THERMAL AND KINETIC PHYSICS (PHY 214)

Coursework 6 : Week 6

These problems are **REVISION PROBLEMS** to be used to help prepare for the mid-term test on November 15. These are **NOT REQUIRED TO BE HANDED IN** but you may attempt them in your own time and take any difficulties to your tutors.

The answers to the problems will appear on the web late on Tuesday in reading week or shortly thereafter.

The mid-term exam will take place on the first Monday after reading week (week 8) on Monday 15 November in Eng 325 at 10am. The exam will be of 50 minutes duration

You will need to settle quickly in order to get the full 50 minutes fitted into a 1 hour period. You should bring writing implements (pen, ruler, eraser, calculator but paper will be provided.

QUESTION 1: (1998 examination question 1)

a) State the Zeroth Law of Thermodynamics. Explain briefly what is meant by ideal gas temperature and by kinetic temperature. Calculate the root mean square speed of a molecule of Nitrogen (N_2, molecular mass 28 amu) in a gas at equilibrium with temperature 300 K.

b) State the First Law of Thermodynamics in infinitesimal form explaining clearly any notation or conventions used .

c) Explain what is meant by an adiabatic process? An ideal rigid diatomic gas with pressure P and volume V has internal energy $U = \frac{5}{2}PV$. Use the First Law to show that for an adiabatic transformation of the gas from state 1 to state 2 we have

$$P_1V_1^{7/5} = P_2V_2^{7/5}$$

d) The ideal diatomic gas in c) above undergoes a cyclic transformation a $\rightarrow$ b $\rightarrow$

c $\rightarrow$ a starting with pressure P_a and volume V_a. The process a $\rightarrow$ b is an isobaric process finishing with $V_b = \frac{1}{2}V_a$ The process b $\rightarrow$ c is an isochoric process finishing with Pc = $2P_a$. The final step, c $\rightarrow$ a, is an isothermal process.

i) Sketch the cycle on a P - V diagram.

ii) Determine the heat absorbed by the gas in the cycle. Express your answer in terms of P_a and V_a.

e) The thermal expansion coefficient β is defined by

$$\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$$

The equations of state for one mole of an ideal gas and one mole of a van der Waals gas respectively are

$$PV = RT$$

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

where a and b are constants. Calculate β for each gas.

QUESTION 2: (1999 examination question 1) .

a) State the Zeroth Law of Thermodynamics. Explain briefly how empirical temperature is defined for a constant volume gas thermometer. Explain how this temperature differs from ideal gas temperature. In a sample of atmosphere in equilibrium at a temperature T = 290 K we find nitrogen (N_2, atomic weight 28 amu), oxygen (O_2, atomic weight 32 amu) and argon (A, atomic weight 40 amu). Calculate the mean kinetic energy of an oxygen molecule. Calculate the ratio of the root mean square speeds of the fastest and slowest of these three molecular species.

b) State the First Law of Thermodynamics in infinitesimal form explaining clearly any notation or conventions used .

c) Explain what is meant by an adiabatic process. The internal energy of a photon gas is related to its pressure and volume by the equation

$$U = 3PV$$

Use the First Law of Thermodynamics to show that for an adiabatic process undergone by a photon gas the following adiabatic law holds,

$$PV^{4/3} = const$$

d) Two identical samples of ideal gas are prepared in state a at pressure P_a and volume V_a. The first sample undergoes a two-stage process, a $\rightarrow$ b $\rightarrow$ c, where a $\rightarrow$ b is an isobaric expansion from volume V_a to volume $V_b = 2V_a$ and b $\rightarrow$ c is an isochoric change from pressure P_b to pressure $P_c = 12P_b$. The second sample undergoes a single-step isothermal process a $\rightarrow$ c.

i) Sketch the two processes on a P - V diagram.

ii) For each of the two processes calculate the work done ON the gas.

iii) For each process calculate the heat absorbed BY the gas.

In ii) and iii) express your answer in terms of P_a and V_a.

e) The isothermal compressibility κ is defined by

$$\kappa = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T$$

The equations of state for one mole of an ideal gas and one mole of a van der Waals gas respectively are

$$PV = RT$$

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

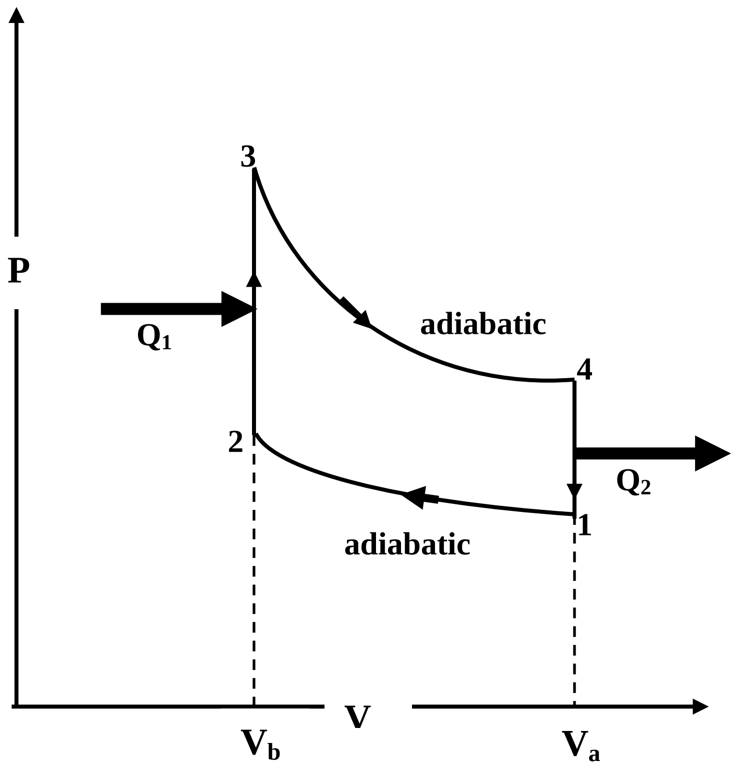
where a and b are constants. Calculate κ for each gas.

QUESTION 3: (1998 examination question 2)

a) State the Second Law of Thermodynamics in the form due to Kelvin and Planck.

b) A petrol powered engine can be approximately represented by the reversible Otto cycle shown below where the processes $1 \to 2$ and $3 \to 4$ are adiabatic while the processes $2 \to 3$ and $4 \to 1$ are at constant volume. The working substance of the engine is an ideal rigid diatomic gas for which the internal energy is $U = \frac{5}{2}PV$

During the cycle the working substance absorbs heat Q_1 and expels heat Q_2.



Define what is meant by the engine efficiency η_E and show that it can be expressed as

$$\eta_E = 1 - \left(\frac{1}{r}\right)^{\gamma - 1}$$

where γ = 7/5 is the adiabatic constant for the working substance and $r = \frac{V_1}{V_2}$ is the compression ratio.

c) For the particular engine considered above you may assume that $V_1 = 400\ cm^3$, $T_1 = 300$ K, $P_1 = 1$ atm, $V_2 = 50\ cm^3$ and $P_3 = 24$ atm. Determine the following:

i) the engine efficiency η_E,
ii) the pressure P_2 and temperature T_2 at point 2,
iii) the temperature T_3 at point 3,
iv) the heat energy Q_1 absorbed in $2 \rightarrow 3$,
v) the power output of the engine if the cycle is traversed 60 times per second

QUESTION 4: (1999 examination question 3)

a) State (without derivation) the definition of entropy in terms of reversible heat flows. State the entropy form of the Second Law of Thermodynamics for processes undergone by a thermally isolated system.

b) A container with perfect heat insulating walls has 5 kg of water at 300 K placed in it together with 0.5 kg of ice at its melting point 273.15 K.

i) What is the final state and final temperature of the water-ice system when it has reached thermodynamic equilibrium?
ii) What is the net entropy change of the water-ice system between the initial and final states?
iii) Is the Second Law as stated above obeyed? (Latent heat of fusion for water is 3.33×10^5 J kg^{-1}, specific heat of liquid water c_P =4.2 × 103 J K^{-1} kg^{-1})

c) A monatomic ideal gas has internal energy $U = \frac{3}{2}nRT$ and obeys the equation of state $PV = nRT$. Using the First Law and the definition of entropy given in **a)**, show that, when the gas undergoes a process taking it from initial equilibrium state T_1, V_1, to a final equilibrium state T_2, V_2, the entropy change is given by

$$\Delta S = \frac{3}{2} nR\, ln\left(\frac{T_2}{T_1}\right) + nR\, ln\left(\frac{V_2}{V_1}\right)$$

d)

i) Explain briefly Boltzmann's microscopic interpretation of the entropy.

ii) Suppose that the ideal gas in **b)** expands reversibly and isothermally from volume V_1 to final volume $3V_1$. What is the entropy change of the gas?

iii) If, instead, the gas is heated reversibly at constant volume from temperature T_1 to final temperature $2T_1$, what is the entropy change?

iv) Explain how both of these entropy changes maybe understood from the standpoint of the Boltzmann interpretation.