Extrasolar Planets and Astrophysical Discs
Problem Set 1

January 2010

The virial theorem for an isolated magnetic but pressureless cloud gives

$$\frac{\mathrm{d}^2 I}{\mathrm{d}t^2} = 2\mathcal{M} + 4\mathcal{K} + 2E_g \ ,$$

where I is the moment of inertia of the cloud, t is time,

$$\mathcal{M} = \int_V \frac{B^2}{2\mu_0}\,\mathrm{d}V \ , \quad E_g = -\frac{1}{2}G\int_V\int_V \frac{\rho(\mathbf{r})\rho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}\,\mathrm{d}V\,\mathrm{d}V' \quad ,$$

and $\mathcal{K}$ is the kinetic energy, provided that the magnetic field is zero on the surface.

Let the cloud be at rest and such that the maximum distance between two points on the surface be D. Suppose that it is permeated by an internal magnetic field with maximum magnitude B_0, which vanishes on the cloud surface and exterior to it. Show that

$$\frac{\mathrm{d}^2 I}{\mathrm{d}t^2} < -\frac{GM^2}{D} + \frac{4\pi D^3 B_0^2}{3\mu_0} \ .$$

Given that a condition for gravitational collapse of this cloud is

$$-\frac{GM^2}{D} + \frac{4\pi D^3 B_0^2}{3\mu_0} < 0 \ ,$$

and defining a parameter $\mathcal{F} = \pi D^2 B_0$, show that collapse ensues if

$$\pi^2 D^4 B_0^2 = \mathcal{F}^2 < \frac{3\pi\mu_0 G M^2}{4} \ .$$

Deduce that the maximum magnetic flux through any surface in the cloud $F_m < \mathcal{F}$, so collapse will occur if

$$F_m^2 < \frac{3\pi\mu_0 G M^2}{4} \ .$$

Estimate the magnitude of the field required to start inhibiting the collapse of a cloud of mass 1 $\mathrm{M}_\odot$ and dimension $D = 10^{16}$ m. How big would this field be if the cloud collapsed to a size 1 $\mathrm{R}_\odot$ conserving magnetic flux ?

(The mass of the Sun is $1\,\mathrm{M}_\odot = 1.989 \times 10^{30}$ kg, the radius of the Sun is $1\,\mathrm{R}_\odot = 6.96 \times 10^8$ m, the constant of gravitation is $G = 6.67 \times 10^{-11}\,\mathrm{m}^3\,\mathrm{kg}^{-1}\,\mathrm{s}^{-2}$, and the permeability of free space is $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{H\,m}^{-1}$.)