Thermal & Kinetic Physics (PHY-214) Exam 2009 Solutions.

SECTION A

A1. $$dU = TdS - PdV$$

dU is the incremental change in internal energy.

dS is the incremental change in entropy.

$-PdV$ is the incremental work done ***on the system*** where P is the pressure and dV the incremental change in volume.

A2. $$C_P = \left(\frac{\partial H}{\partial T}\right)_P$$

$H = U + PV$ is the enthalpy.

Or $$C_P = T\left(\frac{\partial S}{\partial T}\right)_P$$

A3. $U = \frac{5}{2}PV$

A4. The work done in one cycle is on the environment and is therefore negative and given as -100J following convention.

$$\Delta W_{Cycle} = \Delta W_{a\to b} + \Delta W_{b\to c} + \Delta W_{c\to a} = \Delta W_{b\to c} + \Delta W_{c\to a} = -100J$$

Where I use the fact that a → b is isochoric and therefore $\Delta W_{a\to b} = 0$.

Now use the fact that c → a is an isotherm and that therefore $\Delta U_{c\to a} = 0$

$$\Delta Q_{c\to a} = -\Delta W_{c\to a} = -40J$$

$$\Delta W_{b\to c} = -100J - \Delta W_{c\to a} = -140J$$

A5. During the process a → b → c there is no temperature change and therefore no change in internal energy. Therefore the first law reads;

$$\Delta Q_{a\to b\to c} = -\left(\Delta W_{a\to b} + \Delta W_{b\to c}\right) = -\left(\Delta W_{cycle} - \Delta W_{c\to a}\right) = -\left(-100J - 40J\right)$$

$$\Delta Q_{a\to b\to c} = +140J$$

The minus sign indicates that ***this is heat expelled by the system***.

A6. $$dU = TdS + B_0 dM$$

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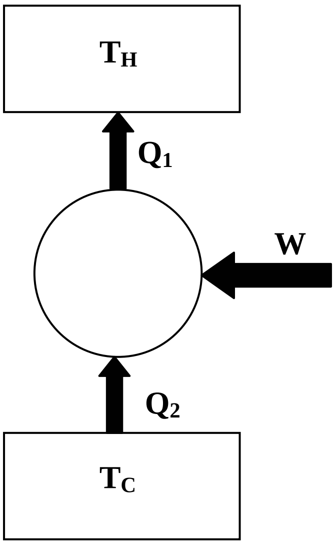
dU is the incremental change in internal energy.

dS is the incremental change in entropy at temperature T.

$B_0 dM$ is the incremental work done ***<u>on the system</u>*** where B_0 is the applied magnetic field and dM the incremental change in total magnetic moment of the paramagnet.

A7.



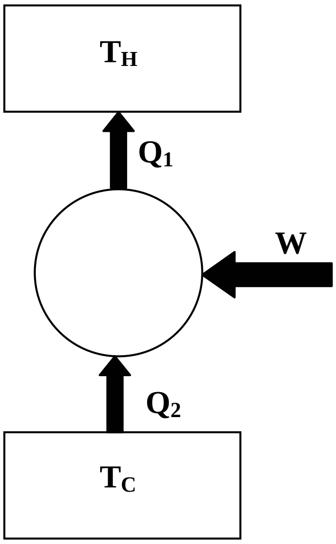
Looking at the schematic diagram

$$Q_1 = Q_2 + W$$

A8.

$$\eta = \frac{Q_1}{W} = \frac{Q_1}{Q_1 - Q_2}$$

$$\eta_C = \frac{T_1}{T_1 - T_2}$$



A9. For an ideal gas $PV = RT$ and therefore we can write

$$P = \frac{RT}{V} \qquad V = \frac{RT}{P} \qquad P = \frac{RT}{V}$$

$$\left(\frac{\partial \mathrm{P}}{\partial \mathrm{V}}\right)_{\mathrm{T}} = \frac{-\mathrm{RT}}{\mathrm{V}^2} \qquad \left(\frac{\partial \mathrm{V}}{\partial \mathrm{T}}\right)_{\mathrm{P}} = \frac{\mathrm{R}}{\mathrm{P}} \qquad \left(\frac{\partial \mathrm{T}}{\partial \mathrm{P}}\right)_{\mathrm{V}} = \frac{\mathrm{V}}{\mathrm{R}}$$

$$\left(\frac{\partial \mathrm{P}}{\partial \mathrm{V}}\right)_{\mathrm{T}}\left(\frac{\partial \mathrm{V}}{\partial \mathrm{T}}\right)_{\mathrm{P}}\left(\frac{\partial \mathrm{T}}{\partial \mathrm{P}}\right)_{\mathrm{V}} = \frac{-\mathrm{RT}}{\mathrm{V}^2}\frac{\mathrm{R}}{\mathrm{P}}\frac{\mathrm{V}}{\mathrm{R}} = \frac{-\mathrm{PV}}{\mathrm{V}^2}\frac{\mathrm{R}}{\mathrm{P}}\frac{\mathrm{V}}{\mathrm{R}} = -1$$ **(QED)**

A10. $P_i V_i^{\gamma} = P_f V_f^{\gamma}$ and $P = \frac{RT}{V}$

Substituting to replace P with T

$$T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}$$

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SECTION B

B1

a)
i)

$$dS = \frac{\delta Q_R}{T}$$

Where dS is the incremental change in the entropy of a system as heat δQ_R flows reversibly at temperature T. If the heat flows from the system then following sign convention it is negative and the entropy change is negative.

ii) The second law for a thermally isolated system may be written as an inequality involving the entropy change ΔS of that system

$$\Delta S \geq 0$$

The equality holds for a reversible process.

b)

i) There is no net heat flow as the process takes place in an adiabatic container so we calculate the final temperature

$$\Delta Q_{H_2O} = 5kg \times C_P \times \left(T_f - 298K\right)$$

$$\Delta Q_{Ice} = 1.0kg \times C_P \times \left(T_f - 273.15K\right) + 1.0kg \times l^{\ melting}$$

And

$$\Delta Q_{Ice} + \Delta Q_{H_2O} = 1.0kg \times C_P \times \left(T_f - 273.15K\right) + 1.0kg \times l^{\ melting} + 5kg \times C_P \times \left(T_f - 298K\right) = 0$$

$$6kg \times C_P \times T_f = C_P\left(273.15 \times 1kg + 298 \times 5kg\right) - 1kg \times l^{\text{ melting}}$$

$$T_f = \frac{\left(273.15 \times 1kg + 298 \times 5kg\right)}{6kg} - \frac{1kg}{6kg}\frac{l^{melting}}{C_P}$$

$$T_f = \frac{1.76315 \times 10^3 kgK}{6kg} - \frac{1kg}{6kg}\frac{3.33 \times 10^5 Jkg^{-1}}{4.2 \times 10^3 Jkg^{-1}K^{-1}} = 2.938 \times 10^2 - 13.214$$

$$T_f = 280.58K$$

The final state of the system is 6kg of water at 280.58 K and at 1 atmosphere

B1 b) cont

ii) The net entropy change for the water initially at 300k is

$$\Delta S_{H_2O} = 5kg \times 4.2 \times 10^3 JK^{-1}kg^{-1} \int_{298}^{280.6} \frac{dT}{T} = 2.1 \times 10^4 \ln\frac{280.6}{298} = -1263 \ \ JK^{-1}$$

$$\Delta S_{Ice} = 1kg \times 4.2 \times 10^3 JK^{-1}kg^{-1} \int_{273.15}^{280.6} \frac{dT}{T} + \frac{1kg \times 3.33 \times 10^5}{273.15}$$

$$\Delta S_{Ice} = \left(4.2 \times 10^3 \ln\frac{280.6}{273.15} + 1219.11 \right) JK^{-1}$$

$$\Delta S_{Ice} = 113.02 + 1219.11 \ \ JK^{-1} = +1332.13 \ \ JK^{-1}$$

$$\Delta S_{Net} = \Delta S_{Ice} + \Delta S_{H_2O} = (1332.13 - 1263) \ \ JK^{-1} = +69.13 \ \ JK^{-1}$$

iii) The mixing process is not reversible as without further input 1kg of 6kg of water at 280.6 K will not turn into ice.

iv) As the process is irreversible we expect from the Second Law that $\Delta S > 0$ and this is what is seen to be the case

c)
i) The final temperature of the mix can be found by making the total heat flow zero

$$mC_P\left[(T_f - T_1) + (T_f - T_2)\right] = 0$$

$$T_f = \frac{T_1 + T_2}{2}$$

The entropy change of the universe is simply the entropy changes of the two masses of water combined;

$$\Delta S = mc_P\left[\int_{T_1}^{T_f} \frac{dT}{T} + \int_{T_2}^{T_f} \frac{dT}{T}\right] = mc_P\left[ln\left(\frac{T_f}{T_1}\right) + ln\left(\frac{T_f}{T_2}\right)\right]$$

$$\Delta S = mc_P\left[ln\left(\frac{T_f^2}{T_1T_2}\right)\right] = mc_P\, 2\, ln\left(\frac{(T_1+T_2)/2}{\sqrt{T_1T_2}}\right)$$

QED

ii) For the Second Law to be obeyed requires that $\Delta S > 0$ as we have an irreversible process. This requires that the argument of the logarithm is greater than 1

$$\frac{T_1 + T_2}{2\sqrt{T_1 T_2}} > 1$$

B1. c) cont

iii) To demonstrate that the second Law is obeyed it is necessary to demonstrate the inequality of part ii) is generally true for all T_1 and T_2 as follows.

$$T_1 + T_2 > 2\sqrt{T_1 T_2}$$

Squaring both sides of the inequality

$$(T_1 + T_2)^2 > 4T_1T_2$$

$$T_1^2 + T_2^2 + 2T_1T_2 > 4T_1T_2$$

$$T_1^2 + T_2^2 - 2T_1T_2 = (T_1 - T_2)^2 > 0$$

This is always true if T_1 and T_2 are real numbers and therefore the Second Law is obeyed.

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Solutions

B2.

a)

i) The work is given by $W = -\int_{V_i}^{2V_i} PdV = -P_i \int_{V_i}^{2V_i} dV = -P_i V_i$ and this is negative and therefore represents ***work done by the gas***.

ii) The work is given by $W = -\int_{V_i}^{V_i/2} PdV$

As it is adiabatic we have $PV^{\gamma} = const$

$$W = -P_i V_i^{\gamma} \int_{V_i}^{V_i/2} V^{-\gamma} dV = \frac{-P_i V_i^{\gamma}}{1-\gamma}\left[\left(\frac{V_i}{2}\right)^{1-\gamma} - V_i^{1-\gamma}\right]$$

$$W = \frac{-P_i V_i^{\gamma}}{1-\gamma} V_i^{1-\gamma}\left[2^{\gamma-1} - 1\right]$$

$$W = \frac{P_i V_i}{1-\gamma}\left(1 - 2^{\gamma-1}\right)$$

$\gamma = \frac{C_P}{C_V}$ is always greater than 1 and therefore the denominator is negative as is the term in the brackets making W positive ie. ***work is done on the gas***.

iii) The work is given by $W = -\int_{V_i}^{V_i/2} PdV$

$$P = \frac{RT}{V}$$

$$W = -RT \int_{V_i}^{V_i/2} \frac{dV}{V} = -RT\, ln\left(\frac{V_i/2}{V_i}\right)$$

$$W = -RT\, ln\left(\frac{1}{2}\right) = RT\, ln\, 2$$

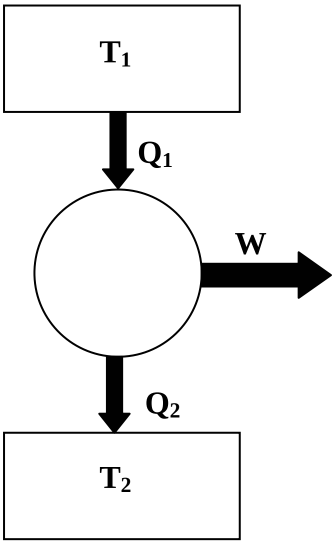
W is positive and therefore represents ***work done on the gas***.

B2cont

b)

i) The efficiency of a heat engine is given by

$$\eta_E = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$



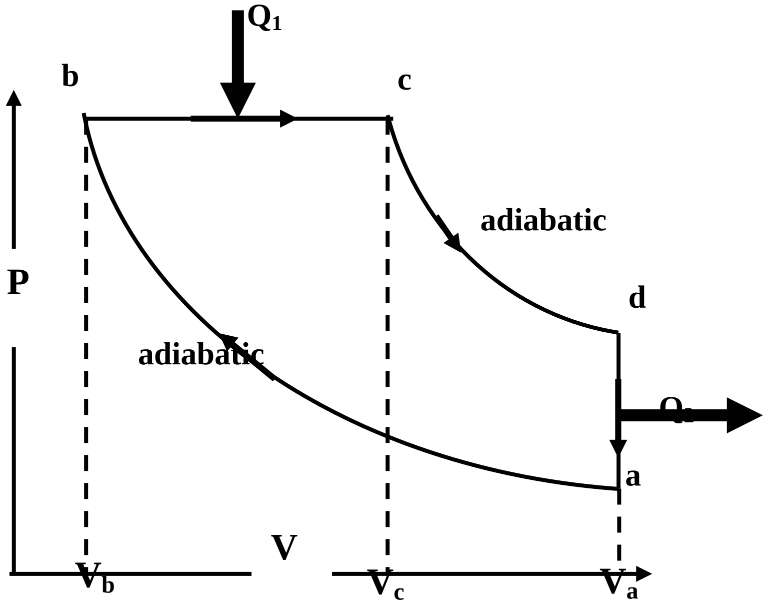


Figure 1

ii) In an isobaric process we use the heat capacity at constant pressure to find Q_1

$$Q_1 = Q_{b\to c} = C_P\left(T_c - T_b\right)$$

iii) In an isochoric process we use the heat capacity at constant volume to find Q_2

$$Q_2 = -Q_{d\to a} = C_V\left(T_d - T_a\right)$$

B2. b) cont

iv) In part i) we have $\eta_E = 1 - \frac{Q_2}{Q_1}$

$$\eta_E = 1 - \frac{C_V(T_d - T_a)}{C_P(T_c - T_b)} = 1 - \frac{1}{\gamma}\frac{(T_d - T_a)}{(T_c - T_b)} \qquad \underline{\textbf{\textit{QED}}}$$

c)

i) Considering the two adiabatic processes $c \to d$ and $a \to b$ and using the adiabatic rule

c → d is an adiabatic process thus

$$\mathrm{T_c V_c^{\gamma-1} = T_d V_d^{\gamma-1}} \quad \Rightarrow \quad \mathrm{T_d = T_c\left(\frac{V_c}{V_d}\right)^{\gamma-1}}$$

a → b is also an adiabatic process and similarly

$$\mathrm{T_a V_a^{\gamma-1} = T_b V_b^{\gamma-1}} \quad \Rightarrow \quad \mathrm{T_a = T_b\left(\frac{V_b}{V_a}\right)^{\gamma-1}}$$

We also know that

$\mathrm{P_b V_b = nRT_b}$ $\mathrm{P_c V_c = nRT_c}$ and $\mathrm{P_b = P_c}$

$$\mathrm{\frac{V_b}{V_c} = \frac{T_b}{T_c}} \quad \Rightarrow \quad \mathrm{T_c = T_b \frac{V_c}{V_b}}$$

$$T_d - T_a = T_c\left(\frac{V_c}{V_a}\right)^{\gamma-1} - T_b\left(\frac{V_b}{V_a}\right)^{\gamma-1}$$

$$T_d - T_a = T_b\left(\frac{V_c}{V_b}\right)\left(\frac{V_c}{V_a}\right)^{\gamma-1} - T_b\left(\frac{V_b}{V_a}\right)^{\gamma-1} = \frac{T_b V_a}{V_b}\left[\left(\frac{V_c}{V_a}\right)^{\gamma} - \left(\frac{V_b}{V_a}\right)^{\gamma}\right]$$

And similarly

B2. c) cont

$$\mathrm{T_c - T_b = T_b\left(\frac{V_c}{V_b}\right) - T_b = \frac{T_b V_a}{V_b}\left[\left(\frac{V_c}{V_a}\right) - \left(\frac{V_b}{V_a}\right)\right]}$$

Including these in the earlier expression for η_E

$$\eta_E = 1 - \frac{1}{\gamma}\frac{T_d - T_a}{T_c - T_b} = 1 - \frac{1}{\gamma}\frac{\left[\left(\frac{V_a}{V_c}\right)^{-\gamma} - \left(\frac{V_a}{V_b}\right)^{-\gamma}\right]}{\left(\frac{V_a}{V_c}\right)^{-1} - \left(\frac{V_a}{V_b}\right)^{-1}}$$

We have $\gamma_\mathrm{e} = \frac{\mathrm{V_a}}{\mathrm{V_c}}$ is the expansion ratio and $\gamma_\mathrm{c} = \frac{\mathrm{V_a}}{\mathrm{V_b}}$ is the compression ratio and when these are used in the above

$$\eta_E = 1 - \frac{1}{\gamma}\left[\frac{\gamma_e^{-\gamma} - \gamma_c^{-\gamma}}{\gamma_e^{-1} - \gamma_c^{-1}}\right]$$

QED

ii) For a rigid diatomic gas $\gamma = \frac{C_P}{C_V} = \frac{7}{5}$

Also we are given $\gamma_e = \frac{V_a}{V_c} = \frac{5000}{3000} = 1.666$ and $\gamma_c = \frac{V_a}{V_b} = \frac{5000}{500} = 10$

$$\eta_E = 1 - \frac{1}{\gamma}\left[\frac{\gamma_e^{-\gamma} - \gamma_c^{-\gamma}}{\gamma_e^{-1} - \gamma_c^{-1}}\right] = 1 - \frac{5}{7}\left[\frac{1.666^{-1.4} - 10^{-1.4}}{1.666^{-1} - 10^{-1}}\right]$$

$$\eta_E = 1 - 0.71 \times \left[\frac{0.49 - 3.98 \times 10^{-2}}{6 \times 10^{-1} - 1 \times 10^{-1}}\right] = 1 - 0.71 \times \frac{0.45}{0.5} = 1 - 0.639 = 0.361$$

iii) From our definition of η_E

$$\eta_E = \frac{W}{Q_1} = 0.361$$

If Q_1 = 50J then W = 0.361 × 50J = 18J per cycle.

The required output is 1kW = 1000J/sec requiring the engine to undergo 55 cycles per second.

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Solutions

<u>B3.</u>

a)

i) Using the definition of F as given

$$F = U - TS$$

To write the incremental for F

$$dF = dU - TdS - SdT$$

Using the thermodynamic identity, $dU = TdS - PdV$, to simplify this

$$\mathrm{dF = TdS - PdV - TdS - SdT}$$

$$\mathrm{dF = -PdV - SdT}$$

The natural variables are then, volume, V and temperature T and $F = F(V,T)$

ii) We use the natural variables to write the incremental dF in an alternative way

$$\mathrm{dF} = \left(\frac{\partial \mathrm{F}}{\partial \mathrm{V}}\right)_{\mathrm{T}} \mathrm{dV} + \left(\frac{\partial \mathrm{F}}{\partial \mathrm{T}}\right)_{\mathrm{V}} \mathrm{dT}$$

And comparing this to the incremental form in i)

$$S = -\left(\frac{\partial F}{\partial T}\right)_V \qquad \text{and} \qquad P = -\left(\frac{\partial F}{\partial V}\right)_T \qquad \text{as required.}$$

iii) It follows that as dF is a perfect differential we may make use of the fact

$$\frac{\partial^2 \mathrm{F}}{\partial \mathrm{T}\partial \mathrm{V}} = \frac{\partial^2 \mathrm{F}}{\partial \mathrm{V}\partial \mathrm{T}} \qquad \Rightarrow \qquad \frac{\partial}{\partial \mathrm{T}}\left(\frac{\partial \mathrm{F}}{\partial \mathrm{V}}\right) = \frac{\partial}{\partial \mathrm{V}}\left(\frac{\partial \mathrm{F}}{\partial \mathrm{T}}\right)$$

Implying from our expressions for P and S in ii) that

$$\left(\frac{\partial \mathrm{P}}{\partial \mathrm{T}}\right)_{\mathrm{V}} = \left(\frac{\partial \mathrm{S}}{\partial \mathrm{V}}\right)_{\mathrm{T}}$$

<u>QED</u>

B3. b) cont.

b)
i) A particular gas has a Helmoltz function given by

$$F = -\frac{na}{V} - RT\,ln(V - nb) + j(T)$$

We can use $P = -\left(\frac{\partial F}{\partial V}\right)_T$ from a) to obtain an expression for P

$$P = -\left(\frac{\partial F}{\partial V}\right)_T = -\left[\frac{n^2 a}{V^2} - \frac{nRT}{V - nb}\right]$$

Re-arrange

$$P + \frac{n^2 a}{V^2} = \frac{nRT}{V - nb}$$

$$\left(P + \frac{n^2 a}{V^2}\right)(V - nb) = nRT$$ ***QED***

ii) To find the change in entropy after an expansion at constant temperature we use the Maxwell relation from a)

$$\left(\frac{\partial \mathrm{P}}{\partial \mathrm{T}}\right)_{\mathrm{V}} = \left(\frac{\partial \mathrm{S}}{\partial \mathrm{V}}\right)_{\mathrm{T}}$$

$$\left(\frac{\partial P}{\partial T}\right)_V = \frac{nR}{V - nb} = \left(\frac{\partial S}{\partial V}\right)_T$$

$$\Delta S = nR\int_{V_i}^{V_f} \frac{1}{V - nb}dV$$

$$\Delta S = nR\ln\left(\frac{V_f - nb}{V_i - nb}\right)$$

iii) If $\Delta S = \frac{2nk_B}{T_0}\left(T_f - T_i\right)$ when the gas temperature is raised from T_i to T_f at constant volume we can use the previously found $S = -\left(\frac{\partial F}{\partial T}\right)_V$ to find $j(T)$ from

$$F = -\frac{n^2 a}{V} - nRT\ln(V - nb) + j(T)$$

$$\Delta S = -\left[-nR\ln(V - nb) + \frac{dj}{dT} \right]_{T_i}^{T_f} = \frac{nk_B}{T_o}\left(T_f - T_i\right)$$

B3. b) cont

$$\frac{dj}{dT} = \frac{nk_B}{T_0} T$$

$$j(T) = \frac{1}{2}\frac{nk_B}{T_0} T^2 + const$$

c)

i) The heat capacity at constant volume is given in terms of partial differential of a state function as

$$C_V = \left(\frac{\partial U}{\partial T} \right)_V$$

Using the ideal gas equation of ststе for a monatomic gas, $U = \frac{3}{2} PV = \frac{3}{2} nRT$

From the expression for C_V we have

$$C_V = \left(\frac{\partial U}{\partial T} \right)_V = \frac{3}{2} nR$$

ii) The thermodynamic identity for a P-V-T system is;

$$dU = TdS - PdV$$

From the thermodynamic identity

$$\left.\frac{dU}{dV}\right|_{T\ const} = T\left.\frac{dS}{dV}\right|_{T\ const} - P\left.\frac{dV}{dV}\right|_{T\ const}$$

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial S}{\partial V}\right)_T - P$$

iii) We can use the Maxwell relation $\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$ as found in a) to write

<u>B3. c) cont</u>

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P$$

Rearrange vdW equation of state

$$P = \frac{nRT}{V-nb} - \frac{n^2 a}{V^2}$$

$$\left(\frac{\partial P}{\partial T}\right)_V = \frac{nR}{V-nb}$$

Therefore

$$\left(\frac{\partial U}{\partial V}\right)_T = T\frac{nR}{V-nb} - P = \frac{nRT}{V-nb} - \frac{nRT}{V-nb} + \frac{n^2 a}{V^2} = \frac{n^2 a}{V^2}$$

We also have

$$\left(\frac{\partial U}{\partial T}\right)_V = C_V = \frac{3}{2}nR$$

Integrating these two equations wrt to dV and dT respectively

$$U(T,V) = -\frac{n^2 a}{V} + g(T) \qquad\qquad U(T,V) = \frac{3}{2}nRT + f(V)$$

where *g(T)* and *f(V)* are constants of integration.

The only consistent solution is

$$U(T,V) = \frac{3}{2}nRT - \frac{n^2 a}{V} + const$$

<u>QED</u>

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Solutions

B4

a)

i) We are given $P(v) = \left(\frac{m}{2\pi k_B T}\right)^{3/2} 4\pi v^2 \, exp\left[-\frac{mv^2}{2k_B T}\right]$

$$\bar{v} = \int_0^\infty vP(v)dv$$

$$\bar{v} = 4\pi\left[\frac{m}{2\pi k_B T}\right]^{3/2} \int_0^\infty v^3 \, exp\left(-\frac{mv^2}{2k_B T}\right)dv$$

Making the suggested substitutions $v^2 \rightarrow x$, $dv = \frac{dx}{2v}$ and $\frac{m}{2k_B T} \rightarrow a$

$$\bar{v} = 4\pi\left[\frac{a}{\pi}\right]^{3/2} \frac{1}{2}\int_0^\infty x\,\exp(-ax)dx = 2\pi\left[\frac{a}{\pi}\right]^{3/2} \int_0^\infty x\,\exp(-ax)dx$$

We now find this integral by integrating by parts;

$$u = x \qquad dv = e^{-ax}$$

$$du = dx \qquad v = -\frac{1}{a}e^{-ax}$$

$$\int udv = \int xe^{-ax} = -\frac{x}{a}e^{-ax} - \int -\frac{1}{a}e^{-ax}dx$$

$$\int_0^\infty xe^{-ax}dx = \left[-\frac{x}{a}e^{-ax} - \frac{1}{a^2}e^{-ax}\right]_0^\infty = \frac{1}{a^2}$$

$$\bar{v} = 2\pi\left[\frac{a}{\pi}\right]^{3/2} \frac{1}{a^2} = \sqrt{\frac{4}{\pi a}} = \sqrt{\frac{8k_B T}{\pi m}}$$

QED

ii) We are given the flux, $\Phi = \frac{1}{4}n\bar{v}$ and number density $n = \frac{N}{V}$

B4. a) cont

The equation of state for the ideal gas that we need is

$$PV = n_m RT = Nk_B T$$

$$n = \frac{N}{V} = \frac{P}{k_B T}$$

$$n_{O_2} = 0.2 \times n = \frac{P}{5k_b T}$$

$$n_{O_2} = \frac{1.01 \times 10^5 Pa}{5 \times 1.4 \times 10^{-23} \times 300K} = 4.8 \times 10^{24}$$

iii) First we need to find $\bar{v}$ as follows

$$\bar{v} = \sqrt{\frac{8k_B T}{\pi m}} = \sqrt{\frac{8 \times 1.4 \times 10^{-23} JK^{-1} \times 300K}{3.14 \times 32 \times 1.66 \times 10^{-27} kg}} = 2.0 \times 10^5 ms^{-1}$$

Now we use $\Phi = \frac{1}{4} n\bar{v} = \frac{1}{4} \times 4.8 \times 10^{24} \times 2 \times 10^5 m^{-2} s^{-1}$

$$\Phi = 2.4 \times 10^{29} \;\; m^{-2} s^{-1} = 2.4 \times 10^{25''} \;\; cm^2 s^{-1}$$

b)

i) The ratio of fluxes incident upon the aperture of the pipe is

$$\frac{\Phi_1}{\Phi_2} = \frac{N_1^e}{N_2^e} = \frac{n_1}{n_2} \frac{v_1}{v_2} = \frac{n_1}{n_2} \sqrt{\frac{m_2}{m_1}}$$ ***QED***

ii)

$$\frac{N^e_{{}^3He}}{N^e_{{}^4He}} = \frac{n_{{}^3He}}{n_{{}^4He}} \sqrt{\frac{m_{{}^4He}}{m_{{}^3He}}} = \frac{10}{90} \sqrt{\frac{4}{3}} = 1.28 \times 10^{-1}$$

<u>B4. b) cont</u>

iii) If this process is repeated through j cycles

$$\frac{N^e_{^3He}}{N^e_{^4He}}(j)) = \frac{10}{90}\left(\sqrt{\frac{4}{3}}\right)^j$$

We require to find value for j that gives 30% ^{3}He

$$\frac{N^e_{^3He}}{N^e_{^4He}}(j)) = \frac{10}{90}\left(\sqrt{\frac{4}{3}}\right)^j = \frac{30}{70}$$

$$\left(\sqrt{\frac{4}{3}}\right)^j = \frac{30\times 90}{70\times 10}$$

$$j\, ln\left(\sqrt{\frac{4}{3}}\right) = ln\left(\frac{27}{7}\right)$$

$$j = \frac{ln\, 3.86}{ln\, 1.155} = \frac{1.351}{0.1437} = 9.4 \approx 10 \quad cycles$$