Queen Mary
University of London

M. Sc. Examination by course unit 2011

ASTM108 Cosmology

Duration: 3 hours

Date and time: 1st June 2011, 1430-1730

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

You may attempt as many questions as you wish and all questions carry equal marks. Except for the award of a bare pass, only the best 4 questions answered will be counted.

Calculators ARE permitted in this examination. The unauthorized use of material stored in pre-programmable memory constitutes an examination offence. Please state on your answer book the name and type of machine used.

Complete all rough workings in the answer book and cross through any work which is not to be assessed.

Candidates should note that the Examination and Assessment Regulations state that possession of unauthorized materials by any candidate who is under examination conditions is an assessment offence. Please check your pockets now for any notes that you may have forgotten that are in your possession. If you have any, then please raise your hand and give them to an invigilator now.

Exam papers must not be removed from the examination room.

Examiner(s): J. E. Lidsey

You are reminded of the following information, which you may use without proof:

The following constants may be assumed:

Speed of light, $c = 3.0 \times 10^8\,\mathrm{m\,s^{-1}}$

Gravitational constant, $G = 6.67 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$

Boltzmann's constant, $k_B = 1.38 \times 10^{-23}\,\mathrm{JK^{-1}}$

Radiation constant, $\alpha = 7.565 \times 10^{-16}\,\mathrm{J\,m^{-3}\,K^{-4}}$

Proton mass–energy, $m_p c^2 = 938.3\,\mathrm{MeV}$

Neutron mass–energy, $m_n c^2 = 939.6\,\mathrm{MeV}$

Mega Parsec, $1\,\mathrm{Mpc} = 3.09 \times 10^{22}\,\mathrm{m}$

Hubble time, $H_0^{-1} = 9.8 \times 10^9 h^{-1}\,\mathrm{yr} = 3.09 \times 10^{17} h^{-1}\,\mathrm{s}$

The Conversion Factor, $1\,\mathrm{eV} = 1.602 \times 10^{-19}\,\mathrm{J}$

The Conversion Factor, $1\,\mathrm{J} = 1\,\mathrm{kg\,m^2\,s^{-2}}$

The Conversion factor, $1\,\mathrm{year} = 3.15 \times 10^7$ seconds

The following formulae may be assumed:

Friedmann Equation

$$H^2 = \frac{8\pi G}{3}\rho + \frac{8\pi G}{3}\Lambda - \frac{kc^2}{a^2},$$

where $H = \dot{a}/a$ is the Hubble parameter, a is the scale factor of the universe, ρ is the mass density, Λ is the cosmological constant, k is a constant and overdots denote time derivatives.

Conservation Equation

$$\dot{\rho} + 3H\left(\rho + \frac{p}{c^2}\right) = 0,$$

where p represents the pressure of the matter in the universe.

Question 1

(a) *[3 marks]* Explain what is meant by the isotropy and homogeneity of the universe. What is the main observational evidence that the universe is isotropic? Prove, with the aid of a diagram, that if the universe is isotropic, it is also homogeneous.

(b) *[6 marks]* Consider a small mass of fluid in an homogeneous, expanding medium. By applying energy conservation and Newtonian mechanics to such a system, and further assuming the cosmological constant Λ vanishes, derive the Friedmann equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}\,. \qquad (1)$$

(c) *[5 marks]* A university professor proposes a theory whereby the universe is dominated by a form of matter with a density that varies as $\rho = 1/a$, where a is the scale factor of the universe. Solve the Friedmann equation (1) when the universe contains such matter and has a spatially flat geometry.

(d) *[6 marks]* By employing the conservation equation, determine the equation of state for the type of matter described in part (c).

(e) *[5 marks]* Describe, qualitatively, the history of a negatively-curved universe that is dominated by the type of matter described in part (c).

Question 2

(a) *[3 marks]* Define the Ω-parameter, Ω. The Friedmann equation can be written in terms of the Ω-parameter such that

$$\Omega - 1 = \frac{kc^2}{a^2H^2}\,.$$

Sketch how Ω varies with time in the standard, hot big bang theory for the cases where $kc^2 = -1$, $kc^2 = 0$, and $kc^2 = +1$, respectively. What is a necessary condition for Ω to become infinite?

(b) *[6 marks]* Why is the evolution of Ω as the universe expands regarded as a problem of the standard, hot big bang theory?

(c) *[4 marks]* Describe, qualitatively, how a phase of inflationary expansion can in principle resolve the problem discussed in part (b).

(d) *[7 marks]* Assuming inflation started when $\Omega = 2$ and the universe was $t = 10^{-36}$ seconds old, determine the value of Ω when the volume of the universe during inflation had increased by a factor of 10^{80}.

(e) *[5 marks]* Given your answer to part (d), calculate the value of Ω when the universe was 1000 years old.

Question 3

(a) *[4 marks]* Starting from the Friedmann equation given on page 2, derive the acceleration equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{8\pi G}{3}\Lambda\,,$$

where ρ and p represent the density and pressure of the matter, respectively, and Λ is the cosmological constant.

(b) *[3 marks]* Show that a pressureless universe with $\Lambda = 0$ has a finite age.

(c) *[5 marks]* If $\rho + 3p/c^2 > 0$ and $\Lambda = 0$, derive an upper limit for the age of a negatively-curved universe in terms of Hubble's constant.

(d) Explain, qualitatively, why a positive cosmological constant enhances the age of the universe for a given value of Hubble's constant.

(e) *[5 marks]* Derive a numerical lower limit for the age of a spatially flat universe containing pressureless matter and a cosmological constant, $\Lambda > 0$.

Question 4

(a) *[4 marks]* State the condition for an elementary particle in the early universe to behave relativistically. How does the energy density of such a particle vary with the volume of the universe? If the early universe was dominated by such particles, how did the scale factor vary with time?

(b) *[5 marks]* Explain why it is valid to assume that the density of the very early universe was very close to the critical density.

(c) *[5 marks]* Given that the temperature of the universe was about 2×10^6 K when it was approximately 1 year old, calculate the age of the universe (in seconds) when its temperature was 10^{27} K.

(d) *[5 marks]* Compute a numerical estimate for the age of the universe at the epoch of decoupling.

(e) *[6 marks]* A MSc student develops a theory predicting that very massive, stable particles formed in the early universe 10^{-20} seconds after the big bang, with a density 10^{-18} that of the critical density. Assuming that all other particles in the universe at that time were behaving relativistically, calculate the density of these massive particles, relative to the critical density, when the universe's temperature was 10^{10} K. Further calculate the age of the universe when these massive particles came to dominate the density.

Question 5

(a) *[3 marks]* What are the key properties of the cosmic microwave background radiation?

(b) *[6 marks]* The Robertson-Walker metric for a spatially flat universe is given by

$$ds^2 = -c^2dt^2 + a^2(t)\left[d\chi^2 + \chi^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right],$$

where $a(t)$ is the scale factor of the universe.

Taking care to justify any assumptions you make, show that when the universe has an age, t, the corresponding horizon distance is given by

$$d_H(t) = a(t)\int_0^t \frac{c}{a(t)}dt\,. \qquad (2)$$

Hence, determine the horizon distance at the present time.

(c) *[6 marks]* Explain what is meant by the 'rescaled horizon distance'. Calculate the rescaled horizon distance for the epoch of decoupling relative to the present horizon distance.

(d) *[6 marks]* Why is the answer you find in part (c) regarded as the horizon problem of the standard, hot big bang theory?

(e) *[4 marks]* By using the integral (2), explain briefly how a finite period of exponential expansion of the very early universe can resolve the horizon problem.

Question 6

Write short notes on the following *[5 marks each]*:

(a) Compare and contrast the key physical processes that occurred during the decoupling and primordial nucleosynthesis eras.

(b) Describe how the physical process that ended inflation led to the temperature anisotropies in the cosmic microwave background.

(c) How do the temperature anisotropies in the cosmic microwave background provide a measurement of the current density of the universe.

(d) Explain how the Hubble constant can be measured observationally. What is the physical significance of this constant?

(e) Summarize the observational evidence that the present universe is undergoing a phase of accelerated expansion.

End of Paper

Q1:

a) 3 marks, b) 6 marks, c) 6 marks, d) 5 marks, e) 5 marks,

Q2:

on p.7 after Question 2. should be: a) last line instead (see. c)) should be : see (Q1c)

a) 7 marks, b) 3 marks, c) 4 marks, d) 5 marks, e) 6 marks,

Q3:

a) 5 marks, b) 5 marks, c) 3 marks, d) 6 marks, e) 6 marks,

Q4:

a) 5 marks, b) 3 marks, c) 4 marks, d) 6 marks, e) 7 marks,

Q5:

This question is very easy in comparison with other questions, but according to rubric it also should be of 25 Marks

a) 4 marks, b) 5 marks, c) 5 marks, d) 5 marks, e) 6 marks,

Q6:

a) 5 marks, b) 5 marks, c) 5 marks, d) 5 marks, e) 5 marks,

Question 1

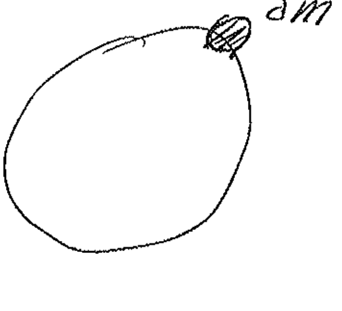
(a) In average the Universe is observed the same in all directions (isotropy). In average the Universe is the same everywhere (homogeneity).

Let assume that Universe contains inhomogeneity in form of some cloud and say the temperature of this inhomogenity, T, is higher than average temperature of the Universe, T_0. Then according to the diagram below, an observer sees that the Universe is anysotropic.

Thus if Universe is isotropic it also should be homogeneous

6) Energy conservation:

$$\dot{\rho} + 3H\left(\rho + \frac{p}{c^2}\right) = 0$$



$$\ddot{a} r = -\frac{GM}{a^2 r^2}$$

But this is not Newt. $\left(\rho + \frac{3p}{c^2}\right)$, $p = 0$ for Newt. Mechanics (or $c \to \infty$)

$$M = \frac{4\pi}{3}\rho r^3 a^3$$

$$\ddot{a} r = -\frac{4\pi G}{3}\rho r a$$

Multiplying by $\dot{a}$ and taking into account that $\ddot{a}\dot{a} = \frac{1}{2}(\dot{a}^2)^{\cdot}$ and

$$\dot{\rho} = -3H\rho$$

$$\frac{\dot{\rho}}{\rho} = -\frac{3\dot{a}}{a} \qquad \rho = \rho_0\left(\frac{a_0}{a}\right)^3$$

$$\ddot{a} = -\frac{4\pi G\rho_0 a_0^3}{3a^2}$$

$$\frac{\dot{a}}{a^2} = -\left(\frac{1}{a}\right)^{\cdot}$$ one obtains:

$$\left(\frac{\dot{a}^2}{2}\right)^{\cdot} = \frac{4\pi}{3} G \rho_0 a_0^3 \left(\frac{1}{a}\right)^{\cdot}$$

$$\frac{\dot{a}^2}{2} = \frac{4\pi}{3} G \rho_0 \frac{a_0^3}{a} + C, \text{ where}$$

C is a constant of integration, which could be presented as $C = -\frac{kc^2}{2}$

$$\dot{a}^2 = \frac{8\pi G}{3} \left(\rho_0 \frac{a_0^3}{a^3}\right) a^2 - kc^2$$

$\rho_0 \frac{a_0^3}{a^3} = \rho$

i.e.

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G \rho}{3} - \frac{kc^2}{a^2}$$

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c) $\rho = \frac{1}{a}$ (This is wrong!!)

Should be $\rho \propto \frac{1}{a}$ or

$\rho = \rho_0 \frac{a_0}{a}$, or $\rho = \frac{A}{a}$, where

dimensions of A: $[A] = [\rho] \cdot [a$

Spatinall flat geometry means

$k = 0$, hence

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3} \rho_0 \frac{a_0}{a}$$

$$\frac{\dot{a}^2}{a} = \frac{8\pi G \rho_0 a_0}{3}$$

$$\frac{\dot{a}}{\sqrt{a}} = \sqrt{\frac{8\pi G \rho_0 a_0}{3}}$$

$$2\sqrt{a} = \sqrt{\frac{8\pi G \rho_0 a_0}{3}}\, t + C$$

C – constant of integration

if $t \to 0$ then $a \to 0$, hence $C = 0$.

$$\boxed{a = \frac{2\pi G \rho_0 a_0 t^2}{3}}$$ this solution.

d) From $\dot{\rho} + 3H\left(\rho + \frac{P}{c^2}\right) = 0$,
and $P = \alpha \rho c^2$
one obtains,

$$\frac{\dot{\rho}}{\rho} = -\frac{3\dot{a}}{a}(1+\alpha)$$

$$\rho \propto a^{-3(1+\alpha)}$$

But it is given that $\rho \propto a^{-1}$,

Hence $3(1+\alpha) = 1$

$$1+\alpha = \frac{1}{3}$$

$$\alpha = -\frac{2}{3} \Rightarrow \text{Equation of state}$$

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e) $P = -\frac{2}{3}\rho c^2$ this is some sort of dark energy, such Universe expands with acceleration.

A negatively-curved Universe $\Rightarrow$ $k = -1$

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G \rho_0}{3}\left(\frac{a_0}{a}\right) + \frac{c^2}{a^2}$$

For very small a the second term dominates $\frac{c^2}{a^2} \gg \frac{8\pi G\rho_0}{3}\frac{a_0}{a}$

$$\frac{\dot{a}^2}{a^2} \simeq \frac{c^2}{a^2} \Rightarrow \dot{a} = c \quad a = ct$$

This is expansion without acceleration (so called Milne Universe)

For large a the first term dominates, i.e. $\frac{8\pi G\rho_0}{3}\frac{a_0}{a} \gg \frac{c^2}{a^2}$, and Universe expands with constant acceleration, according solution of d): $a = At^2 \quad \dot{a} = 2At \quad \ddot{a} = 2A > 0$

Question 2.

!! Define the Ω-parameter! Ω - should be reworded. $\Omega = \frac{\rho}{\rho_{crit}}$ where $\rho_{crit} = \frac{3H^2}{8\pi G}$, corresponds to spatially flat Universe
$\Omega > 1$ corresponds to closed Universe
$\Omega < 1$ corresponds to the open Universe

$kc^2 = -1$

$$\Omega = 1 - \frac{1}{a^2 H^2}$$

$H \sim \frac{1}{t}$ $a \sim t^{\beta}$ In standard Big bang theory $\beta < 1$, because $\ddot{a} < 0$, indeed $\dot{a} \sim \beta t^{\beta - 1}$, $\ddot{a} \sim \beta(\beta-1)t^{\beta-2}$

$\beta > 0$, $t^{\beta-2} > 0$, $\ddot{a} < 0 \Rightarrow \beta - 1 < 0 \Rightarrow \beta < 1$

Hence

$$\Omega = 1 - A t^{2(1-\beta)}$$

Taking into account that by definition $\Omega \geq 0$, Ω decreasing first and then remains to be constant, when $\beta = 1$, i.e. in Milne Universe (See. c))

$kc^2 = 1$

$$\Omega = 1 + \frac{1}{a^2H^2}$$

is increasing until expansion is replaced by contraction and only on stage of contraction asymptotically ~~goes to 1~~ tends to 1 (all the time $\Omega > 1$).

$kc^2 = 0$ $\quad \Omega = 1$ all the time.

~~$\frac{1}{a^2H^2} \sim t^{2(1-\beta)}$, hence $\Omega \to \infty$, if and only if $\beta > 1$, $kc^2 = 1$ and $t \to 0$, i.e. contraction~~

If $kc^2 = 1$

$$\Omega = 1 + \frac{1}{a^2H^2} = 1 + \frac{1}{\dot{a}^2}$$

Hence $\Omega = \infty$ if $\dot{a} = 0$ this is the moment when expansion turns to contraction.

(b) According to Big Bang $|\Omega-1|$ is actually increases with expansion, while According to observations Ω is close to 1 now, which means that in the past it should be extreamly close to 1, say $|\Omega-1| < 10^{-17}$ at the moment of nucleosyntesis. There is no explanation of such "tuning" in standard model.

(c) In inflationary models $H \simeq const$, and $a \sim e^{Ht}$. Thus $|\Omega-1| \sim \frac{Kc^2}{a^2}$ i.e. Ω exponentially approaches to 1. Does not matter how big it was in the past, today it should be close to 1 in agreement with observations

d) ~~Ω~~ $\Omega_0 = 2 \Rightarrow kc^2 = 1$

$H_0 = \text{const} \qquad 2 - 1 = \frac{1}{H_0^2 a_0^2}$

$a_0 = \frac{1}{H_0} \qquad t.$

at $t = t_1$ ~~Ω~~ $\left(\frac{a_1}{a_0}\right)^3 = 10^{80}$

$a_1 = a_0 \cdot 10^{\frac{80}{3}}$

$\Omega_1 - 1 = \frac{1}{H_0^2 a_0^2 \cdot 10^{\frac{160}{3}}} = 10^{-53}$

$H_0^2 a_0^2 = 1$

(e) $\Omega_{1000\,year} - 1 = \frac{1}{H_0^2 a_0^2} e^{-2H_0(t_1 - t_0)} =$

$= \frac{e^{-2H_0 \cdot 10^3 \cdot 3 \cdot 10^7 \, sec}}{H_0^2}$

$\frac{1}{H_0^2} e^{-2H_0 \cdot 10^{-36} \, sec} = 1$

$$\Omega_{1000yer} - 1 \simeq e^{-6 \cdot H_0 \cdot 10\, s \;+ 2H_0 \cdot 10}$$

$$H_0 = \frac{1}{t_0} = 10^{36}\, s^{-1}$$

$$\Omega_{100ye} - 1 = e^{-6 \cdot 10^{46}}$$

Very close to 1!

Question 3 -12-

a)

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho + \frac{8\pi G}{3}\Lambda - \frac{kc^2}{a^2}$$

$$\left(\frac{\dot{a}^2}{a^2}\right)^{\cdot} = \frac{8\pi G}{3}\dot{\rho} + \frac{2kc^2}{a^3}\dot{a}$$

~~$$\frac{\ddot{a}}{a^2} - \frac{2\dot{a}^2}{a^3} = \frac{8\pi G}{3} \Big| -\frac{3\dot{a}}{a}$$~~

$$2\frac{\dot{a}}{a}\left(\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2}\right) = \frac{8\pi G}{3}\left[-\frac{3\dot{a}}{a}\left(\rho + \frac{p}{c^2}\right)\right] + \frac{2kc^2}{a^2}\frac{\dot{a}}{a}$$

Dividing by $\frac{2\dot{a}}{a}$

$$\frac{\ddot{a}}{a} = \frac{\dot{a}^2}{a^2} - 4\pi G\left(\rho + \frac{p}{c^2}\right) + \frac{kc^2}{a^2} =$$

$$= \frac{8\pi G}{3}\rho + \frac{8\pi G}{3}\Lambda - 4\pi G\left(\rho + \frac{p}{c^2}\right) - \frac{kc^2}{a^2} + \frac{kc^2}{a^2}$$

$$\frac{\ddot{a}}{a} = -4\pi G\left(\rho + \frac{p}{c^2} - \frac{2}{3}\rho\right) + \frac{8\pi G}{3}\Lambda = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{8\pi G}{3}\Lambda$$

b) The age of the Universe is

$t_0 = \int_0^{t_0} dt = \int_0^{a_0} \frac{da}{aH}$ in the Universe with

$\Lambda = 0$, $p = 0$

$$H = \sqrt{\frac{8\pi G}{3}\frac{\rho_0 a_0^3}{a^3} - \frac{kc^2}{a^2}}, \text{ hence}$$

$$aH = \sqrt{\frac{B}{a} - kc^2}, \quad B = \frac{8\pi G \rho_0 a_0^3}{3}$$

$$t_0 = \int_0^{a_0} \frac{\sqrt{a}\,da}{\sqrt{B - kc^2 a}} = I = \int_0^{a_0} f(a)\,da$$

When $a \to 0$ $f(a) \to \frac{a^{1/2}}{B^{1/2}} \sim t^{1/3}$

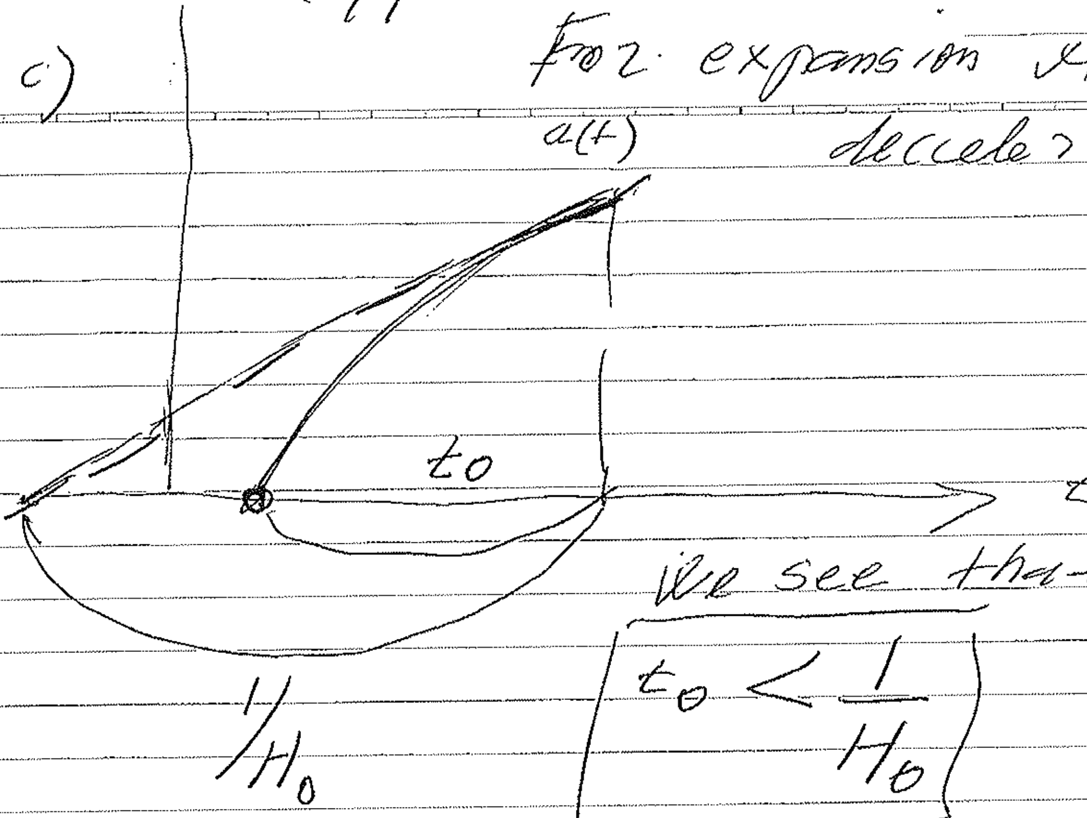
To-day $\frac{8\pi G \rho_0 a_0^3}{3 a_0^3} > \frac{1}{a_0}$, $B > \frac{1}{a}$, $|kc^2| = 1$ Which means that even if $kc^2 = 1$ $B - a > 0$ in the past, Hence I converges and is finite.

~~c) As was shown in notes [illegible] in one of problem sheets~~

~~$t_0 = \frac{2}{3H_0}\left[1 + \frac{1}{5}(1 - \Omega_0)\right]$~~

~~Even if we don't know Ω_0~~

c) For expansion with ( ) deceleration



We see that

$$t_0 < \frac{1}{H_0}$$

d) From a) one can see that Λ-term for positive Λ corresponds to repulsive force, hence for given H deceleration is smaller and the Universe needs more time to slow down for observed to-day rate of expansion.

e) In this case — Replacing $a < a_0$, $I > \ldots$

$$H^2 = \frac{8\pi G \rho_0}{3}\left(\frac{a_0}{a}\right)^3 + \frac{8\pi G}{3}\Lambda = \frac{B}{a^3} + \frac{8\pi G}{3}\Lambda$$

$$\frac{da}{dt} = \sqrt{\frac{B}{a} + \frac{8\pi G}{3}\Lambda a^2} \quad \text{(see d)} \quad t_0 = \int_0^{a_0} \frac{\sqrt{a}\, da}{\sqrt{B + \frac{8\pi G \Lambda a^3}{3}}}$$

$$> \frac{1}{\sqrt{B + \frac{8\pi G \Lambda a_0^3}{3}}} \cdot \frac{2}{3} a_0^{3/2}$$

/ / ( )

$$t_0 > \frac{2}{3} \frac{a_0^{3/2}}{\left[\frac{8\pi G}{3}\rho_0 a_0^3 + \frac{8\pi G \Lambda a_0^3}{3}\right]} =$$

$$= \frac{1}{\left[6\pi G(\rho_0 + \Lambda)\right]^{1/2}}$$

Question 4.

a) $p > mc$, or $k_B T > mc^2$

Mistake: the energy density of such a particle – does not make any sense, could be

the energy density of such particle

(the answer $\rho \sim kT \cdot n \sim$ ↑ number density always $\sim 1/a^3$

$\sim \frac{1}{a} \cdot \frac{1}{a^3} \sim \frac{1}{a^4}$ or $\frac{1}{V^{4/3}}$ or

the energy of such a particle

(the answer

$E = \sqrt{m^2c^4 + p^2c^2}$ where m and p are mass and momentum of the particle.

$\sim pc \propto T \sim \frac{1}{a} \sim \frac{1}{V^{1/3}}$

$$\varepsilon \sim \frac{1}{t^2} \sim \frac{1}{a^4} \Rightarrow a \sim t^{1/2}$$

b) $$|\Omega - 1| = \frac{1}{a^2 H^2} \sim t^{2-1} \sim t$$ when $t \to 0$ $\Omega \to 1$

c) $$T \sim \frac{1}{a} \sim \frac{1}{t^{1/2}} \qquad \frac{T_1}{T_2} = \left(\frac{t_2}{t_1}\right)^{1/2}$$

$$\frac{t_2}{t_1} = \left(\frac{T_1}{T_2}\right)^2 \qquad t_2 = t_1 \left(\frac{T_1}{T_2}\right)^2 = 3$$

$$= 1\,\text{year} \left(\frac{2 \cdot 10^{6} K}{10^{27} K}\right)^2 = 3 \cdot 10^7 s \cdot 4 \cdot 10^{-42} =$$

$$\approx 10^{-34} s$$

d) As follows from kinetic of ionization and recombination

$$\frac{n_{ion}}{n_e} = \frac{n_\gamma}{n_B} e^{-\frac{E_{ion}}{k_B T}}$$

Taking into account that $\frac{n_\gamma}{n_B} \approx 10^9$

For $n_{ion} \ll n_e$ we got

H^+ $E_{ion} \simeq 13.6\,eV$

neutral atom which allows to interact with radiation

hence $T_{rec} \lesssim \frac{13.6}{k_B \ln(10^9)}$

$\approx 7300\,K$ / proper value actually is $3000\,K$ and it would be hard to explain to students that the factor 2 comes from 2-photon ionization !!!

We know from c) that

$$\frac{t_2}{t_1} = \left(\frac{T_1}{T_2}\right)^2, \quad t_2 \simeq 1\,year$$

$$\left(\frac{2 \cdot 10^6 K}{3 \cdot 10^3 K}\right)^2 \simeq \frac{4}{9} \cdot 10^6 \simeq 400\,000$$

Total Ω in the very early universe $\Omega \simeq 1$

e) ρ_m is the density of very massive particles

$$\rho_m = (\Omega_m)_*\left(\frac{a_*}{a}\right)^3 \simeq \Omega_m \left(\frac{t_*}{t}\right)^{3/2}$$

ρ_R is the density of relativistic particles

$$\rho_R \simeq (\Omega_R)_* \left(\frac{a_*}{a}\right)^4 \simeq \Omega_R \left(\frac{t_*}{t}\right)^2$$

$\Omega = \Omega_m + \Omega_R$ ( )

$$\frac{\rho_m}{\rho_R} \simeq \frac{\Omega_m}{\Omega_m}\left(\frac{a}{a_x}\right) \sim 10^{-18}\left(\frac{t}{t_x}\right)^{1/2}$$

$$T = 2 \cdot 10^{6} K \sqrt{\frac{1\ \text{year}}{t}} \simeq$$

$$\simeq 2.10^{6} \, 5.10^{3} \sqrt{\frac{1s}{t}} \simeq$$

$$\simeq 10^{10} K \left(\frac{t}{1s}\right)^{-1/2}$$

$$T_{t=10^{-20}s} \approx 10^{10} K . 10^{10} \simeq$$

$$\simeq 10^{20} K$$

$$\frac{t_2}{t_1} = \left(\frac{T_1}{T_2}\right)^2 \simeq \left(\frac{10^{20}}{10^{10}}\right)^2 =$$

$$= 10^{20}$$

$$t_2 = 10^{20} . 10^{-20} = 1s$$

$$\frac{\rho_m}{\rho_R} \simeq \frac{\rho_m}{\rho_{crit}} \simeq 10^{-18}\left(\frac{1}{10^{-20}}\right)^{1/2} \simeq 10^{-8}.$$

$\frac{\rho_m}{\rho_R} \sim 1$ $\quad 10^{8}\left(\frac{t}{15}\right)^{1/2} \simeq 1$ ( )

$t \simeq 10^{16} s \simeq 3 \cdot 10^{8}$ years!

Actually This means that the will never dominate because relativistic stage should be over $\sim 10^{4}$ years. But how students can know this? $10^{4} \sim t^{2/3}$ (6)

10^{4} years

Q5, 9) 1) CMB is nearly isotropic

2) The spectrum of CMB is perfect black body spectrum.

b) Assume that the Universe is always transparent, i.e. short period ~~when universe expa~~ before decoupling is negligible in comparison with present horizon. Assume also that the main contribution to integral comes from dust-stage when $a \sim t^{2/3}$, hence

$$d_H(t) = ct^{2/3} \int_0^t \frac{dt'}{t'^{2/3}} =$$

$$3ct^{2/3} \cdot t^{1/3} = 3ct$$

c) 'Rescaled horizone distance' d_{RH} is the size of the part of the Universe within present horizon it had in the moment of decoupling. Taking into account $d_H \sim t$, while physical distances $d \sim \frac{1}{z} \sim t^{2/3}$

$$\frac{d_{RH}}{d_H} \sim \left(\frac{t_d}{t_0}\right)^{2/3} \sim z_d^{-1} \sim 10^{-3}$$

/ / ( )

d) Temperature is the same ~~was~~ from all parts of ~~las~~ the surface of last scattering dispite the fact that are not causally ~~casualy~~ related with each other because real horizon at the moment of decoupling much smaller than rescaled horizon.

e) For inflation $a \sim e^{+Ht}$

$$d_H(t) = c e^{Ht} \int_0^{t_0} \frac{dt'}{e^{Ht'}} = \frac{c e^{Ht}}{H}\left(1 - e^{-Ht}\right) =$$

$= \frac{c e^{Ht}}{H}$ is getting to be large even for small t

Question 6

a) As the Universe expanded its temperature dropped. Eventually photons lacked sufficient energy to keep hydrogen ionized. The electrons where therefore able to combine with the nuclei with the result that no naked electric charge remained in the Universe. Consequently the photons were no longer able to interact directly with the matter and were propagate unhindered. This is decoupling epoch, since the matter and radiation effectively decoupled from on at this time. The radiatio has remained essentially undisturbed through to the present day, although it has lost energy due to the expansion of the Universe. This is CMB

This is analogous to the process that prevented the formation of ~~[illegible]~~ nucleons during epoch of nucleosyntesis. But this epoch the typical temperature is about 10^{10} K

/ / ( )

while during decoupling it is about $3.10^3 K$.

6) The overal effect of the quantum fluctuations is that the cause inflation to end at different times in different regions of the Universe. It implies that different regions inflate by different amounts, hence, the density of matter will vary throughout the Universe after inflation. The denser regions would have had a slightly higher temperature than the less dense regions. That is why CMB is slightly anisotropic

c) The position of the first peak in the distribution of Anisotropy of CMB over spherical multipoles depends linear on $\sqrt{\Omega_0}$, hence measuring the position of this peak we can determine the density of the Universe.

d) From spectroscopic measurements of spectral lines from remote galaxies is possible to determine redshifts of the lines, which is explained by Doppler effect, hence it is possible to determine radial velocities of Galaxies. By comparison with measuments of distances to remote object it is possible to confirm observationally Hubble's law: $v = HR$.

e) A cosmological constant ~~changes~~ effects the value of the deceleration parameter leading to ~~[illegible]~~ observable deviations from Hubble's law. The evidence comes from the type Ia supernovae redshift surveys. It was found that supernovae are dimmer than expected on the basis of a standard cosmology.