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Queen Mary
University of London

M. Sci. Examination by course unit 2009

MTH703U Advanced Cosmology

Duration: 3 hours

Date and time: 5th May 2009, 1430-1730

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

You may attempt as many questions as you wish and all questions carry equal marks. Except for the award of a bare pass, only the best 4 questions answered will be counted.

Calculators ARE permitted in this examination. The unauthorized use of material stored in pre-programmable memory constitutes an examination offence. Please state on your answer book the name and type of machine used.

Complete all rough workings in the answer book and cross through any work which is not to be assessed.

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Exam papers must not be removed from the examination room.

Examiner(s): J. E. Lidsey

You are reminded of the following information, which you may use without proof:

The following constants may be assumed:

Speed of light, $c = 3.0 \times 10^8\,\mathrm{m\,s^{-1}}$
Gravitational constant, $G = 6.67 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$
Boltzmann's constant, $k_B = 1.38 \times 10^{-23}\,\mathrm{JK^{-1}}$
Radiation constant, $\alpha = 7.565 \times 10^{-16}\,\mathrm{J\,m^{-3}\,K^{-4}}$
Proton mass-energy, $m_p c^2 = 938.3\,\mathrm{MeV}$
Neutron mass-energy, $m_n c^2 = 939.6\,\mathrm{MeV}$
Mega Parsec, $1\,\mathrm{Mpc} = 3.09 \times 10^{22}\,\mathrm{m}$
Hubble time, $H_0^{-1} = 9.8 \times 10^9 h^{-1}\,\mathrm{yr} = 3.09 \times 10^{17} h^{-1}\,\mathrm{s}$
The Conversion Factor, $1\,\mathrm{eV} = 1.602 \times 10^{-19}\,\mathrm{J}$
The Conversion Factor, $1\,\mathrm{J} = 1\,\mathrm{kg\,m^2\,s^{-2}}$

The following formulae may be assumed:

Friedmann Equation

$$H^2 = \frac{8\pi G}{3}\rho + \frac{8\pi G}{3}\Lambda - \frac{kc^2}{a^2},$$

where $H = \dot{a}/a$ is the Hubble parameter, a is the scale factor of the universe, ρ is the mass density, Λ is the cosmological constant, k is a constant and overdots denote time derivatives.

Conservation Equation

$$\dot{\rho} + 3H\left(\rho + \frac{p}{c^2}\right) = 0,$$

where p represents the pressure of the matter in the universe.

Acceleration Equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right).$$

Question 1

(a) *[4 marks]* Give the definition of the Ω-parameter in terms of the critical density of the universe. Show that in terms of Ω, the Friedmann equation can be expressed in the form

$$\Omega - 1 = \frac{kc^2}{\dot{a}^2}.$$

(b) *[4 marks]* A university professor develops a theory where the universe is dominated by a new form of matter with an equation of state $p = -\frac{2}{3}\rho c^2$. Starting from the conservation equation, derive how the density of this matter varies with the scale factor of the universe.

(c) *[5 marks]* Assuming that the universe is spatially flat and dominated by the new form of matter described in part (b), show that the scale factor grows with time as $a \propto t^2$.

(d) *[6 marks]* Suppose that the universe expands as $a \propto t^2$ for all values of Ω. Sketch how Ω would vary with time for the three cases where initially $\Omega > 1$, $\Omega = 1$ and $\Omega < 1$, respectively. (Explain your reasoning).

(e) *[6 marks]* What is the flatness problem of the big bang theory? Explain how this problem could be solved if the universe did indeed expand as $a \propto t^2$ for a finite time in its very distant past. By what factor must the volume of the universe increase during this era to solve the flatness problem?

Question 2

(a) *[5 marks]* Explain why the inverse of the Hubble constant, H_0^{-1}, provides an upper limit to the age of the universe when $\Lambda = 0$.

(b) *[4 marks]* For a spatially flat, pressureless universe with vanishing cosmological constant, write down how the scale factor varies with time. Hence, derive an expression for the age of such a universe in terms of the Hubble constant.

(c) *[6 marks]* Verify that the solution to the Friedmann equation for a positively-curved, pressureless universe with vanishing cosmological constant can be expressed in the parametric form

$$a(\theta) = \frac{4\pi G\rho_0 a_0^3}{3}(1 - \cos\theta), \qquad t(\theta) = \frac{4\pi G\rho_0 a_0^3}{3}(\theta - \sin\theta),$$

where ρ_0 and a_0 represent the present-day values of the matter density and scale factor, respectively. (You may assume $kc^2 = +1$.)

(d) *[5 marks]* Given the solution in part (c), derive an expression relating the age of a positively-curved, pressureless universe to its size, $a_{\max}$, at the moment when such a universe starts to recollapse. What is the lifetime of such a universe when expressed in terms of $a_{\max}$?

(e) *[5 marks]* Explain, briefly, why the age of a negatively-curved universe is greater than that of a spatially flat universe with the same value of the Hubble constant.

Question 3

(a) *[4 marks]* Explain what is meant by isotropy and homogeneity in the universe.

(b) *[5 marks]* Summarize the key properties of the cosmic microwave background radiation.

(c) *[5 marks]* Describe, briefly, the key physical processes that resulted in the formation of the cosmic microwave background at the epoch of decoupling.

(d) *[6 marks]* The horizon distance at a time t (corresponding to the furthest distance a photon could have travelled since the big bang) is given by

$$d_H(t) = a(t)\int_0^t \frac{c}{a(t)}dt.$$

Explain what is meant by the 'rescaled horizon distance'. For a pressureless universe with critical density and vanishing cosmological constant, verify that the rescaled horizon distance corresponding to the decoupling era, $\tilde{d}_H(t_{\rm dec})$, satisfies the condition

$$\tilde{d}_H(t_{\rm dec}) \approx 0.1ct_0.$$

(e) *[5 marks]* Explain why the approximate equality in part (d) is a problem for the big bang theory.

Question 4

(a) *[3 marks]* State how the density of a relativistic particle species varies with the scale factor and temperature of the universe, respectively.

(b) *[4 marks]* What is the physical reason why the density of radiation decreases more rapidly than that of non-relativistic matter as the universe expands?

(c) *[4 marks]* Given that the universe was dominated by relativistic matter when it was one second old and had a temperature $T \approx 10^{10}$K at that time, show that at some earlier time, t, the temperature was given by

$$\frac{T}{10^{10}\,\mathrm{K}} \approx \sqrt{\frac{1\,\mathrm{sec}}{t}}.$$

(d) *[4 marks]* Estimate the age and temperature of the universe when it was the size of a golf ball.

(e) *[4 marks]* Estimate the age and temperature of the universe when the neutrons started to behave non-relativistically.

(f) *[6 marks]* A cosmologist predicts that non-relativistic, stable particles are formed when the universe was 10^{-15} seconds old. Assuming that the density of these particles at the time of formation was 10^{-6} times that of the critical density, estimate the age of the universe when these particles come to dominate the density. (You may assume that all other particle species in the universe were behaving relativistically.)

Question 5

(a) *[3 marks]* Explain what is meant by cosmological redshift and state how it is related to the scale factor of the universe.

(b) *[5 marks]* The proper distance to a galaxy with redshift z is given by the integral

$$D_P = c\int_0^z \frac{dz}{H(z)},$$

where $H(z)$ denotes the Hubble parameter as a function of z. Consider a spatially flat, pressureless universe with vanishing cosmological constant. Use the Friedmann equation to determine how the Hubble parameter varies with redshift and hence show that the proper distance to a galaxy with a redshift $z = 3$ is $D_P = c/H_0$.

(c) *[6 marks]* Give the definition of the deceleration parameter. Show that in a negatively curved universe with a general equation of state $p = (\gamma - 1)\rho c^2$, where $0 \leq \gamma \leq 2$ is a constant, the deceleration parameter satisfies

$$q < \frac{3\gamma - 2}{2}.$$

(d) *[6 marks]* Summarize the main observational evidence that the universe today is dominated by a non-zero cosmological constant with a density about 70% the critical density.

(e) *[5 marks]* Prove that if the present density of pressureless matter is less than the critical density, the universe will never recollapse if such a cosmological constant does indeed exist.

Question 6

(a) *[6 marks]* Describe, briefly, the three primary sources of anisotropy in the temperature of the cosmic microwave background radiation.

(b) *[5 marks]* What is the main evidence that there is baryonic dark matter in the universe?

(c) *[5 marks]* Describe the most important steps in the formation of primordial helium.

(d) *[4 marks]* List the following events in chronological order: the formation of large-scale structure in the universe, the decoupling era, the end of inflation, the primordial nucleosynthesis era, the epoch of matter-radiation equality.

(e) *[5 marks]* Outline, quantitatively, how the density perturbations in the dark matter evolve during the radiation- and matter-dominated eras. What restrictions must be satisfied by the density perturbation if galaxies are to have formed by the present time?

End of Paper

(book)

ASTM 108

2009 EXAM

1 a) $\Omega \equiv \frac{\rho}{\rho_{crit}}$, $\rho_{crit} = \frac{3H^2}{8\pi G}$

$$H^2 = \frac{8\pi G}{3}\rho_{total} - \frac{kc^2}{a^2} \Rightarrow 1 = \Omega - \frac{kc^2}{a^2H^2} \Rightarrow \Omega - 1 = \frac{kc^2}{\dot{a}^2} \quad 2$$

(4)

b) & c) [new similar to c/w]

b) $P = -\frac{2}{3}\rho c^2$

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + P/c^2) \Rightarrow d\rho + 3\frac{da}{a}\frac{1}{3}\rho = 0 \quad 2 \Rightarrow \int\frac{d\rho}{\rho} = -\int\frac{da}{a}$$

$$\Rightarrow \rho \propto \frac{1}{a} \quad 2$$

(4)

c) $$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho \Rightarrow \frac{\dot{a}^2}{a^2} \propto \frac{1}{a} \Rightarrow a^{-1/2}\dot{a} = \text{constant} \quad 2$$

$$a \propto t^n,\ \dot{a} \propto t^{n-1} \Rightarrow t^{-n/2}\,t^{n-1} = \text{const} \rightarrow n = 2$$

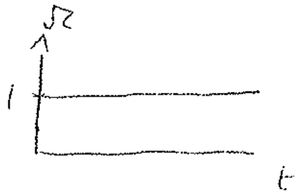
2 (5)

d) Need to realise $\ddot{a} > 0$ if $a \propto t^2 \Rightarrow$ expansion rate getting faster $\Rightarrow \dot{a}^2$ is increasing as $t \uparrow \Rightarrow |\Omega - 1|$ decreases as $t \uparrow$

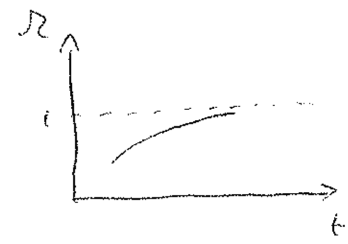
ie Ω moves towards unity

[discussed in lectures]

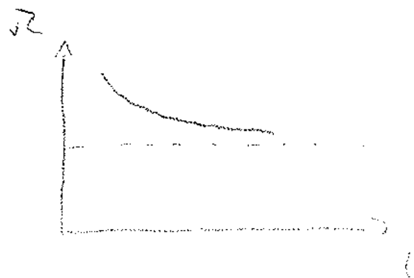
$\Omega = 1, \Rightarrow \Omega = 1$ always



$\Omega < 1 \Rightarrow \Omega < 1$ always since sign $(\Omega - 1)$ = sign k



$\Omega > 1 \Rightarrow \Omega > 1$ always



1e) In hot big bang, expansion decelerates due to gravitational attraction of matter $\Rightarrow$ $\dot{a}$ decreases $\Rightarrow$ $\frac{1}{\dot{a}}$ increases $\Rightarrow$ $|\Omega - 1|$ moves away from zero $\Rightarrow$ Ω moves away from unity.

However, if $\Omega_0 \simeq O(1)$ today $\Rightarrow$ must have been extremely close to unity at very early times $\Rightarrow$ a problem of fine tuning

If $a \propto t^2$ for long enough in early universe, Ω pushed so close to unity, that it can still be close to one today.

To solve flatness problem, need $\frac{a_{end}}{a_{start}} > 10^{26}$ [from lectures] $\Rightarrow$

1 mark

volume increase by factor $> 10^{78}$

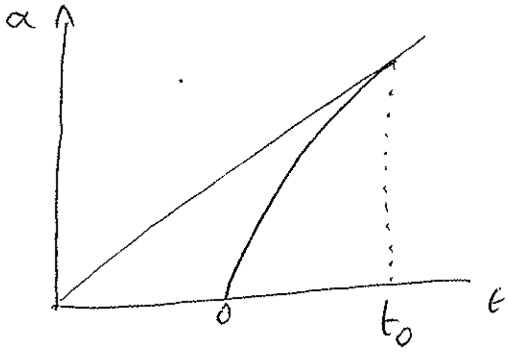
[discussed in lectures]

2 a) Consider linear expansion $a(t) = a_0 \left(\frac{t}{t_0}\right)$ [book]

$\Rightarrow H = \frac{\dot{a}}{a} = \frac{1}{t} \Rightarrow t_0 = \frac{1}{H_0}$

Real universe, if $\rho \geq 0$ & $p > 0$, acc^n equation $\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$

$\Rightarrow \ddot{a} < 0 \Rightarrow a(t)$ is always concave downwards :



2

$\Rightarrow$ age of universe always less than linearly expanding universe.

[book]

b) Need to recall $a \propto t^{2/3}$ 2 $\Rightarrow H = \frac{\dot{a}}{a} = \frac{2}{3t} \Rightarrow t_0 = \frac{2}{3} H_0^{-1}$ 2

c) $H^2 = \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2}$ [new similar to c/w]

Need to recall $\rho = \rho_0 \left(\frac{a_0}{a}\right)^3$ 1 $\Rightarrow \dot{a}^2 = \frac{8\pi G}{3}\rho a^2 - kc^2 = \frac{8\pi G}{3}\frac{\rho_0 a_0^3}{a} - 1$

$\dot{a} = \frac{da}{d\theta}\left(\frac{dt}{d\theta}\right)^{-1} = \frac{\sin\theta}{1-\cos\theta}$ 2 $\Rightarrow \frac{\sin^2\theta}{(1-\cos\theta)^2} = \frac{8\pi G}{3}\rho_0 a_0^3 \cdot \frac{3}{4\pi G \rho_0 a_0^3} \frac{1}{1-\cos\theta} - 1$

$\Rightarrow \sin^2\theta = 2(1-\cos\theta) - (1-\cos\theta)^2$

$\Rightarrow \sin^2\theta = 1 - \cos^2\theta$ 3

2 d) $a = a_{max}$ when $\cos\theta = -1$ ie $\theta = \pi$ [new]

$\Rightarrow a_{max} = \frac{8\pi G}{3} \rho_0 a_0^3$.

Occurs when $t = t_{max} = \frac{4\pi G}{3} \rho_0 a_0^3 \, \pi = \frac{\pi}{2} a_{max}$ 3

Big crunch when $\cos\theta = 1 \Rightarrow \theta = 2\pi$

$\Rightarrow$ lifetime of this universe is $2 t_{max} = \pi a_{max}$ 2

[OK if students realise $a(\theta)$ is symmetric about $\theta = \pi$ & then quote result].

~~2 e) For given H_0^{-1}, decreasing~~

[new +ve curvature case discussed in lectures]

2 e) Negative - curvature $\Rightarrow$ density < critical density

For given H_0^{-1}, decreasing present-day value of density ~~decreases~~ increases age since less matter in universe $\Rightarrow$ deceleration of expansion is lower $\Rightarrow$ takes more time for expansion rate & $\Rightarrow$ Hubble parameter to decrease to present values.

[book]

3a) Isotropy implies no preferred direction in universe &

$\Rightarrow$ universe looks same regardless of direction when viewed

from particular point. 2

Homogeneity $\Rightarrow$ at a given instant universe appears same

everywhere 2

[discussed lectures]

b) CMB spectrum is blackbody with a temperature $T = 2.728K$.
It appears to originate from all directions in the universe
It is highly isotropic ie temperature is ~~same~~ almost same regardless

of direction of origin

Not precisely isotropic & contains temperature fluctions of the

order $\frac{\Delta T}{T} \approx 10^{-5}$.

Anisotropy is peaked over angular scales separated by $\simeq 1°$. (5)

c) Decoupling : important points are that before decoupling temp [discussed lectures]

of radiation so high that any neutral Hydrogen was immediately

ionized. Universe expands, temp drops $\Rightarrow$ energy of radiation drops.

Eventually e^- & p^+ could combine to form neutral H $\Rightarrow$ no naked

electric charge in universe $\Rightarrow$ radiation not interact with matter

$\Rightarrow$ became decoupled. (5)

3d) Rescaled horizon distance is horizon distance corresponding to some earlier time $t_1 < t_0$ scaled to its present-day value:

$$\tilde{d}_H(t_1) = \frac{a_0}{a(t_1)} d_H(t_1) = a_0 \int_0^{t_1} \frac{c}{a(t)} dt \quad (2)$$

[new similar to c(w)]

Assume universe dominated by pressureless matter for most of its history $\Rightarrow a \propto t^{2/3}$ (1)

$$\Rightarrow d_H(t_0) = a \int_0^{t_0} \frac{c}{a} dt = t_0^{2/3} c \, 3 t_0^{1/3} = 3ct_0 \quad (1)$$

Use $a(t) = a_{dec} \left(\frac{t}{t_{dec}}\right)^{2/3}$

$$\Rightarrow \tilde{d}_H(t_{dec}) = c a_0 \int_0^{t_{dec}} dt \frac{t_{dec}^{2/3}}{a_{dec} t^{2/3}} = 3ct_0 \cdot \left(\frac{t_{dec}}{t_0}\right)^{1/3} \quad (2)$$

$$\left.\begin{array}{l} t_{dec} \simeq 300{,}000 \text{ yrs} \\ t_0 \simeq 10^{10} \text{ yrs} \end{array}\right\} \quad \frac{t_{dec}}{t_0} \simeq 3 \times 10^{-5} \quad (1)$$

$$\Rightarrow \tilde{d}_H(t_{dec}) \simeq 0.1 \, ct_0 \qquad (6)$$

e) $\tilde{d}_H(t_{dec}) < 2 d_H(t_0)$. This is horizon problem. [discussed in lecture]

Due to finite age of universe, ∃ limit to how far light could have travelled since big bang ⇒ how far we can see. This corresponds to horizon distance @ a given time. When we look in different directions in universe at regions very far away, horizon distance much smaller then it today ⇒ these regions could not have been able to communicate ⇒ [illegible] processes could not have operated between them ⇒ could not [illegible] [illegible] state. [illegible]
[illegible]

4 a) $\rho_{rel} \propto \frac{1}{a^4}$ 2 $\rho_{rel} \propto T^4$ 1 [book] (3)

b) density decreases by factor $1/a^3$, due to expansion. [book]
extra factor of a^{-1} since the wavelength of radiation stretched in proportion to scale factor ^{2}, $\lambda \propto a$, due to cosmic expansion, and energy of wave $E \propto \frac{1}{\lambda} \propto \frac{1}{a}$. 1 (4)

c) early universe, can assume $\Omega = 1$ & $\Rightarrow a \propto t^{1/2}$, 1 (discussed in lectures)
since aT = constant 2 $\Rightarrow t^{1/2} T$ = const
normalizing on in $T \simeq 10^{10}$ K @ $t \simeq 1$ sec $\Rightarrow$

$$\frac{T}{10^{10} K} \simeq \left(\frac{t}{sec}\right)^{-1/2}$$ 1 (4)

d) golf ball $\simeq$ 5 cm in diameter. [d & e) new similar to c/w]
Present size of universe $\simeq 10^{28}$ cm
$aT = a_0 T_0$ = constant $\Rightarrow T_{golf} \simeq \frac{a_0}{a_{golf}} T_0 \simeq \frac{10^{28}}{5} \times 3$ K 2

$\Rightarrow T_{golf} \simeq 6 \times 10^{27}$ K
$\Rightarrow t_{golf} \simeq \left(\frac{6 \times 10^{27}}{10^{10}}\right)^{-2} \simeq 3 \times 10^{-36}$ s (4)
2

e)

e) n^0 non-relativistic when $mc^2 \simeq k_B T$ 2 new similar to c/w

From rubric, $mc^2 \simeq \cancel{934}\ 940\ \text{MeV} \simeq 940 \times 10^6 \times 1.6 \times 10^{-19}\ \text{J}$

$$\simeq 1.5 \times 10^{-10}\ \text{J}$$

$$\Rightarrow \quad T \simeq \frac{mc^2}{k_B} \simeq \frac{1.5 \times 10^{-10}}{1.38 \times 10^{-23}} \simeq 10^{13}\ \text{K}$$

$$t \simeq \left(\frac{T}{10^{10}\text{K}}\right)^{-2} \simeq \left(\frac{10^{13}}{10^{10}}\right)^{-2} \simeq 10^{-6}\ \text{sec.}$$ (4)

f) $t_{form} \simeq 10^{-15}$ s new similar to c/w

early universe arena $\Rightarrow$ effects of spatial curvature negligible.

$\Rightarrow \Omega_{tot} = \Omega_{rel} + \Omega_{non\text{-}rel}$.

$$\frac{\rho_{non\text{-}rel}}{\rho_{rel}} \propto a \Rightarrow \frac{\rho_{non\text{-}rel}}{\rho_{rel}} = \left(\frac{\rho_{nr}}{\rho_r}\right)_{form} \frac{a}{a_{form}} \simeq \left(\frac{\Omega_{nr}}{\Omega_r}\right)_{form} \frac{a}{a_{form}}$$ 2

But $\Omega_{rel,form} \gg \Omega_{non\text{-}rel,form} \Rightarrow \Omega_{rel,form} \simeq 1$

$$\Rightarrow \quad \frac{\rho_{nr}}{\rho_r} = 10^{-6} \frac{a}{a_{form}} \simeq 10^{-6}\left(\frac{t}{t_{form}}\right)^{1/2} \simeq 10^{-6}\left(\frac{t}{10^{-15}}\right)^{1/2}$$ 2

non-rel dominate when $\rho_{nr} \simeq \rho_r$

$$\Rightarrow \quad t_{dom} \simeq \left(\frac{10^{-15}}{10^{-12}}\right)^{2} \simeq 10^{-18}\ \text{s}$$ 2

(6)

5a) stretching of wavelength of rad^n due to cosmic expansion

$1+z = \frac{a_0}{a}$ 2 [book] (3)

b) $\Lambda = k = 0 \Rightarrow \rho \propto \frac{1}{a^3} \Rightarrow H^2 \propto \rho \propto \frac{1}{a^3} \propto (1+z)^3$

new similar to c/w

Normalize at present time, $H(z) = H_0 (1+z)^{3/2}$ 3

$$D_p = c\int_0^z \frac{dz}{H(z)} = \frac{c}{H_0}\int_0^z (1+z)^{-3/2} = -\frac{2c}{H_0}\left\{\left[1+z\right]^{-1/2} - 1\right\}$$

$z=3 \Rightarrow D_p = \frac{2c}{H_0}\left(1 - \frac{1}{\sqrt{1+3}}\right) = \frac{c}{H_0}$ 2 (5)

c) $q = -\frac{a\ddot{a}}{\dot{a}^2} = -\frac{\ddot{a}}{aH^2}$ 2 [book]

From rubric, $\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3P/c^2\right)$ [discussed in lectures]

$P = (\gamma-1)\rho c^2 \Rightarrow q = -\frac{\ddot{a}}{aH^2} = \frac{4\pi G}{3H^2}\left(\rho + (3\gamma - 3)\rho\right)$ 2 $\Rightarrow q = \frac{\Omega}{2}(3\gamma - 2)$ at once!

$H^2 = \frac{8\pi G}{3}\rho + \frac{1}{a^2}$ (open universe) $\Rightarrow q = \frac{4\pi G(3\gamma\rho - 2\rho)}{8\pi G\rho + 3/a^2}$

$\Rightarrow q < \frac{4\pi G(3\gamma - 2)\rho}{8\pi G\rho}$ since $3/a^2 > 0$

$\Rightarrow q < \frac{3\gamma - 2}{2}$ 2 (6) [discussed lectures].

d) luminosity - distance redshift surveys of type Ia supernovae (SN)

Type Ia thought to be good standard candles, Redshift $z \gtrsim 0.1$, deviations from linear Hubble law seen. Deviations $\Rightarrow$ SN are dimmer than expected if $p=0$ $\Rightarrow$ further away $\Rightarrow$ expansion rate higher. explain with cosmological constant.

CMB anisotropies & position of first peak $\Rightarrow \Omega_{tot} = 1$

Galaxy clusters $\Rightarrow \Omega_{matter} \simeq 0.3$ only. Together imply $\Omega_{\Lambda,0} \simeq 0.7$. (6)

e) Recollapse only possible if $k = +1$ 1 $\Rightarrow H^2 = \frac{8\pi G \rho_{mat}}{3} + \frac{8\pi G \Lambda}{3} - \frac{1}{a^2}$

never recollapse if $\frac{8\pi G \Lambda}{3} > \frac{1}{a_0^2}$ (since H^2 positive definite $\forall\, t > t_0$) [derived in lectures]2

$$\Rightarrow \quad \frac{8\pi G \Lambda}{3H_0^2} > \frac{1}{a_0^2 H_0^2}$$

$$\Rightarrow \quad \Omega_{\Lambda,0} > \frac{1}{a_0^2 H_0^2} \qquad \left(\Omega_\Lambda = \frac{8\pi G \Lambda}{3H^2} \text{ etc}\right)$$

2

Friedmann Equation ~~H_0~~ $1 = \Omega_{mat,0} + \Omega_{\Lambda,0} - \frac{1}{a_0^2 H_0^2}$

$\Rightarrow$ no recollapse if $\Omega_{mat,0} < 1$

2.

6 a) Primary Sources : ① gravitational (Sachs-Wolfe) · arises because photons moving out of overdense region on last scattering surface loses energy & is redshifted. 2

② Density (adiabatic) perturbations : coupling of baryonic matter and radiation can result in compression of radiation ⇒ increase in temperature 2

③ Velocity (Doppler) perturbations : ionized plasma has nonzero velocity & undergoes coherent oscillations. Results in Doppler shift of observed frequency of radiation & corresponding shift in temperature 2 (6)

b) density of visible matter ⇔ determined by counting # density of stars in sufficiently large & nearby region ⇒ estimate density ⇒ find $\Omega_{vis} \simeq 0.01$ 2

Primordial nucleosynthesis constrains density of baryons to be 2

$$0.011 < \Omega_B h^2 < 0.015$$

~~Since h...~~

Since $h \simeq 0.65$ ⇒ $\Omega_B \gtrsim 0.03 > \Omega_{vis}$ ⇒ most of baryonic matter is dark. 1

(5)

c) @ $t \ll 1$ sec, $T \gg 10^{10}$K. Temp so high, rad^{n} (photons) break apart any nuclei that may form. n^0 & p^+ in equilibrium:

$n^0 \Leftrightarrow p^+ + e^- + \bar{\nu}_e + 0.8$ meV. Temp falls, reverse reaction harder $\Rightarrow$ # p^+ grows relative to number of n^0, when $k_BT \lesssim 0.8$ meV.

when $k_BT \lesssim 0.1$ meV, ^{4}He can form (and not before) since must wait for ^{2}D to become stable. p^+ & n^0 combine to form D^2 and $D^2 + D^2$ combine to produce He^4_2:

$$p^+ + n^0 \rightarrow D + \gamma \sim \text{photon}$$

$$D^2 + D^2 \rightarrow He^4 + \gamma$$

(5)

d) ~~inflation → matter-radiation equality → decoup~~ !

d) inflation → nucleosynthesis → matter-radiation equality → decoupling → formation large-scale structure (4)

e) dark matter & radiation uncoupled. 1

δ grows logarithmically with time during rad^{n} era 1

$\delta \propto a \propto t^{2/3}$ during matter domination 2

must have $\delta \gtrsim 1$ by present-era (at least) for galaxies to have formed by t_0. 1

(5)