

# Extrasolar Planets and Astrophysical Discs

## Problem Set 3

January 2010

Consider a dwarf–nova binary system consisting of two equal mass stars, the donor star being a  $1 M_{\odot}$  solar–type star, and the compact object being a  $1 M_{\odot}$  white dwarf. The system is observed to be quiescent for time scales of 1 month, and to undergo periodic outbursts with duration of 5 days.

Assume that the disc mass builds up to  $10^{-11} M_{\odot}$  before the outburst ensues, that the disc has a radius equal to  $2/3$  of the Roche lobe radius, and that the effective temperature is given by

$$T_{eff} = 6.6 \times 10^4 \left( \frac{R}{10^7 \text{ m}} \right)^{-3/4}$$

during the outburst. Assume also that the disc surface density is constant, and that  $H/R = 0.03$ .

Estimate the atomic mean free path,  $L$ , according to

$$L = \frac{1}{n\sigma}$$

where  $n$  is the particle number density and  $\sigma$  is the collision cross section for the atoms. For simplicity use  $\sigma = \pi a^2$  where  $a = 10^{-10}$  m is the radius of a hydrogen atom. Estimate the kinematic ‘molecular’ viscosity assuming a characteristic temperature of  $6.6 \times 10^4$  K, and hence calculate the evolutionary time scale of a dwarf–nova disc evolving under the influence of a viscosity of this magnitude. Given this estimate of the evolutionary time, argue that a source of anomalous viscosity is required to explain the duration of dwarf–novae outbursts.

Using an ‘ $\alpha$ ’ model of turbulent viscosity, estimate the value of ‘ $\alpha$ ’ required to obtain an outburst duration of 5 days.

[The radius of the Sun is  $1 R_{\odot} = 6.96 \times 10^8$  m, the mass of the Sun is  $1 M_{\odot} = 1.99 \times 10^{30}$  kg, the mass of a hydrogen atom is  $1 m_H = 1.67 \times 10^{-27}$  kg, and the gas constant is  $\mathcal{R} = 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$ .]