example problems

Here, we will attempt a very common problem in spherical astronomy, which is also probably the most important one of all to understand - the conversion of equatorial coordinates into horizontal coordinates.

1. **A star, *X*, of declination δ = 42°21´N is observed when its hour angle, *H* = $8^h16^m42^s$. If the observer's latitude, ϕ, is 60°N, calculate the star's altitude, *a* and azimuth, *A*, at the time of observation.**

The most important step is to sketch as accurately as possible a celestial sphere of the problem, as shown in Figure 25.

figure 25: Example problem 1 - the conversion of equatorial coordinates to horizontal coordinates.

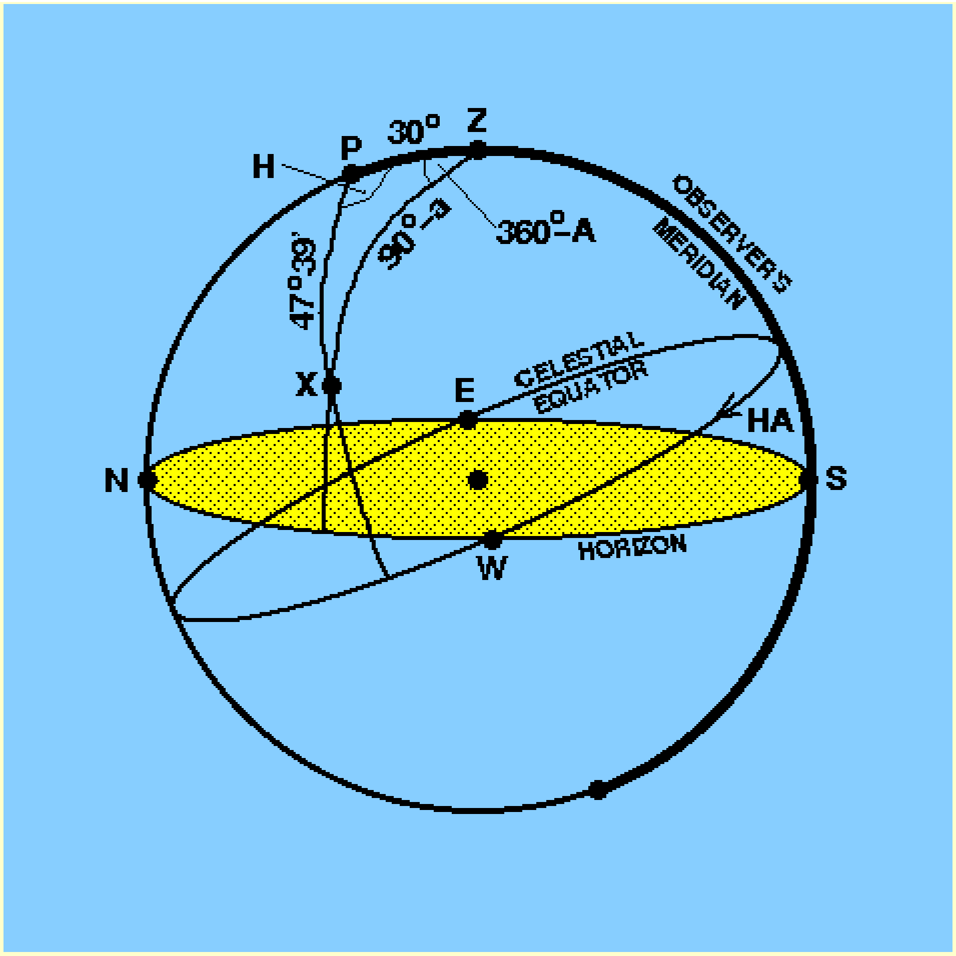

Figure 25 shows that the spherical triangle we need to work with is triangle *PZX*. We have:

$PX = 90° - \delta = 90° - 42°21' = 47°39'$,
$ZPX = H = 8^h16^m42^s = 124°10'30''$, and
$PZ = 90° - \phi = 90° - 60° = 30°$.

Applying the cosine formula, we obtain

$\cos ZX = \cos PZ \cos PX + \sin PZ \sin PX \cos ZPX$

and hence

$\cos ZX = \cos 30° \cos 47°39' + \sin 30° \sin 47°39' \cos 124°10'30''$.

Recalling that, for any angle x,

$\sin x = \cos (90° - x)$ and
$\cos x = \sin (90° - x)$,

we can write

$\sin a = \cos 30° \cos 47°39' + \sin 30° \sin 47°39' \cos 124°10'30''$,

which gives

$a = 22°04'34''$.

To determine the azimuth we can apply the sine formula to triangle *PZX*, which gives

$\sin H / \sin (90° - a) = \sin (360° - A) / \sin 47°39'$, or

$\sin (360° - A) = \sin H \sin 47°39' / \cos a$.

Thus

$\sin (360° - A) = \sin 124°10'30'' \sin 47°39' / \cos 22°04'34''$.

The sine formula must be used with care since it is not possible to say whether *A* is acute or obtuse, unless other information is available, i.e. the sine formula gives *A* or 180° - *A*. Hence we have

$360° - A = 41°17'6''$ or $138°42'54''$,

that is

$A = 318°42'54''$ or $221°17'6''$.

An inspection of Figure 25 suggests that the former answer is correct, but we should check it anyway. We can do this by using the cosine formula again:

$\cos 47°39' = \cos 30° \cos (90° - a) + \sin 30° \sin (90° - a) \cos (360° - A)$

$\cos 47°39' = \cos 30° \sin 22°04'34'' + \sin 30° \cos 22°04'34'' \cos (360° - A)$.

This gives

$360° - A = 41°17'6''$ and so

$A = 318°42'54''$ east of north.

Two things should be noted here. The first is that although the cosine of an angle, *A*, suffers from an ambiguity between *A* and 360°-*A*, it does not suffer from the acute/obtuse problem that the sine of an angle suffers from, which is why the above check works. Secondly, you may have wondered why the *altitude* we calculated above does not suffer from the acute/obtuse problem. Strictly speaking, it does, but because altitude is simply the angular distance of the star above the horizon, 180° - *a* is equivalent to *a*.
