THERMAL AND KINETIC PHYSICS (PHY 214)

Coursework 9: Week 10

ISSUE: Tuesday 30 November, 2010 HAND-IN: Tuesday 07 December, 2010

Students name (top left corner), course title & exercise number and exercise group (top right corner) should appear on every sheet of the submitted coursework and sheets should be firmly held together. A stapler is available if needed from the secretary's office.

Hand-in of worked exercises must take place by 4:00 p.m. on the above date at the labeled box provided outside the Teaching Administrators office on the first floor.

This time will be strictly adhered to and no late working will be accepted without written explanation to the course organiser. The solutions will appear shortly after this time on the TKP website.

Each coursework is worth 40 marks and the aggregate coursework mark will count 10% towards the final mark. An indicative mark scheme is given with each question. **Note**: ***I want to see the method of solution. No credit will be given for simply writing down the answer.***

Students should collect new exercise sheets in the Tuesday lecture or download them from the Web. Marked exercises will be returned in exercise classes or via the box outside the Teaching Administrators office on the first floor.

QUESTION 1: (24 marks) (Based on question 8.6 in Appendix 4/D of Finn)

The equation of state of an elastic band may be written as

$$\mathcal{F} = aT\left[\frac{L}{L_0} - \left(\frac{L_0}{L}\right)^2\right]$$

where $\mathcal{F}$ is the tension in the stretched band, L is the stretched length, L_0 is the resting or unstretched length of 1 m and a is a constant with the value 1.3×10^{-2} N K^{-1} .

(a) Using the appropriate expression for the infinitesimal work done in stretching the band by a distance dL, write down the First Law for the elastic band and obtain the thermodynamic identity for elastic. From the thermodynamic identity show that the infinitesimal change in the Helmholtz free energy $F = U - TS$ has the form

$$dF = -SdT + FdL$$ **[2mks]**

Hence obtain an appropriate Maxwell relation from dF and show that for a stretched band

$$\left(\frac{\partial S}{\partial L}\right)_T < 0$$ **[5mks]**

An elastic band is made of an entangled jumble of very long macromolecules. Explain in a few brief sentences why should its entropy decrease when it is stretched?

[2mks]

(b) From the First Law and the Maxwell relation above derive the so-called energy equation for the band.

$$\left(\frac{\partial U}{\partial L}\right)_T = \mathcal{F} - T\left(\frac{\partial \mathcal{F}}{\partial T}\right)_L$$ **[4mks]**

(c) Use the equation of state together with (b) to show that *U* is a function of *T* only

[5mks]

(d) The heat capacity at constant length of the band is constant and has the value C_L = 1.2 J K^{-1} . Express C_L in terms of an appropriate derivative of the entropy. If the band is stretched adiabatically (ΔS = 0) from 1.5 m to 1.6 m in length, with an initial temperature of 300 K, what is its final temperature? **[6mks]**

HINT: You could estimate this change of temperature if you knew the partial derivative $\left(\frac{\partial T}{\partial L}\right)_S$. See if you can combine the cyclical identity for partial derivatives with the information given previously to evaluate this derivative. Alternatively, you could use the information above in (a) and (d) to find the entropy *S* as a function of *T* and *L*, *S* = *S*(*T*, *L*).

QUESTION 2: (16 marks)

One form of low temperature thermometry depends on accurate measurement of the vapour pressure of liquid Helium-4 along the liquid-vapour coexistence curve at temperatures of about 1 K and below. Some standard calibration points are given in the literature as follows:

P (Torr)	**T (K)**
0.70	**1.216**
0.65	**1.206**
0.60	**1.194**
0.5	**1.17**
0.4	**1.14**
0.3	**1.10**
0.2	**1.06**
0.1	**0.98**
0.01	**0.79**

(a) Plot ln P against 1/T to see how well this data obeys the Clausius-Clapeyron equation for a first order phase transition coexistence boundary **[8mks]**.

(b) From the data estimate the liquid-vapour latent heat **[8mks]**.