equation of radiative transport

If we assume for the moment that the condition for the occurrence of convection is not satisfied, we can write down the fourth equation of stellar structure, which expresses the rate of change of temperature with radius in a star assuming that all energy is transported by radiation (i.e. ignoring the effects of convection and conduction).

The energy carried by radiation per square metre per second, i.e. the flux F_{rad}, can be expressed in terms of the temperature gradient and a coefficient of radiative conductivity, λ_{rad}, as follows:

$$F_{rad} = - \lambda_{rad} \, dT / dr,$$

where the minus sign indicates that heat flows down the temperature gradient. Assuming that all energy is transported by radiation, we will now drop the subscript *rad* from the remainder of this discussion.

The radiative conductivity measures the readiness of heat to flow. Astronomers generally prefer to work with an inverse of the conductivity, known as the *opacity*, which measures the resistance of material to the flow of heat. Detailed arguments (see Appendix 2 of Tayler) show that the opacity, κ, is defined by the relation

$$\kappa = 4acT^3 / 3\rho\lambda,$$

where a is the radiation density constant and c is the speed of light. Combining the above equations we obtain:

$$F = - (4acT^3 / 3\kappa\rho) \, (dT / dr).$$

Recalling that flux and luminosity are related by

$$L = 4\pi r^2 F,$$

we can write:

$$L = - (16\pi acr^2 T^3 / 3\kappa\rho) \, (dT / dr).$$

On rearranging, we obtain:

$dT / dr = - 3\kappa\rho L / 16\pi acr^2T^3$.

This is known as the *equation of radiative transport* and is the temperature gradient that would arise in a star if all the energy were transported by radiation. It should be noted that the above equation also holds if a significant fraction of energy transport is due to conduction, but in this case L refers to the luminosity due to radiative *and* conductive energy transport and κ refers to the opacity to heat flow via radiation *and* conduction.

$$\frac{dT}{dr} = -\frac{3\kappa\rho L}{16\pi acr^2T^3}$$

the equation of radiative transport

Clearly, the flow of energy by radiation can only be determined if an expression for κ is available. How such an expression can be obtained will be discussed later in the course.
