I. Course work 5

CW5: due to 18.03.10:

Q1. *30 Marks*

a) Derive the Friedman-Lemetre-Robertson-Walker metric for spatially flat Universe.

b) Derive the Friedman-Lemetre-Robertson-Walker metric in the case of constant negative curvature of the three-dimensional space.

Q2. *30 Marks*

a) Show that a positive Λ - term corresponds to a repulsive force whose strength is proportional to distance. Show that when Λ - term dominates

$$R \sim exp[(\Lambda/3)^{1/2}t].$$

b) Verify that the substitution

$$\sigma = A^{-1}\sin A\chi$$

turns the metric

$$ds^2 = -c^2dt^2 + R(t)^2[d\chi^2 + (A^{-1}\sin A\chi)^2(d\theta^2 + \sin^2\theta d\phi^2)]$$

into the form

$$ds^2 = -c^2dt^2 + R(t)^2[(1 - A^2\sigma^2)^{-1}d\sigma^2 + \sigma^2(d\theta^2 + \sin^2\theta d\phi^2)].$$

Q3. *40 Marks*

a) Starting from the first law of thermodynamics (in cosmological context it means the change of energy is equal to work done by forces of pressure)

$$dE = -pdV,$$

where E is total energy in some volume V and p is pressure and ρ is mass density, derive the energy conservation equation

$$\dot{\rho} = -(3\dot{R}/R)(\rho + p/c^2).$$

b) Show that the Einstein equation

$$(3\ddot{R}/R) = -4\pi G(\rho + \frac{3p}{c^2})$$

can be combined with the energy conservation equation and integrated to give the Friedman equation

$$(\dot{R}/R)^2 - 8\pi G\rho/3 = -kc^2/R^2.$$

c) Assume that $p = \alpha\rho c^2$, where α is equation of state parameter, show that at some times

$$R \sim t^{2/[3(1+\alpha)]}$$

and find the range of time when this equation is valid for different α, which is supposed to be $-1 \leq \alpha \leq 1$.