Queen Mary
University of London

BSc/MSci Examination

PHY214 Thermal and Kinetic Physics

Time allowed: 2hours 30 minutes

Date: 3rd June 2010

Time: 10:00 – 12:30

Instructions:

Answer ALL questions in section A. Answer ONLY TWO questions from section B. Section A carries 50 marks, each question in section B carries 25 marks. An indicative marking-scheme is shown in square brackets [ ] after each part of a question. Course work comprises 20% of the final mark

Numeric calculators are permitted in this examination. Please state on your answer book the name and type of machine used. Complete all rough workings in the answer book and cross through any work which is not to be assessed.

Important note: The academic regulations state that possession of unauthorised material at any time when a student is under examination conditions is an assessment offence and can lead to expulsion from the college. Please check now to ensure you do not have any notes in your possession. If you have any then please raise your hand and give them to an invigilator immediately. Exam papers cannot be removed from the exam room

You are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

Examiners: **K.J.Donovan**

G.Thompson

DATA SHEET

You may wish to use some of the following data.

c = speed of light in vacuum = 3×10^{8} m s^{-1}

k_B = Boltzmann's constant = 1.38×10^{-23} J K^{-1}

N_A = Avagadro's number = 6.02×10^{23} mol^{-1}

R = Gas constant = 8.31 J mol^{-1} K^{-1}

P_{atm} = Atmospheric pressure = 1atm = 1.01×10^{5} Pa

σ = Stefan-Boltzmann constant = 5.67×10^{-8} W m^{-2} K^{-4}

T_S = Ice point of water = 273.15 K

ρ_{Eth} = Density of alcohol 0.789 gm.cc^{-1}

c_P = Specific heat of water at constant pressure = 4.2×10^{3} J K^{-1} kg^{-1}

c_{Eth} = Specific heat of alcohol at constant pressure = 2.4×10^{3} J K^{-1} kg^{-1}

c_P^{Ice} **= Specific heat of ice at constant pressure = 2.1×10^{3} J kg^{-1}**

l^{SL} **= Latent heat of melting ice = 3.33×10^{5} J kg^{-1}**

l^{LV} **= Latent heat of evaporating water = 2.26×10^{6} J kg^{-1}**

1 *amu* **= One atomic mass unit = 1.66×10^{-27} kg**

SECTION A. Attempt answers to all questions

Question A1

Write down the thermodynamic identity for a P-V system explaining all symbols used. **[5]**

Question A2

Write down Boltzmann's equation describing the relationship between entropy, S, and the number of distinct microstates, Ω, that can make the macrostate in question. **[5]**

Question A3

A Carnot engine operates between hot and cold reservoirs at temperatures T_1 and T_2 respectively. Write down the efficiency of the engine in terms of these temperatures. **[5]**

Question A4

Write down the 2nd Law of Thermodynamics expressed as an inequality involving only state variables. **[5]**

Question A5

A refrigerator operating between a hot and cold reservoir delivers heat Q_2 from the cold reservoir to the working system whilst giving up heat Q_1 to the hot reservoir. Write down the refrigerator efficiency, η_R , in terms of these heat flows. **[5]**

Question A6

During an adiabatic process on an ideal gas what is the relationship between initial pressure and volume P_i and V_i and final pressure and volume P_f and V_f? Explain any terms used. **[5]**

Question A7

Express the heat capacity of a gas at constant pressure, C_P, in terms of a partial differential ***using only state variables***. Define all of the symbols used. **[5]**

Question A8

What is the internal energy of a gas of N rigid diatomic molecules at equilibrium at temperature T in terms of the temperature? What is the average kinetic energy of one of the molecules in terms of T? **[5]**

Question A9

The isothermal bulk modulus is defined as $\kappa = -V\left(\frac{\partial P}{\partial V}\right)_T$. For the case of an ideal gas write down an algebraic expression for κ. **[5]**

Question A10

An ideal gas is compressed in an isobaric process at pressure P from volume V to a volume $V/2$. Write down the work done ***including sign*** and ***state whether this is work done on or by the gas***.

[5]

Turn Over

SECTION B. Attempt two of the four questions in this section

B1) **a)**

i) In an isothermal process one mole of an ideal gas at temperature T is expanded from an initial volume V to a final volume $2V$. What is the work done in this process in terms of T and V ***including sign*** and state whether the work was done by or on the gas.

ii) In an isochoric process one mole of a monatomic ideal gas of volume V_1 has its pressure reduced from initial pressure P_1 to a final pressure $P_1/2$. What is the heat transfer during this process in terms of P_1 and V_1 ***including sign.*** State whether the heat was absorbed or released by the gas.

iii) In an isobaric process one mole of a diatomic ideal gas at pressure P_1 is compressed from initial volume V_1 to a final volume $V_1/2$. Write down the internal energy change during this process in terms of P_1 and V_1 ***including sign***.

[4 marks]

b) One mole of a rigid diatomic ideal gas where the ratio of specific heat capacities, $\gamma = c_P/c_V = 7/5$, is compressed adiabatically from an initial volume V_1 and initial temperature $T_1 = 25\ ^{o}\mathrm{C}$ to a final volume . $\dfrac{V_1}{2}$.

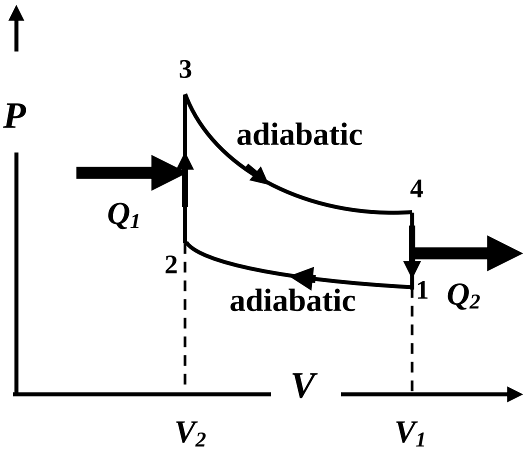
i) Calculate the value of the final temperature of the gas.

ii) Calculate the change in internal energy of the gas during this process ***including sign.***

iii) What is the work done on the gas during this process ***including sign***?

[6 marks]

c) A petrol engine may be approximated by the reversible Otto cycle shown below. The processes **1** → **2** and **3** → **4** are adiabatic whilst **2** → **3** and **4** → **1** are carried out at constant volume. The working substance of the engine is an ideal rigid diatomic gas. During one cycle the gas absorbs heat Q_1 and expels heat Q_2.



B1 c) cont.

i) Write down an expression for the engine efficiency η_E in terms of heat flows.

ii) For the Otto cycle described above show that η_E may be expressed as

$$\eta_E = 1 - \left(\frac{1}{r}\right)^{\gamma - 1}$$

where $r = {}^{V_1}\!/_{V_2}$, is the compression ratio.

iii) For the engine under discussion you may assume that V_1 = 300cm^3 , V_2 = 75cm^3 ,

T_1 = 300K, P_1 = 1atm and P_3 = 25atm.

Determine a value for;

1. The engine efficiency,
2. The pressure P_2 and
3. The heat absorbed, Q_1**.**

and hence

4. Determine the power output of the engine if the cycle is traversed 50 times per second.

[15 marks]

Turn Over

B2) **a)**

i) Write down an expression for the entropy change, ΔS, of a system as it goes from an initial equilibrium state to a final equilibrium state in terms of reversible heat transfers, ΔQ_R and temperature, T. Explain the sign convention.

ii) 3kg of water at 30 ^{o}C is placed in the garden where the temperature is 7 ^{o}C. Calculate the entropy change, ***including sign***, of the water.

iii) 3kg of ice from a freezer at -10 ^{o}C is placed in a bucket in the garden where the temperature is 7 ^{o}C. Calculate the entropy change, ***including sign***, of the ice/water.

[9 marks]

b)

i) By considering the thermodynamic identity show that for one mole of an ideal monatomic gas the entropy change as volume and temperature are changed is given by;

$$\Delta S = S(V_f, T_f) - S(V_i, T_i) = \frac{3}{2} R \ln\left[\frac{T_f}{T_i}\right] + R \ln\left[\frac{V_f}{V_i}\right]$$

ii) An ideal gas is compressed isothermally from V_{i} to $V_i/3$. Calculate the entropy change of the gas ***including the sign***.

iii) An ideal gas is heated in an isochoric process from temperature T_{i} to $2T_{\mathrm{i}}$. Calculate the entropy change of the gas ***including the sign***.

[9 marks]

c) A system with heat capacity C_{P} and mass m at an initial temperature T_{i} is held at constant volume and placed in contact with a heat reservoir at a higher temperature T_{R}. The two are isolated from the rest of the world.

i) By considering the change in entropy of both the system and the heat reservoir show that the entropy change of the universe is given by

$$\Delta S_{Universe} = -mC_P\, ln(1 - X) - mC_P X$$

where $X = \left(1 - \frac{T_i}{T_R}\right)$

ii) Show, using *the McClaurens series expansion,* $-ln(1-x) = x + \frac{x^2}{2} + \frac{x^3}{3} + ...$, that the entropy change of the universe described in part i) is positive.

[7 marks]

B3) **a)**

i) Demonstrate the truth of the cyclic relation for partial differentials using a P-V-T system and the equation of state of an ideal gas.

ii) The isothermal bulk modulus for a gas is defined as $\kappa = -V\left(\frac{\partial P}{\partial V}\right)_T$ and the thermal expansion coefficient is defined as $\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$. Using the cyclical and reciprocal relations find a partial differential relationship for the product $\kappa\beta$.

iii) If the pressure of an ideal gas is increased isothermally by 2% calculate the percentage change in volume including sign.

[6 marks]

b) For a P-V-T system the Gibbs free energy is defined as $G = U - TS + PV$

i) Using the above definition of *G* and the thermodynamic identity for a P-V-T system find the natural variables of *G*.

ii) Find the relationships between the partial differentials of *G* with respect to each of the two natural variables and two other thermodynamic variables.

iii) From i) and ii) find a Maxwell relation between partial differentials of *S* and *V* with respect to the natural variables of *G*.

[10 marks]

c) One mole of a van der Waals gas is described by the equation of state

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT$$

i) Derive an expression for the thermal expansion coefficient of a van der Waals gas, β, as defined in a).

ii) The internal energy of one mole of a van der Waals gas is given by

$$U_{VdW} = \frac{3}{2}RT - \frac{a}{V}$$

Show that during an adiabatic process performed on a van der Waals gas the temperature and volume are related by the equation

$$T^{3/2}(V - b) = const$$

[9 marks]

Turn Over

B4)

a)

i) Write down an expression for the root mean square velocity, v_{rms} , of a molecule of mass m in a gas at temperature T.

ii) The atmosphere consists mostly of molecular nitrogen (N_2, molecular weight 28 amu), molecular oxygen (O_2, molecular weight 32 amu). For an atmospheric temperature of 300 K calculate the root mean square velocity, v_{rms} of the molecular nitrogen and of the molecular oxygen.

[6 marks]

b) For a gas of molecules of mass m at temperature T the Maxwell Boltzmann velocity distribution function is given by

$$f(v) = \left(\frac{m}{2\pi k_B T}\right)^{3/2} \exp\left[-\frac{mv^2}{2k_B T}\right]$$

where v is the magnitude of the velocity in a given direction.

i) What is the Maxwell speed distribution?

ii) Form an appropriate integral ***using the speed distribution*** from part i) that will give the mean speed.

iii) By evaluating the integral of part ii) by parts with the substitution $v^2 \to x$, show that the mean speed is given by

$$\bar{v} = \sqrt{\frac{8k_B T}{\pi m}}$$

[10 marks]

c) In a gas the number of atoms colliding with or crossing unit area per unit time or flux, Φ, is given by $\Phi = \frac{1}{4} n\bar{v}$ where $n = \frac{N}{V}$ is the number density of atoms with N being the total number of atoms in a volume V.

i) Use the ideal gas equation of state and the mean speed at temperature T to rewrite the flux equation with $\Phi(P,T)$, as a function of pressure and temperature.

ii) A chamber containing two species of gas with densities, n_1 and n_2 and masses m_1 and m_2 has a small hole with diameter much less than the mean free path of the

Question continued overleaf

B4 c) cont.

molecules created in it. Show that the ratio of species 1 molecule to species 2 that have escaped, $\frac{N_1^e}{N_2^e}$ after some time *t* is given by

$$\frac{N_1^e}{N_2^e} = \frac{n_1}{n_2}\sqrt{\frac{m_2}{m_1}}$$

iii) Suppose we have a mixture of 10% ^{3}He and 90% ^{4}He in the first chamber and it is connected to a second chamber that is under vacuum, by a hole whose diameter is much smaller than the mean free path. What is the percentage of ^{3}He that collects in the second chamber?

[9 marks]

END OF PAPER