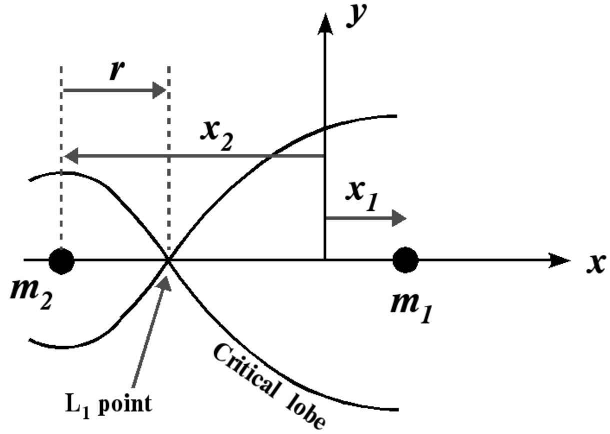
Extrasolar Planets and Astrophysical Discs
Problem Set 2

January 2010

A binary system contains 2 stars with masses m_1 and m_2, where $m_2 \ll m_1$. They rotate with angular velocity Ω in circular orbit about the centre of mass. In a frame corotating with the orbit, the line of centres lies on the x–axis and the origin is at the centre of mass. Thus

$$D = x_1 - x_2, \quad x_1 = \frac{m_2 D}{m_1 + m_2}, \quad x_2 = -\frac{m_1 D}{m_1 + m_2},$$

where D is the distance between the stars, and x_1 and x_2 are their positions on the x–axis.



On the line of centres the gravitational and centrifugal potential is

$$\Phi = -\frac{Gm_1}{|x - x_1|} - \frac{Gm_2}{|x - x_2|} - \frac{1}{2}\Omega^2 x^2 \qquad \text{with} \qquad \Omega^2 = \frac{G(m_1 + m_2)}{D^3}.$$

At the L_1 Lagrangian point, $\frac{\partial \Phi}{\partial x} = 0$. Show that there (for $x > x_2$, $x < x_1$)

$$-\frac{Gm_1}{\left(x - \frac{m_2 D}{m_1 + m_2}\right)^2} + \frac{Gm_2}{\left(x + \frac{m_1 D}{m_1 + m_2}\right)^2} - \frac{G(m_1 + m_2)x}{D^3}) = 0.$$

Set $x = r + x_2 = r - \frac{m_1 D}{m_1 + m_2}$. Then show that

$$-\frac{Gm_1}{(r - D)^2} + \frac{Gm_2}{r^2} - \frac{G(m_1 + m_2)r}{D^3} + \frac{Gm_1}{D^2} = 0,$$

and hence that, for small r and m_2, approximately

$$\frac{Gm_2}{r^2} = \frac{3Gm_1 r}{D^3}$$

and thus

$$r = D\left(\frac{m_2}{3m_1}\right)^{1/3}.$$