C. Course work 2

Q1. *[25 Marks]*

a) Starting with the Newtonian deceleration equation

$$\ddot{R} = -\frac{4\pi G\rho}{3}R, \tag{C.1}$$

derive the Friedman equation in the form

$$\dot{R}^2 = \frac{8\pi G\rho}{3}R^2 - kc^2, \tag{C.2}$$

where ρ is homogeneous density and k is a constant.*[10 Marks]*

b) Show that the following relationships

$$\Omega(t) = 2q(t), \quad \text{and} \quad kc^2 = R(t)^2H(t)^2(\Omega(t) - 1) \tag{C.3}$$

between deceleration

$$q(t) = -\frac{R(t)\ddot{R}(t)}{\dot{R}^2(t)}, \tag{C.4}$$

the dimensionless density

$$\Omega(t) = \frac{8\pi G\rho(t)}{3H^2(t)}, \tag{C.5}$$

the Hubble constant $H(t)$ and the scale factor $R(t)$ are valid for any moment of time (not only at the present moment). *[15 Marks]*

Q2.*[25 Marks]*

a) Show that in the Newtonian cosmological model with k=+1 the scale factor, $R(t)$, as a function of time can be presented in the following parametric form

$$R = a(1 - \cos\eta), \quad t = \frac{a}{c}(\eta - \sin\eta), \tag{C.6}$$

where a is a constant and the parameter η goes from 0(the Big Bang)to 2π (the Big Crunch). *[15 Marks]*

b) Calculate the maximal value of the scale factor R in terms of the main cosmological parameters.*[10 Marks]*

Q3.*[20 Marks]*

Find in a parametric form the dependence of the Hubble constant, $H(t)$, on time in Newtonian cosmological model with $k = 1$.

Q4. *[30 Marks]*

a) Motivate that the fate of the Universe can be predicted on basis of observations. *[10 Marks]*

b) Find the Hubble constant, $H(t)$, as a function of time in Newtonian cosmological model with $k = 0$. *[10 Marks]*

c) Find the age of the Universe predicted by the Newtonian theory in this case and express your results in terms of observable cosmological parameters. *[10 Marks]*