K. Course work 6

CW6: due to 25.03.09:

Q1. *30 Marks*

a) What is the proper area of a sphere centered at the origin.

b) Express your result, first, in terms of χ and then in terms of σ.

c) For a closed Universe, one can scale radial coordinate r so that A=1. Show that the total volume of such a Universe is

$$V = 2\pi^2 R_0^3.$$

Q2. *20 Marks*

a) In a zero-pressure $\Omega_0 = 1$ Friedman model, show that the current physical distance to an object with redshift z is

$$r(z) = r_H[1 - (1+z)^{-1/2}],$$

where r_H is the current particle horizon size.

b) Deduce that objects at the particle horizon have an infinite redshift.

Q3. small*25 Marks*

a) The energy flux received per unit area from a source of bolometric luminosity P at redshift z was shown in the lectures to be $\frac{P}{d_L^2}$, where d_L is the "luminosity distance". Show that

$$d_L = (1+z)R_0 \frac{\sin A\chi}{A}.$$

b) Show that

$$R_0 \frac{\sin A\chi}{A} = \frac{2c}{H_0 \Omega_0^2 (1+z)} \left[\Omega_0 z + (\Omega_0 - 2)(\sqrt{\Omega_0 z + 1} - 1)\right].$$

Q4. *25 Marks*

a) The apparent angular size of an object with linear diameter D at redshift z was shown in the lectures to be

$$\theta = \frac{D(1+z)}{R_0} \frac{A}{\sin A\chi},$$

where χ is the co-moving radial coordinate. Using the Friedman equation show that for $\Omega_0 = 1$

$$\theta(z) = \frac{DH_0(1+z)^{3/2}}{2c(\sqrt{1+z} - 1)}.$$

b) Prove that $\theta(z)$ is non-monotonic function and find z corresponding to the minimum of $\theta(z)$.