows:

Let $\mid A >$ and $\mid B >$ represent state vectors for a particle in state $\mid \Psi >$ to pass through slit A and slit B respectively. Since the particle must pass either through A or B, and there is no other alternative, $\mid \Psi >$ is a linear superposition

$$\mid \Psi >=\mid A >< A \mid \Psi > + \mid B >< B \mid \Psi >$$

The probability amplitude for finding the particle at position $\mathbf{r}$ is

$$< \mathbf{r} \mid \Psi >=< \mathbf{r} \mid A >< A \mid \Psi > + < \mathbf{r} \mid B >< B \mid \Psi >$$

Then the probability $|< \mathbf{r} \mid \Psi >|^2$ of finding the particle at position $\mathbf{r}$ is

$$|< \mathbf{r} \mid A >< A \mid \Psi > |^2 + |< \mathbf{r} \mid B >< B \mid \Psi > |^2 + 2Re[< \mathbf{r} \mid A >< A \mid \Psi >< \mathbf{r} \mid B >< B \mid \Psi >]$$

The first and second terms are the probabilities of finding the particle at position $\mathbf{r}$ and passing through slit A and B respectively; the third is an interference term. The two pathways interfere.

Figure 2.

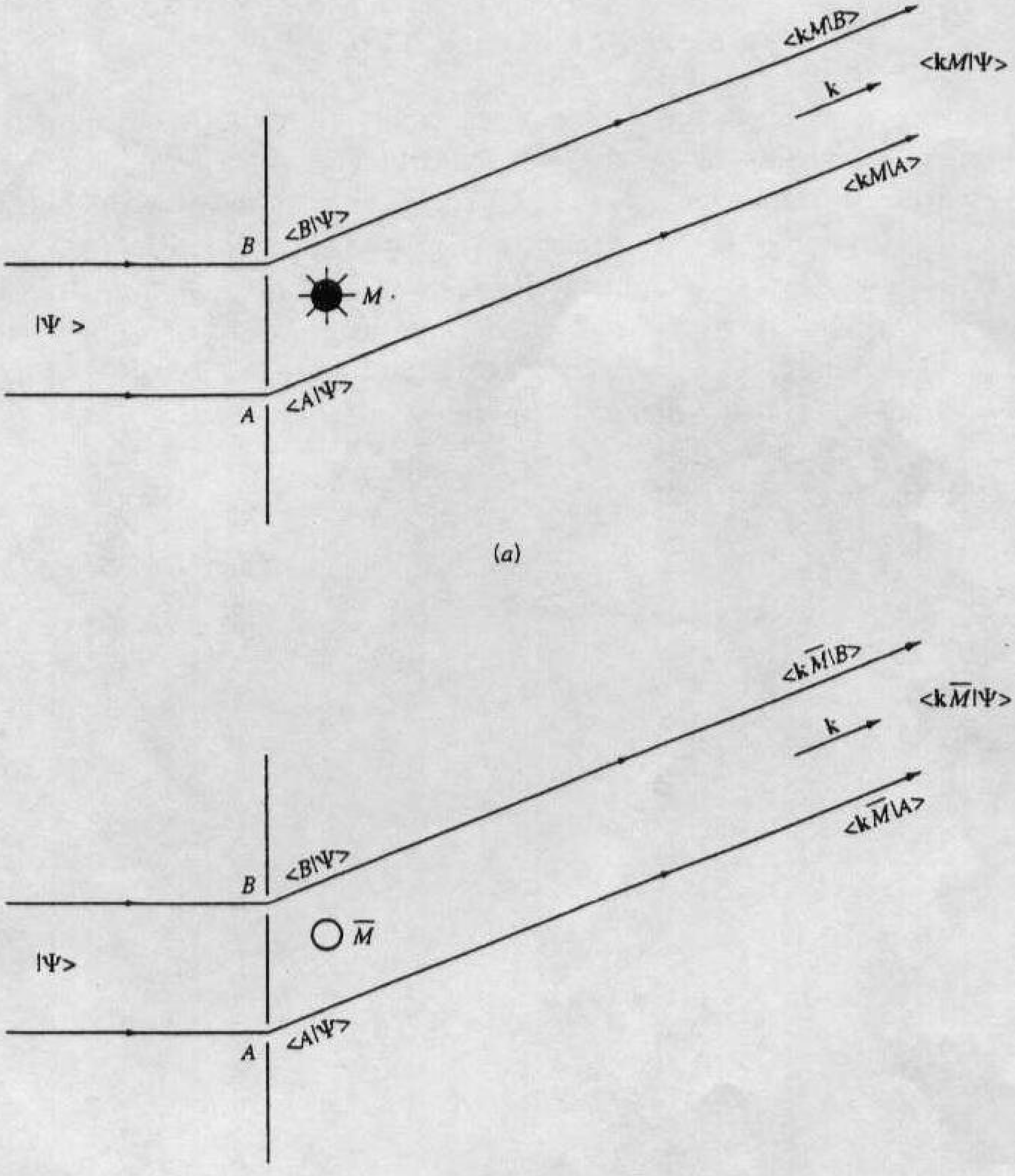


Figure 2 Two-slit interference in the presence of a monitor that has the capacity to respond to the passage of a particle diffracted in direction **k** through slits A or B. In scenario (a) the monitor registers the passage with probability amplitudes $\langle \mathbf{k}M|A\rangle$ and $\langle \mathbf{k}M|B\rangle$, respectively. In scenario (b) the amplitudes $\langle \mathbf{k}\overline{M}|A\rangle$ and $\langle \mathbf{k}\overline{M}|B\rangle$ correspond to the monitor remaining in its latent initial state, while the particle is diffracted. The off-axis placement of the monitor is meant to suggest that it may be more sensitive to particles passing through one slit than the other.

Figure 2.

Suppose (Figure 2) we have a monitor M that registers the passage of a particle through slits A, B with probability amplitudes $< \mathbf{k}M \mid A >$ and $< \mathbf{k}M \mid B >$ with the particle diffracted into the direction $\mathbf{k}$. Let the corresponding amplitudes for the case when the monitor is not triggered, i.e. does not register anything, be $< \mathbf{k}\bar{M} \mid A >$ and $< \mathbf{k}\bar{M} \mid B >$. The amplitude for diffraction into direction $\mathbf{k}$ and the monitor firing is

$$< \mathbf{k}M \mid \Psi > = < \mathbf{k}M \mid A >< A \mid \Psi > + < \mathbf{k}M \mid B >< B \mid \Psi > \qquad (i)$$

and for the monitor not firing is

$$< \mathbf{k}\bar{M} \mid \Psi > = < \mathbf{k}\bar{M} \mid A >< A \mid \Psi > + < \mathbf{k}\bar{M} \mid B >< B \mid \Psi > \qquad (ii)$$

Clearly the probabilities $|< \mathbf{k}M \mid \Psi >|^2$ and $|< \mathbf{k}\bar{M} \mid \Psi >|^2$ both exhibit interference.

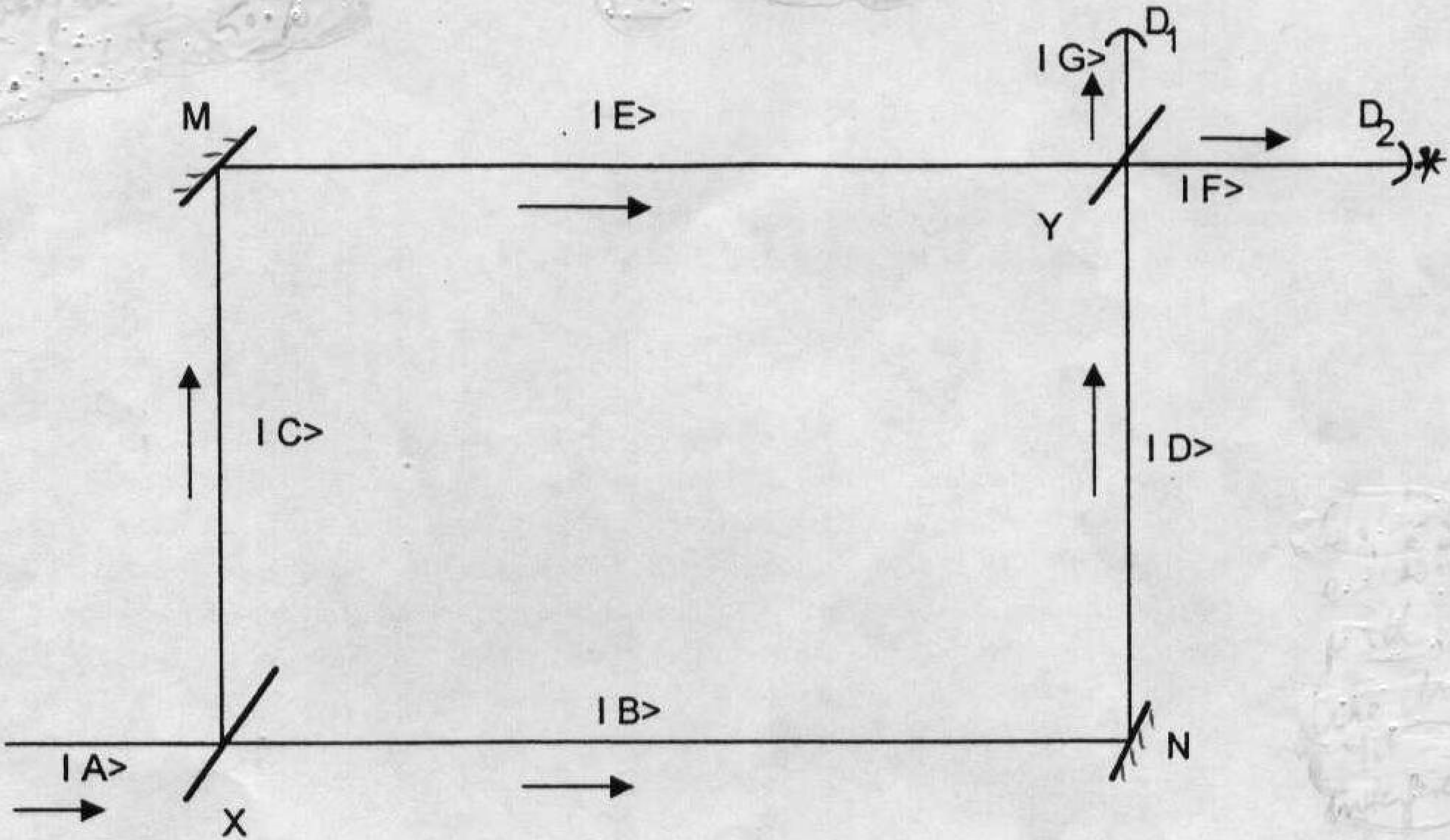
Suppose however the monitor (for example if it is placed near A) only reponds with significant efficiency to passage of a particle through A. Then

$< \mathbf{k}M \mid B >= 0$ and $< \mathbf{k}\bar{M} \mid A >= 0$.

The probability of observing the particle diffracted into direction $\mathbf{k}$ is

$$|< \mathbf{k}M \mid \Psi >|^2 + |< \mathbf{k}\bar{M} \mid \Psi >|^2 = |< \mathbf{k}M \mid A >< A \mid \Psi >|^2 + |< \mathbf{k}\bar{M} \mid B >< B \mid \Psi >|^2$$

and there is no interference term, but we know which slit the particle passed through; it must have passed through A. Analagous conclusions could be drawn if the monitor only responded with significant efficiency to passage of a particle through B. If however the detector monitors passage of a particle through both slits perfectly (e.g. by being placed halfway between them) the corresponding probability is the sum of the squared moduli of (i) and (ii), which clearly exhibits interference. But in this case we have no knowledge of which slit the particle passed through. Note that the monitors may be placed in position after the particle has passed through one or both of the slits and a delayed-choice experiment performed.



In the above diagram, X and Y are half-silvered mirrors, M and N are fully silvered mirrors, and D1 and D2 are particle detectors.

2.2 The Beam Splitter experiment

Figure 3: The Beam-Splitter Arrangement.

The beam splitter illustrates the same features of quantum mechanics as the double-slit arrangement.

A single photon is incident on a half-silvered mirror X which allows partial transmission and reflection of light (a beam-splitter; Figure 3). Which path does it take, that represented by state vector $\mid B >$ or $\mid C >$? If we place a detector in the form of a photocell in either of the two possible paths, a photon is detected with equal probability by each one. Does the photon take either path at random? Does it even take either path $\mid B >$ or path $\mid C >$? In fact, it takes both paths at once, and can interfere with itself if the two paths are re-merged.

Introduce two mirrors M and N that turn the paths through 90° and put a second beam-splitter Y at the cross-over point, to bring the paths back together.

Destructive interference leads to zero beam intensity recorded by detector D1, with 100% of the intensity going into detector D2, as the following simple calculation shows:

Referring to Figure 3, the state vector $\mid A >$ after passage through X evolves into a superposition of $\mid B >$ and $\mid C >$:

$$\mid A > \rightarrow \frac{1}{\sqrt{2}}(\mid B > +i \mid C >)$$

where the factor i takes into account a phase shift of a quarter of a wavelength that arises between the reflected and transmitted beams at such a mirror. At M, $\mid C>\rightarrow i\mid E>$, and at N, $\mid B>\rightarrow i\mid D>$. Thus

$$\mid B>+i\mid C>\rightarrow i\mid D>-\mid E>$$

At Y,

$$\mid D>\rightarrow \frac{1}{\sqrt{2}}(\mid G>+i\mid F>)$$

$$\mid E>\rightarrow \frac{1}{\sqrt{2}}(\mid F>+i\mid G>)$$

therefore

$$\frac{1}{\sqrt{2}}(i\mid D>-\mid E>)\rightarrow -\mid F>$$

The terms involving $\mid G>$ cancel and only D2 is triggered.

If we replace the mirror M by a photocell, it would encounter the state

$$\frac{1}{\sqrt{2}}(\mid B>+i\mid C>)$$

where $\mid C>$ would cause it to register and $\mid B>$ not to register. Thus a photon is detected with 50% probability. The same applies if N is replaced by a photocell.

If we blocked one path by placing an obstruction at N say, the photon's state as it approached Y would be $-\mid E>$ which evolves into $\frac{1}{\sqrt{2}}(-\mid F>-i\mid G>)$ so that the photon strikes D1 or D2 with equal probability. But when both paths are open, the photon somehow "knows" it and D2 alone is always struck. The photon appears to receive some kind of information that prevents it from reaching D1. The photon undergoes single-particle interference: when both paths are open it travels along both paths at once; it is in a coherent superposition of being on both paths at the same time!

If we try to find out which path it took, we get no interference.

If Y is absent, D1 and D2 are struck with equal probability; if it is present, only D2 is struck.

Now the experimenter's decision to insert Y, or not, can be left until a photon has almost arrived at the cross over point. In other words, whether the photon has traversed path $\mid B>$ or path $\mid C>$ or both at the same time is determined only after the traverse has taken place. In this delayed-choice experiment it appears that the experimenter can influence how quantum particles behaved in the past!

Delayed-choice beam-splitter experiments have actually been carried out using photon detectors called Pockels cells which can be switched on and off very rapidly (in nanoseconds), and in particular, after the light has already passed through the half-silvered mirror X. A detector was placed in each of the two possible paths followed by the photons. With the detectors switched on, the light behaved like photons, with a whole photon travelling by either one or the other path, one detector firing at a time. With both detectors off, it behaved like waves, even when one photon at a time was incident on the mirror X, following both routes and producing interference. These studies provided experimental proof that the behaviour of photons at the half-silvered mirror X was changed by how we were **going to** look at them, even when we had not yet made up our minds precisely what we were going to to look for!

We may extend the experiment in principle to gravitational lensing of quasar light by a distant galaxy. The photons started out 5 billion years ago. Yet we can make the decision either to observe interference or to find out which way a photon has come today, after it has accomplished most of its travel time! This is, to quote Wheeler, delayed-choice with a vengeance.

The route taken by a photon, one path or the other, or both at the same time, a billion years ago depends on what an astronomer on earth in 2002 chooses to measure.

3 The Copenhagen Interpretation

Bohr's interpretation of quantum mechanics, which has become known as the "Copenhagen Interpretation" and is widely regarded as the official, orthodox view, though disapproved of by many, puts emphasis on measurement.

According to this interpretation it is meaningless to ask what, for example, an atom or electron "really is" or what is "really happening" in an experiment. We may ask the question, but physics will not supply the answer. However, if a physicist carries out an experiment and gives a full specification of the entire apparatus used, and the precise procedure, then quantum mechanics can make a meaningful prediction of what will be observed, and this may be communicated to fellow human beings in a well-understood language.

Bohr overcame apparent contradictions such as wave/particle duality by means of his **Principle of Complementarity** which states that mutually exclusive descriptions can be applied to a quantum system but not simultaneously.

In double-slit experiment, the particles can be left alone, and an interference pattern observed, or their trajectories can be determined, washing out the pattern. We can measure the position of a particle, making its momentum uncertain, or vice versa. The more it behaves as a wave, the less it behaves like a particle. Wave properties and particle properties are complementary aspects of its nature which, according to Bohr, never come into conflict in an experimental situation.

For experimental physics to make sense, any device used must give reproducible, predictable results, and thus be deterministic. Measurements must take place at the classical level. This leads to the difficulty that in experimental verification of quantum processes, a purely probabilistic theory is being tested by macroscopically deterministic apparatus. The Copenhagen Interpretation implies the existence of two separate domains, the macroscopic, deterministic classical one and the microscopic, quantum one. It is unclear where any frontier between them lies; indeed, in any self-consistent theory covering both classical and quantum mechanics such a frontier should not exist. The passage from classical to quantum physics is described by Bohr in terms of the **Correspondence Principle,** which states that the two formulations should agree in the limit of large quantum numbers or under circumstances for which Planck's constant may be regarded as negligible. But the Correspondence Principle has never been expressed quantitatively in the form of a precise limiting process, in contrast with the transition from relativity theory to Newtonian mechanics in the limit $c \to \infty$.

A further difficulty arises in the interpretation of the Collapse or Reduction of the wave function on measurement. A measurement must be an interaction between the measured (quantum) object and the measuring device, and ought to be describable by a Schrödinger equation, albeit a complicated one. But the collapse process is a non-unitary, irreversible event that cannot be described by a Schrödinger equation; in reduction of the wave function into an eigenstate, the usual, quantum dynamics is surreptitiously replaced by something else. What can be the meaning of an interpretation that is violating the theory it purports to interpret?

Complementarity is a vague concept, introducing ambiguity and impermanence into the definition of an object. Indeed, its validity has been challenged by the results of an experiment first suggested by a group of scientists from India and subsequently carried out in Japan. In this experiment, the half-silvered mirror in a beam splitter arrangement is replaced by a pair of triangular prisms positioned with their hypotenuse surfaces facing each other, separated by a small gap. Incident light is totally reflected at the gap, but if the gap is narrowed to a width smaller than the wavelength of the light, tunnelling through the gap will occur. Only waves can do this. It was found that when the experimental setup was right 50% of the photons were reflected, 50% tunnelled through the gap. Detectors placed in the paths of the reflected and tunnelled photons clicked in anti-coincidence, revealing that photons were behaving as waves and particles at the same time.

The Copenhagen Interpretation comes close to denying the existence of objective reality, that there really are things "out there" existing in their own right whether or not we observe them. It seems to demote microscopic particles and by extension macroscopic objects like chairs, planets and galaxies, to a subjective status. Heisenberg has been quoted as commenting that "the conception of objective reality had evaporated." Einstein, however, was much opposed to this view.

The Copenhagen Interpretation claims that if we apply the rules of quantum mechanics "as if" the electron or atom were real we always get the right answers to well posed questions like "how much energy does an atom have" ? We do not need to assume anything more about the atom than is necessary to obtain correct results of observations: "atom" is merely a convenient way of talking about a set of equations connecting observations. In some cases it is clear that a naive classical picture is inappropriate; if an electron with spin were really a rotating sphere of charge with a radius of about $10^{-15}m$ it is easily calculated that the rotational speed at its surface would exceed that of light.

The Copenhagen Interpretation has something in common with Kant's doctrine that we can never have certain knowledge of any object (*das Ding an sich*) belonging to the material world, all we can know is how it appears to us; our knowledge is veiled. The point of view that nothing has real existence unless it is being observed is akin to the idea of the empiricist philosopher Berkeley, who said much the same thing, but then solved the problem by declaring that in the absence of any other observer, God is always aware of an object, imbuing it with existence.

Before sense can be made of what a quantum system like an electron is doing, the total experimental context – organization of the apparatus, precisely what is being measured, the environment – must be completely specified. This includes the observer. Thus the microworld and macroworld are inextricably interwoven. The part has no meaning except in relation to the whole. Tking this idea to its conclusion implies that the universe has a holistic character, a view that brings quantum mechanics into accord with some mystical philosophies and religions, and to devotees of some contemporary beliefs that also emphasise the holistic nature of the cosmos.

The involvement of the observer in the act of measurement is also similar to Kant's doctrine that when we ask questions about the world we are asking about a totality of which we ourselves form a tiny part, and because of this we can never completely know this totality.

The non-locality of quantum mechanical phenomena further reinforces the idea that the theory favours a holistic view of the physical world.

4 Hidden Variables

Scientists, notably Einstein, who have been averse to the probabilistic *("Jedenfalls bin ich überzeugt, dass Der nicht würfelt")*, indeterministic nature of quantum theory have put forward the idea of "hidden variables" : variables of which we are not yet aware and which are required to determine a system completely, drive its quantum behaviour and which are subject to deterministic laws. Then the indeterminism in quantum mechanics is only apparent, and due to our incomplete knowledge of hidden sub-structure.

A good analogy is with the classical kinetic theory of gases; quantities such as pressure or internal energy are statistical, but we know to be due to the motions of a very large member of individual molecules, which, since the theory is classical, are completely deterministic. We never consider the motions of individual molecules: we leave that undetermined, and consider only their average behaviour.

De Broglie's original interpretation of the wave function, that the particle has well defined position and momentum but is coupled to a "pilot wave" , a real field propagating in space and responsible for diffraction phenomena, belongs to this class of theory.

Bohm and Hiley, working at Birkbeck College, University of London, have developed an elaborate hidden variable theory involving a so-called "quantum potential" , whose action is holistic and encodes information about apparatus, observers, and the rest of the environment. This theory retains objective reality, is deterministic, but like Bohr's, is <u>**non-local**</u>, which means that it admits the possibility that **action can be transmitted from one place to affect the situation at another instantaneously.**

Early on, Von Neumann had claimed to have proved the impossibility of hidden variables, but his calculations have since been shown to have been in error. Bell, as well as Bohm, subsequently produced credible hidden-variable models. Hidden variable theories have much to offer to those who are attracted to deterministic explanations of nature, but they tend to be complicated and to suffer from contextuality, which means that the result of measuring a variable will depend on which other variables are measured simultaneously.

5 Non-locality and EPR effects

In the two-slit experiment, the probability of a particle being detected at a particular point on the screen depends on whether one or both slits are open when it passes through. It is as if the particle's behaviour is influenced by the presence of a slit through which it does not pass: the particle somehow knows if the second slit is open or closed. This implies some kind of "action at a distance" or non-locality.

Despite his famous remark about God not playing dice, Einstein appears to have been less concerned about the probabilistic nature of quantum theory than its apparent denial of objective reality and its non-local character. In a famous paper, Einstein, Podolsky and Rosen (EPR) laid down two criteria for the basis of an acceptable theory:

1. Reality: Physical quantities should be "real" where in this context reality is defined by the criterion "If without in any way disturbing a system, we can predict with certainty the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity"

2. Locality: The theory should be local, with no action at a distance.

They invented a thought experiment that reveals the profound peculiarities of the quantum description of a system extended over a large region of space (the EPR effect).

A single stationary particle explodes into two identical fragments A and B. Simultaneous precise knowledge of the position and momentum of either A or B is ruled out by the Uncertainty Principle. However, because of the law of action and reaction (momentum conservation) a measurement of B's momentum can be used to deduce A's momentum. Also, by symmetry, A will have moved a distance equal to that of B from the point of explosion, so a measurement of B's position reveals that of A.

An observer at B is free to decide whether to observe either the momentum or position of B. As a result, they will know either the momentum or the position of A, depending on their choice, so a subsequent observation of either A's momentum or position will have a predictable result. Einstein therefore concluded that A must possess a real momentum or a real position, according to the choice of the observer at B, but never, according to quantum mechanics both simultaneously.

Also, a measurement carried out on B will affect A instantaneously even if they are billions of light-years apart and the time for a signal to pass between them at the velocity of light is billions of years. This could interpreted as a violation of the special theory of relativity, with a signal being transmitted at superluminary speed. But it is not. Rather, A and B seem to cooperate in their behaviour, even over long distances, in a sort of conspiracy. Einstein found the idea of widely separated particles conspiring to give coordinated results unacceptable, calling it "ghostly action at a distance", and concluded that quantum theory was in some way incomplete. To Bohr, there was no problem, since A and B form an inseparable, correlated quantum system.

David Bohm gave an example of the EPR effect involving spins of a pair of particles which remain correlated even when they are a long distance apart, and the collapse of the wave function acts instantaneously at an arbitrarily large separation.

Take a system composed of a pair of spin-1/2 particles each with zero orbital angular momentum, and total spin S = 0. This is an example of an **entangled state.** The wave function for the system is

$$\Psi(1,2) = \frac{1}{\sqrt{2}} \left(\chi_+^n(1)\chi_-^n(2) - \chi_-^n(1)\chi_+^n(2) \right)$$

where n defines the axis of quantisation and $+$ and $-$ denote spin up or spin down respectively.

If n is the z-axis, the wave function is

$$\chi(1,2) = \frac{1}{\sqrt{2}}(\alpha_1\beta_2 - \beta_1\alpha_2)$$

Spin up for particle 1 is always associated with spin down for particle 2, and vice versa. But neither particle has a specific value for S_z until a measurement only taking account of that particle is made. All that can be said is that the particles have opposite spins. A measurement of the component of spin in the z-direction, $S_z(1)$ causes collapse of the wave function to either $\alpha_1\beta_2$ or $\beta_1\alpha_2$. So if particle 1 has spin up, particle 2 has spin down, and vice versa.

The particles are allowed to move apart and when they are well separated the component S_z of particle 1 is measured. Suppose the result $+\hbar/2$ (spin up) is obtained. Then since $M = m_{s_1} + m_{s_2} = 0$ a measurement of S_z of particle 2 has to give the result $-\hbar/2$. On the other hand, since

$$\beta_2 = \frac{1}{\sqrt{2}} \left(\chi_+^x(2) + \chi_-^x(2) \right)$$

measurement of S_x of particle 2 would yield either $\hbar/2$ or $-\hbar/2$ each with 50% probability.

If initially we had chosen to measure S_x of particle 1 instead, and found it to be $\hbar/2$, S_x of particle 2 would have to be $-\hbar/2$ and a measurement of S_z of particle 2 would give $+\hbar/2$ or $-\hbar/2$ with equal probability. Thus the experimenter's choice of which component of spin of particle 1 is measured alters the result of a subsequent measurement of a component of 2. This alteration takes place instantaneously even if 1 and 2 are arbitrarily widely separated. Moreover, which spin direction we choose to measure on particle 1 appears to fix the direction of the spin axis of particle 2, though the mere act of choosing which direction of spin to measure does nothing that is actually observable. Choice of spin direction could also be delayed until the particles have separated to a distance for which no subluminary signal could pass between them, without affecting the outcome.

This is a classic example of quantum entanglement, a phenomenon with no classical analogue. The two particles are not in communication with each other in the sense of being able to exchange instantaneous messages, but entangled such that they cannot be considered as separate independent objects until the collapse of the wave function brought about by a measurement, disentangles them.

The disentanglement of entangled states is not to be regarded as proceeding via finitely propagated signals constrained by the speed of light. There is thus no violation of special relativity associated with non-locality.

If one wishes to retain locality, one is obliged to dispute the orthodox interpretation of quantum mechanics that the individual components S_z are not defined prior to a measurement.

It is apparent that both the example quoted by Einstein, Podolsky and Rosen, and that by Bohm violate both of the criteria for reality and locality. This is because they both, in slightly different ways, involve quantum entangled states. Similar types of non-local behaviour may be demonstrated with polarisation states of two photons instead of spins of fermions.

The mysterious long range correlations between two widely separated particles and the strange dependence of the expected behaviour of one particle on the subjective fickleness of a distant human experimenter who has no means of interacting with it is one of the most bizarre manifestations of quantum mechanics.

The EPR effect may not be in conflict with the *causality* of relativity, but if we describe an EPR-type experiment from a relativistic viewpoint, we encounter another puzzle. Consider an experiment with two entangled spin-1/2 particles. Neither of them has an individual spin component, but when the spin component of one is measured, collapse of the wave function occurs and the other, unmeasured particle, which may be a long way away from its companion, now has a definite spin component, a fact that may be confirmed by a measurement. The problem is that the two measurements are *spacelike separated*, which means that each lies outside the other's light cone (see Figure 4). The question of which measurement ocurred *first* is not physically meaningful since it depends on an observer's state of motion. If the observer (not necessarily the same person as the one measuring the polarisation) is moving rapidly to the right, then he or she considers the right hand measurement to have ocurred first, and if to the left, then it is the left hand measurement that is judged to be first. For an observer moving to the right, the right hand measurement causes the jump of the spin of the left-hand particle, but for an observer moving to the left, the left hand measurement occurs first and causes the jump of the spin of the right-hand particle. We are thus presented with two different pictures of physical reality!

Figure 4. Light Cones for an EPR experiment: FIRST indicates the measurement that causes the simultaneous JUMP in the state vector: SECOND indicates the confirming measurement. Two different observers form mutually inconsistent pictures of reality in this EPR experiment with two spin-1/2 particles in an $S = 0$ state. The observer moving to the right pictured in (a) judges that the left-hand part of the state jumps before it is measured, the jump being caused by the measurement on the right. The observer moving to the left pictured in (b) has the opposite opinion.

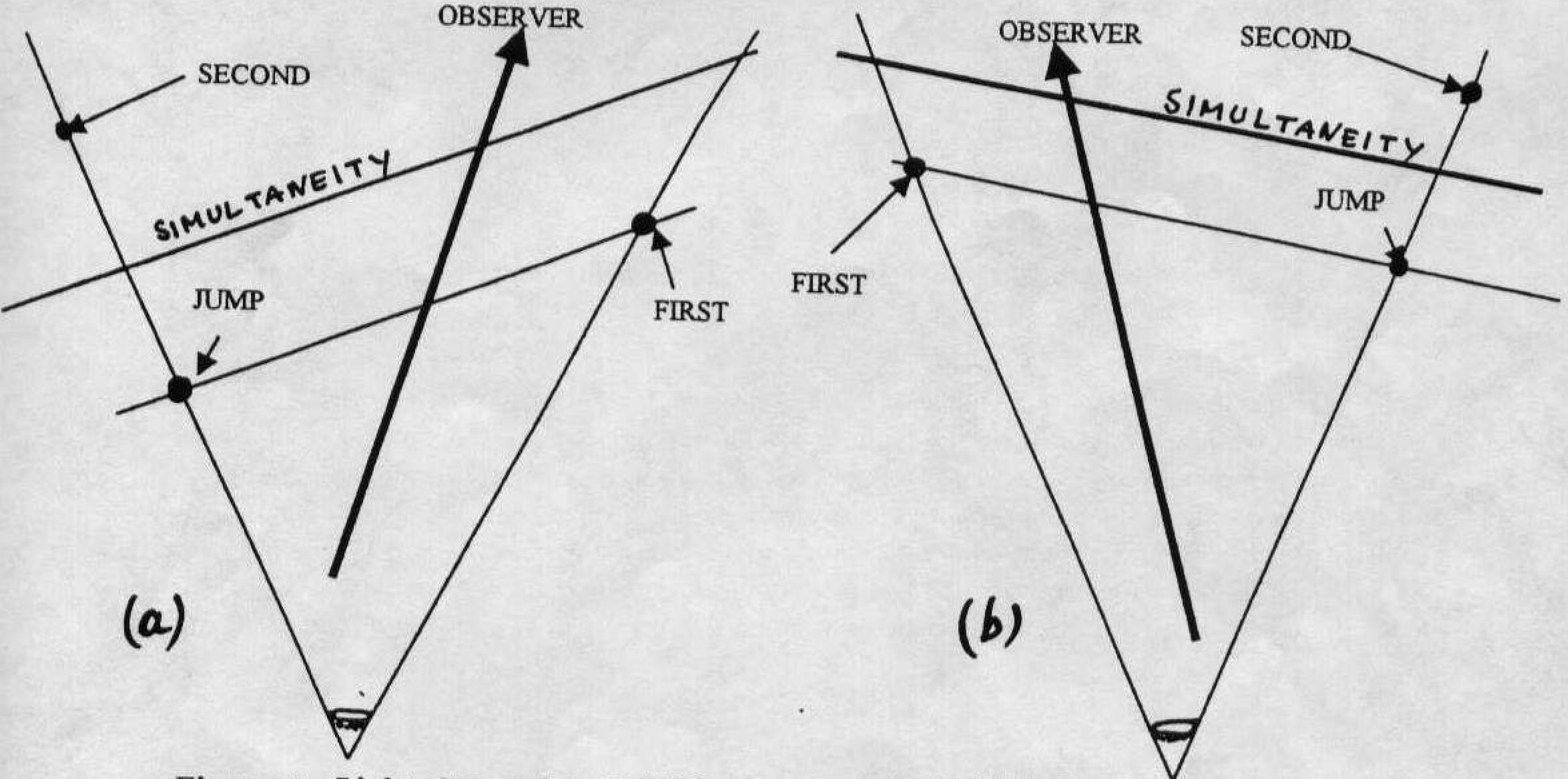


Figure 4. Light Cones for an EPR experiment: FIRST indicates the measurement

6 Bell's inequalities

What was needed was a practical experimental test to decide between Einstein and Bohr and to see if a hidden-variable theory could explain the apparent paradoxes of the Copenhagen interpretation.

Bell derived a number of inequalities that must hold between joint probabilities of spin measurements made on a two-particle system that were a necessary consequence of their being separate, independent entities, such as would be the case in a real, local theory. In conventional quantum mechanics, these inequalities could be violated.

As an example, consider again the S = 0 state of a system of two spin-1/2 particles.

The spin wave function is

$$\chi(1,2) = \frac{1}{\sqrt{2}}(\alpha_1\beta_2 - \beta_1\alpha_2) \tag{1}$$

The component of spin $S_a(1)$ of particle 1 along a direction specified by a unit vector $\hat{a}$ is

$$S_a(1) = \mathbf{S}\cdot\hat{a} = (\hbar/2)\sigma(1)\cdot\hat{a} \tag{2}$$

and a single measurement of $S_a(1)$ will yield $+\hbar/2$ or $-\hbar/2$ with equal probability. The expectation value of $S_a(1)$ is zero.

$$< \chi \mid S_a(1) \mid \chi >= 0 \tag{3}$$

If both $S_a(1)$ and $S_b(2)$, the component of spin of particle 2 along a direction $\hat{b}$ are jointly measured, the corresponding observable is

$$\hat{K} = [(\hbar/2)\sigma(1)\cdot\hat{a}][(\hbar/2)\sigma(2)\cdot\hat{b}] \tag{4}$$

and the average value of joint measurements of $S_a(1)$ and $S_b(2)$ is the expectation value of this operator.

$$E(\hat{a},\hat{b}) =< \chi \mid \hat{K} \mid \chi > \tag{5}$$

$E(\hat{A},\hat{b})$ is also known as the Correlation Coefficient. It is readily shown using the Pauli spin matrices, that

$$E(\hat{a},\hat{b}) = -\hat{a}\cdot\hat{b} = -\cos\gamma \tag{6}$$

where γ is the angle between $\hat{a}$ and $\hat{b}$ and we express spins in units of $\hbar/2$. $E(\hat{a},\hat{b})$ is the Correlation Coefficient obtained on the basis of ordinary, non-local quantum theory.

Now suppose there exists a hidden variable λ which defines the state of the system completely and which determines the value of variables observed in an experiment and whose dynamical evolution is subject to Einstein, Podolsky and Rosen's criteria for reality and locality. Each spin zero system has a definite value of λ and when a large number of such systems is prepared identically a fraction $p(\lambda)$ have values between λ and $\lambda+d\lambda$. Since $p(\lambda)$ is a probability, $p(\lambda) \geq 0$ and

$$\int p(\lambda)d\lambda = 1.$$

Denote the result of a measurement of $S_a(1)$ by $A(\hat{a},\lambda)\hbar/2$ and of $S_b(2)$ by $B(\hat{b},\lambda)\hbar/2$ where A and B can take on the values ± 1 only. Since S = 0, $A(\hat{a},\lambda) = -B(\hat{b},\lambda)$, because the two particles must have opposite spin components. In a real, local theory A and B are entirely specified by the value of λ. Also, for locality to hold, $A(\hat{a},\lambda)$ can only depend on λ and $\hat{a}$ but not on $\hat{b}$: likewise $B(\hat{b},\lambda)$ is independent of $\hat{a}$. The Correlation Coefficient based on a real, local theory, or the average result of many joint measurements of $S_a(1)$ and $S_b(2)$ is

$$\epsilon(\hat{a},\hat{b}) = \int p(\lambda)A(\hat{a},\lambda)B(\hat{b},\lambda)d\lambda \tag{7}$$

Consider joint measurements of $S_a(1)$ and $S_c(2)$ where $\hat{c} \neq \hat{b}$. Then

$$\epsilon(\hat{a},\hat{b}) - \epsilon(\hat{a},\hat{c}) = \int p(\lambda)[A(\hat{a},\lambda)B(\hat{b},\lambda) - A(\hat{a},\lambda)B(\hat{c},\lambda)]d\lambda \tag{8}$$

Using $B(\hat{b},\lambda) = -A(\hat{b},\lambda)$ and $A^2(\hat{b},\lambda) = 1$ we find

$$\epsilon(\hat{a},\hat{b}) - \epsilon(\hat{a},\hat{c}) = -\int p(\lambda)[A(\hat{a},\lambda)A(\hat{b},\lambda)\{1 + A(\hat{b},\lambda)B(\hat{c},\lambda)\}]d\lambda \tag{9}$$

From the fact that for any integral $I = \int f(x)dx$

$$\mid I \mid \leq \int \mid f(x) \mid dx \tag{10}$$

$$\mid \epsilon(\hat{a},\hat{b}) - \epsilon(\hat{a},\hat{c}) \mid \leq \int \mid p(\lambda)A(\hat{a},\lambda)A(\hat{b},\lambda)[1 + A(\hat{b},\lambda)B(\hat{c},\lambda)] \mid d\lambda \tag{11}$$

Now $p(\lambda)$ is non negative, and so is the term in square brackets in (11) : (inspection shows that it is either equal to 2 or 0.) A and B can only have values ± 1 and $|A(\hat{a},\lambda)A(\hat{b},\lambda)| = 1$. Therefore

$$|\epsilon(\hat{a},\hat{b}) - \epsilon(\hat{a},\hat{b})| \leq \int p(\lambda)[1 + A(\hat{b},\lambda)B(\hat{c},\lambda)]d\lambda \tag{12}$$

or

$$|\epsilon(\hat{a},\hat{b}) - \epsilon(\hat{a},\hat{c})| \leq 1 + \epsilon(\hat{b},\hat{c}) \tag{13}$$

or

$$|\epsilon(\hat{a},\hat{b}) - \epsilon(\hat{a},\hat{c})| - \epsilon(\hat{b},\hat{c}) \leq 1 \tag{14}$$

This inequality must be satisfied by a real, local theory. For a non-real, non-local theory, $\epsilon(\hat{a},\hat{b})$ should be replaced by $E(\hat{a},\hat{b})$ in the inequality.

It is easy to find conditions under which the quantum expectation values violate this condition. Let $\hat{a},\hat{b},\hat{c}$ lie in a plane with $\hat{b}$ bisecting the angle between $\hat{a}$ and $\hat{c}$. $E(\hat{a},\hat{b}) = -\cos\gamma$; $E(\hat{a},\hat{c}) = -\cos 2\gamma$ and $E(\hat{b},\hat{c}) = -\cos\gamma$; i.e. Bell's inequality requires

$$|-\cos\gamma + \cos 2\gamma| + \cos\gamma \leq 1 \tag{15}$$

For $0 \leq \gamma \leq \pi/2$ this is violated. For example, when $\gamma = \pi/3$ the left hand side of the inequality is equal to 1.5, certainly greater than 1.

This is one of several inequalities derived by Bell that a quantum theory must satisfy if it is to be real and local.

An important experimental test of Bell's inequalities was carried out by Aspect, Dalibard and Roger in 1982.

7 The Aspect Experiments

For experimental reasons, measurements were made on polarisations of a pair of oppositely-moving photons emitted simultaneously by Ca atoms, rather than spin-1/2 particles. The $4p^2\ {}^1S_o$ state is populated by two-photon excitation using two lasers, a Kr laser at 406 nm and a tunable dye laser at 581nm. The $4p^2$ cascades via the $4s4p^1P_1$ back to $4s^2\ {}^1S_o$ ground state with emission of two photons with wavelengths 551.3 nm and 422.7 nm. (Figure 5). The total angular momentun of both initial and final states is zero, so the total angular momentum of the two-photon system is also zero; the two photons propagate in opposite directions, along, say, the z axis.

Figure 5. Energy levels of Ca and transitions used in Aspect's experiment.

Figure 6. Schematic diagram of the apparatus of Aspect et al.

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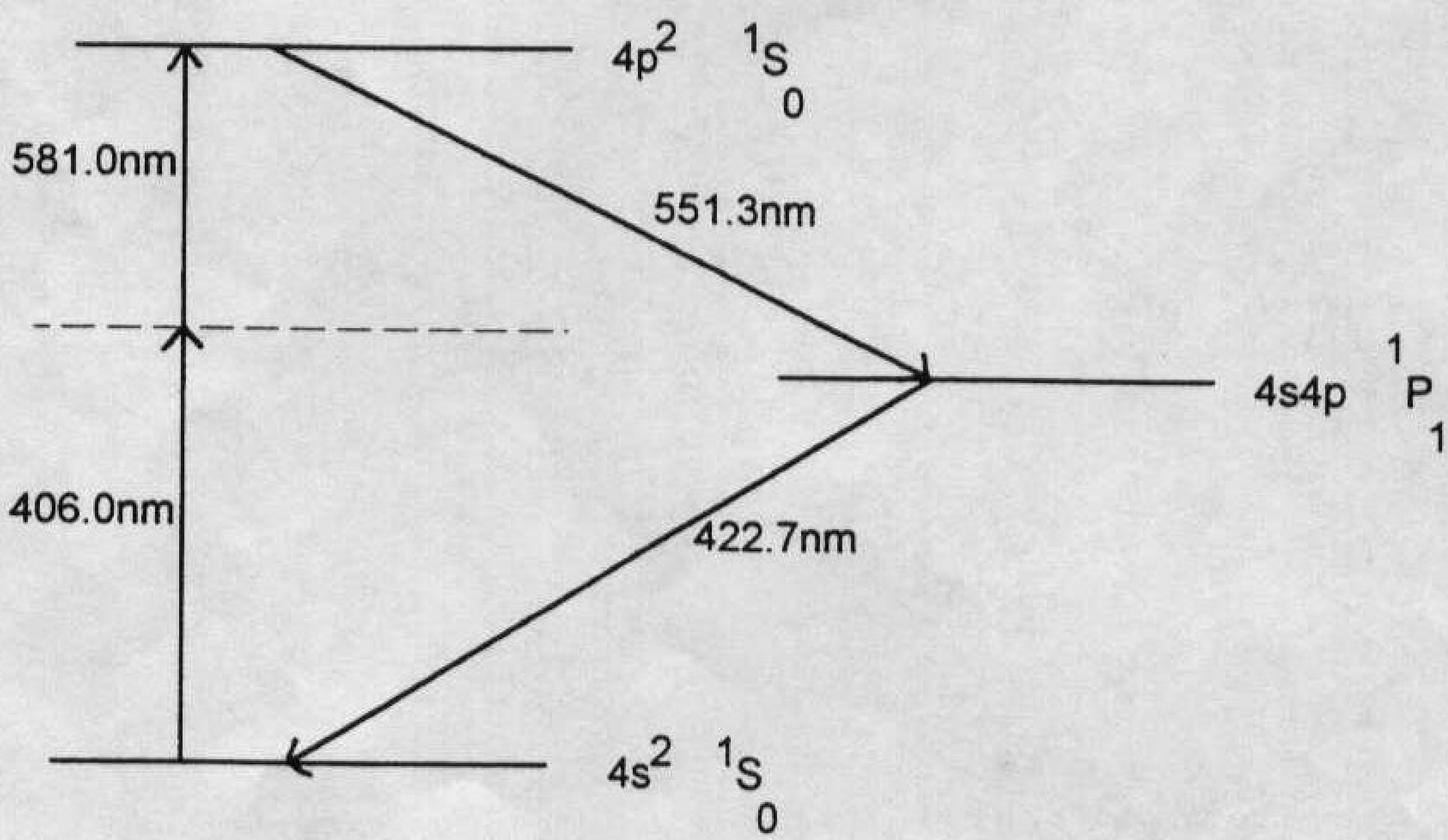
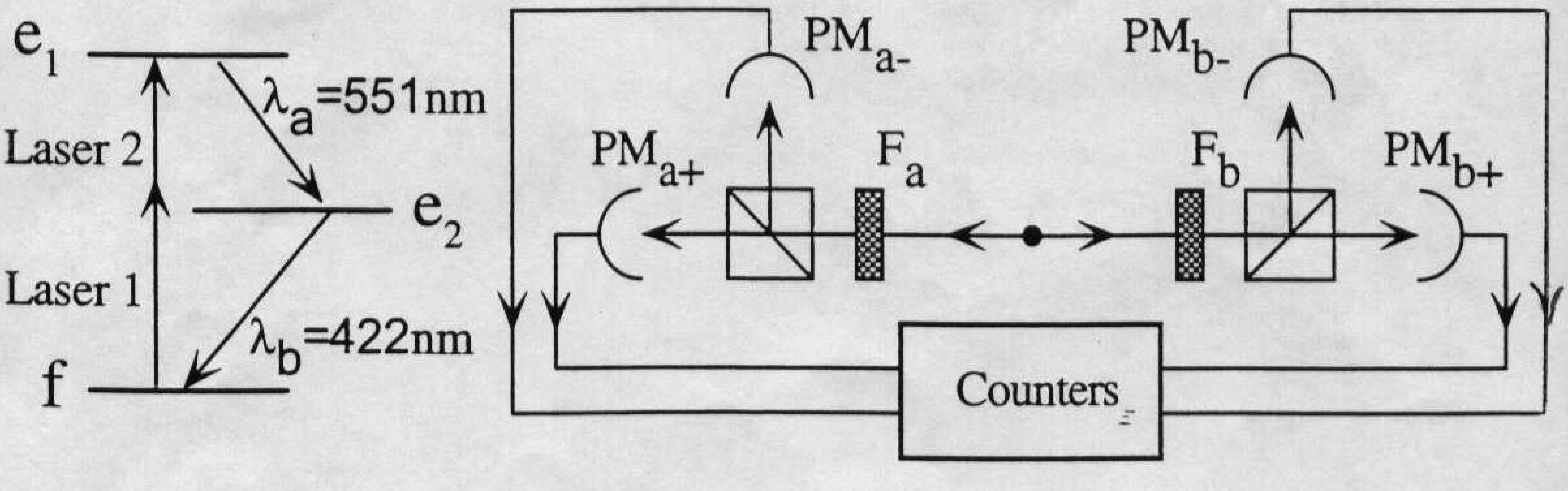


Figure 6. Schematic diagram of the apparatus of Aspect et al.



The apparatus is shown schematically in Figure 6.

About 6m from the source are placed acousto-optical switching devices,(F_a and F_b) which exploit the fact that the refractive index of water varies with compression. In the switch an ultrasonic standing wave at 25 MHz is set up. By arranging for the photons to encounter the switch at near the critical angle for total internal reflection, it is possible to switch from transmission to reflection at each half-cycle of the sound wave.

After the switch, both transmitted or reflected photons meet polarisers, analogs of Stern-Gerlach devices, which either pass or block them with certain definite probabilities. The polarisers are oriented at different angles relative to the polarisation of the photons. What happens to the photons is recorded by detectors (photomultipliers) ($PM_{a-}, PM_{a+}, PM_{b-}, PMb_+$) connected to an electronic coincidence monitor to assess the level of correlation. The polarisers transmit photons with polarisation $|\uparrow>$ towards the detectors PM_{a+}, PM_{b+} and they deflect photons with polarisation $|\rightarrow>$ towards the detectors PM_{a-}, PM_{b-}. The detectors record +1 if the photon is found to be linearly polarised parallel to the orientation of the polariser, -1 if normal to it.

It can be shown that the polarisation state of the emitted photon pair is

$$\frac{1}{\sqrt{2}}(| \, a :\uparrow; b :\uparrow> + \, | \, a :\rightarrow; b :\rightarrow>)$$

This entangled state leads to the same type of correlations as does the $S = 0$ spin state discussed above.

Let $\hat{a}$ and $\hat{b}$ be the orientations of the detectors. We define four coincidence rates, such that, for example, $N_{++}(\hat{a}, \hat{b})$ is the rate for recording +1 in detector 1 and +1 in detector 2. The correlation coefficient is then given by

$$E(\hat{a}, \hat{b}) = \frac{N_{++} + N_{--} - N_{+-} - N_{-+}}{N_{++} + N_{--} + N_{+-} + N_{-+}} \tag{16}$$

As in the case of spin 1/2 particles we can define a correlation coefficient $\epsilon(\hat{a}, \hat{b})$ based on a real local hidden-variable theory.

$$\epsilon(\hat{a}, \hat{b}) = \int p(\lambda) A(\hat{a}, \lambda) B(\hat{b}, \lambda) d\lambda$$

In the Aspect experiment, four sets of measurements with orientations of the polarisers $\hat{a}, \hat{a'}$ in the first wing and $\hat{b}, \hat{b'}$ in the second were taken and the quantity

$$X = \epsilon(\hat{a}, \hat{b}) + \epsilon(\hat{a}, \hat{b'}) - \epsilon(\hat{a'}, \hat{b}) + \epsilon(\hat{a'}, \hat{b'})$$

where $\hat{a}, \hat{a'}, \hat{b}, \hat{b'}$ represent different orientations of the polarisers, was determined. Then for a real, local theory, the appropriate Bell inequality, due to Clauser, Holt, Horne and Shimony is

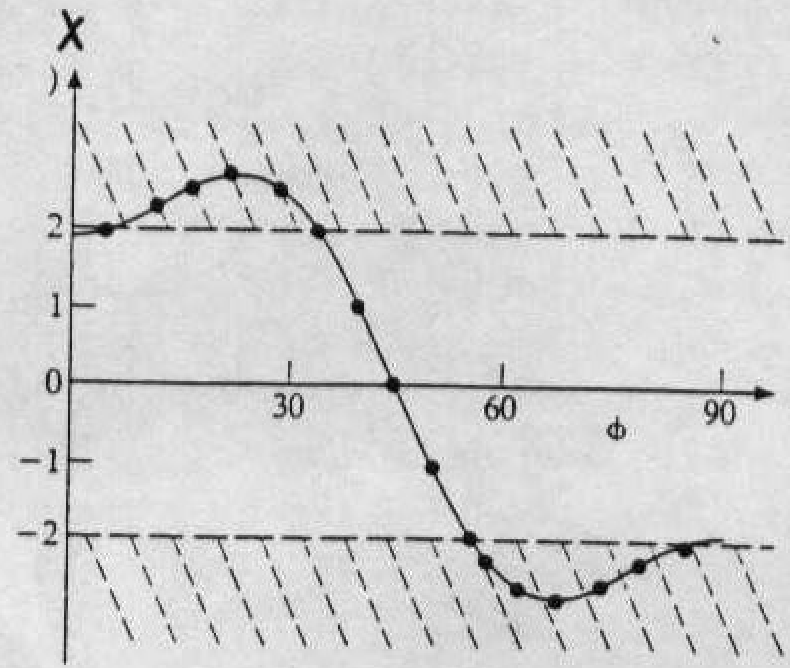
$$-2 \leq X \leq 2$$

However, if ϵ is replaced by the quantum mechanical equivalent E, then for some orientations X should violate this inequality. Aspect et al. chose

$$\hat{a} \cdot \hat{b} = \hat{b} \cdot \hat{a'} = \hat{a'} \cdot \hat{b'} = \cos \phi$$

$$\hat{a} \cdot \hat{b'} = \cos 3\phi$$

Figure 7.



Experimental data obtained for the correlation quantity X by A. Aspect, P. Grangier and G. Roger. The solid curve represents the prediction of quantum mechanics and the shaded areas are those in which Bell's inequality is violated. (*By courtesy of A. Aspect.*)

Figure 7.

The results of the experiment, shown in Figure 7, clearly violate the inequality for a range of angles, and thus lend support to the predictions of non-local quantum mechanics, and are inconsistent with the existence of hidden variables.

The experiment had an additional feature. By use of an ingenious high frequency ultrasonic switch they were able to switch the light beams from one detector to the other at random while the photons were in midflight, equivalent to rapidly switching the orientation of one detector relative to the other. The result showed that if e.g. the left hand detecting apparatus were sending some kind of message to the right hand photon informing it how the left hand one was set up, so that the right hand could interact in the appropriate way with the right hand apparatus, the signal would have to travel faster than the speed of light. There could be no communication between them unless it was superluminary.

The interpretation of the results of the experiments of Aspect et al., and of other experimenters, relies on the Copenhagen Interpretation's insistence that the measuring apparatus is an integral part of the experiment: the results of a measurement of the polarisation of a photon depends on the other photon and the two sets of detectors. Correlation is between photons and pairs of detectors. Before the time of the first measurement, the two photons do not have an independent existence and neither one has a polarisation until it is measured. No signal passes between two independent particles; the system is merely non-local.

What the result of the experiment shows is not that hidden variables do not exist, but if they do, any theory involving them must be non-local. Actually, since the derivation of Bell's inequalities does not depend on quantum mechanics, the experiment shows that all quantum theories must be non-local. The arguments upon which Bell's inequalities are based are founded on common-sense logic, logically equivalent to such propositions as, for example, the number of people in the world under 25 years old must be less than the number of females under 25 plus men of all ages. No one could dispute that. Yet the results of the Aspect experiment are logically equivalent to finding that there are actually more under-25s in the quantum-mechanical world than there are females under 25 plus all men put together.

Clearly, quantum mechanics does not follow "common sense logic."

To Be Continued......