BSc/MSci   Examination by course unit

18 May 2009   2:30pm

PHY214   Thermal and Kinetic Physics   Duration 2hours 30 minutes

YOU ARE NOT PERMITTED TO START READING THIS QUESTION PAPER UNTIL INSTRUCTED TO DO SO BY AN INVIGILATOR.

Answer <u>ALL</u> questions from Section A and <u>ONLY TWO</u> questions from Section B.

Section A carries 40 marks and each question in section B carries 30 marks. An indicative marking scheme is shown in square [ ] brackets after each part of a question. Coursework comprises 20% of the final mark.

CALCULATORS ARE PERMITTED IN THIS EXAMINATION. PLEASE STATE ON YOUR ANSWER BOOK THE NAME AND TYPE OF MACHINE USED.

COMPLETE ALL ROUGH WORKINGS IN THE ANSWER BOOK AND CROSS THROUGH ANY WORK WHICH IS NOT TO BE ASSESSED.

CANDIDATES SHOULD NOTE THAT THE EXAMINATION AND ASSESSMENT REGULATIONS STATE THAT POSSESSION OF UNAUTHORISED MATERIALS AT ANY TIME WHEN A CANDIDATE IS UNDER EXAMINATION CONDITIONS IS AN ASSESSMENT OFFENCE. PLEASE CHECK YOUR POCKETS NOW FOR ANY NOTES THAT YOU MAY HAVE FORGOTTEN THAT ARE IN YOUR POSSESSION. IF YOU HAVE ANY THEN PLEASE RAISE YOUR HAND AND GIVE THEM TO AN INVIGILATOR NOW.

EXAM PAPERS CANNOT BE REMOVED FROM THE EXAM ROOM.

Examiners: K.J.Donovan

G.Thompson

DATA SHEET

You may wish to use some of the following data.

c = speed of light in vacuum   = 3×10^8 m s^{-1}

k_B = Boltzmann's constant   = 1.38×10^{-23} J K^{-1}

N_A = Avagadro's number   = 6.02×10^{23} mol^{-1}

R = Gas constant   =   8.31 J mol^{-1} K^{-1}

P_{atm} = Atmospheric pressure   = 1atm = 1.01×10^5 Pa

σ = Stefan-Boltzmann constant = 5.67×10^{-8} W m^{-2} K^{-4}

T_S = Ice point of water   =   273.15 K

c_P = Specific heat of water at constant pressure = 4.2×10^3 J K^{-1} kg^{-1}

c_{Eth} = Specific heat of alcohol at constant pressure   = 2.4×10^3 J K^{-1} kg^{-1}

c_P^{Ice} = Specific heat of ice at constant pressure   = 2.1×10^3 J kg^{-1}

l^{SL} = Latent heat of melting ice   = 3.33×10^5 J kg^{-1}

l^{LV} = Latent heat of evaporating water   = 2.26×10^6 J kg^{-1}

1 amu = One atomic mass unit   = 1.66×10^{-27} kg

Turn Over

SECTION A **(Answer ALL QUESTIONS in section A)**

Question A1

Write down the first law of thermodynamics in infinitesimal form for a *P-V* system _**using only state variables**_. Explain all of the symbols used. **[4]**

Question A2

Express the heat capacity of a gas at constant pressure, C_P, in terms of a partial differential _**using only state variables**_. Define all of the symbols used. **[4]**

Question A3

Write down an equation of state for a rigid diatomic gas relating the internal energy, U, to the pressure and volume, *P* and *V*. **[4]**

Question A4

A system consisting of a gas contained in a cylinder with a frictionless piston is taken around the closed path a $\rightarrow$ b $\rightarrow$ c $\rightarrow$ a as shown in figure 1 where the process c $\rightarrow$ a is isothermal. During one cycle the system does work 100J _**on**_ the environment.
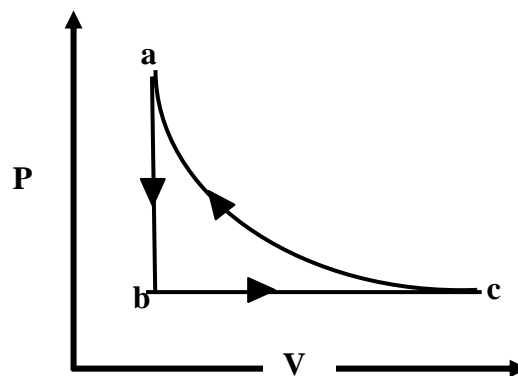

Figure 1

If the heat expelled during the isothermal leg of the cycle is -40J what is the work done by/on the system in the isobaric leg of the cycle going from b to c? _**Be careful to include the sign and make clear whether work is done by or on the system.**_ **[4]**

Question A5

During the combined process, a $\rightarrow$ b $\rightarrow$ c, shown in figure 1, how much heat is expelled/absorbed by the gas? _**Be careful to include the sign and make clear whether heat is expelled by or absorbed by the system.**_ **[4]**

Question A6

Write down the Thermodynamic Identity for a paramagnet and define all of the terms. **[4]**

Turn Over

Question A7

A refrigerator operating between a hot and cold reservoir delivers heat Q_2 from the cold reservoir to the working system whilst work W is performed on the system. How much heat Q_1 does the system give up to the hot reservoir? **[4]**

Question A8

A Carnot heat pump operates between hot and cold reservoirs at temperatures T_1 and T_2 respectively. Write down the efficiency of the Carnot heat pump. **[4]**

Question A9

Demonstrate the cyclic relation, $\left(\dfrac{\partial P}{\partial V}\right)_T \left(\dfrac{\partial V}{\partial T}\right)_P \left(\dfrac{\partial T}{\partial P}\right)_V = -1$, for the case of an ideal gas where P,

V and T have their usual meanings. **[4]**

Question A10

During an adiabatic process on a monatomic gas what is the relationship between initial temperature and volume T_i and V_i and final temperature and volume T_f and V_f in terms of the ratio of specific heat capacities $c_P/c_V = \gamma$? **[4]**

Turn Over

SECTION B **(Answer <u>TWO QUESTIONS ONLY</u> from section B)**

B1) **a)**

i) Write down an expression for the incremental entropy change, dS, of a thermodynamic system as an amount of heat, δQ_R, flows to or from that system at temperature T. Make clear how the sign of dS is determined. **[2]**

ii) Write down the second law as an inequality involving entropy. **[2]**

b) *1kg of ice at 0 ^{o}C is mixed with with 5kg of water at 25 ^{0}C in an adiabatic container at 1 atmosphere.*

i) What is the final state of the system? **[4]**

ii) What is the net entropy change of the water-ice system, ΔS ? **[4]**

iii) Is the mixing process reversible? **[2]**

iv) State with reason whether the Second Law of Thermodynamics has been obeyed?

[2]

c) *A mass of water, m, at temperature T_1 is mixed with an equal mass of water at temperature T_2 under isobaric and adiabatic conditions.*

i) Demonstrate that the entropy change of the universe is given by;

$$\Delta S_{Uni} = 2mc_P \ln\left[\frac{(T_1 + T_2)/2}{\sqrt{T_1 T_2}}\right]$$
[6]

ii) Considering the above expression for the entropy change of the universe express the requirement that the Second Law is obeyed in terms of T_1 and T_2. **[4]**

iii) Demonstrate that the Second Law has been obeyed. **[4]**

Hint: Use the fact that $(a - b)^2 > 0$ for the case when a and b are both real numbers

Turn Over

B2)   **a)**

     i)   A gas is expanded isobarically from pressure P_i and volume V_i to volume $2V_i$. What is the work done in this process and is it done by or on the gas?   **[3]**

     ii)   A gas is compressed adiabatically from pressure P_i and volume V_i to volume $\dfrac{V_i}{2}$. What is the work done in this process and is it done by or on the gas?   **[3]**

     iii)   A gas is compressed isothermally from pressure P_i and volume V_i to pressure $2P_i$ and volume $\dfrac{V_i}{2}$. What is the work done and is it done by or on the gas?   **[3]**

     b)   A diesel engine cycle may be approximated by the cycle shown in figure 1
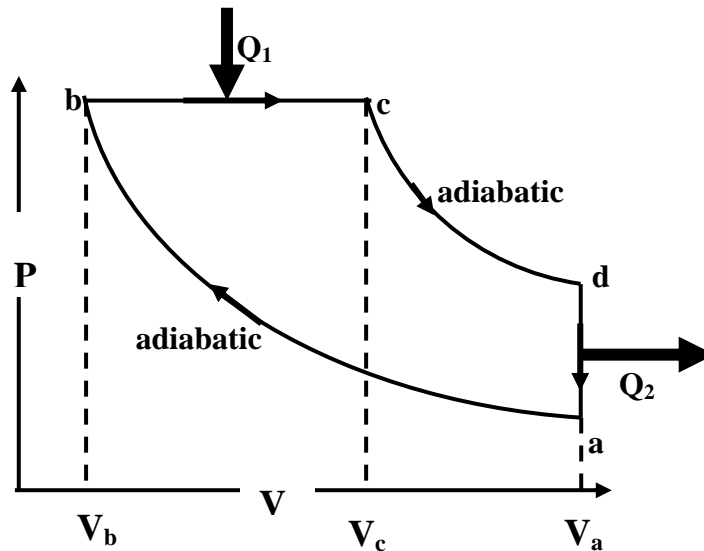

Figure 1

     i)   Write down an expression for the efficiency of the engine, η_E , in terms of heat flows Q_1 and Q_2 .   **[2]**

     ii)   Find an expression for the heat absorbed by the system, Q_1, in the isobaric process $b \rightarrow c$ in terms of the ***relevant*** heat capacity and the temperatures T_c and T_b **[2]**

     iii)   Find an expression for the heat rejected by the system, Q_2, in the isochoric process $d \rightarrow a$ in terms of the ***relevant*** heat capacity and the temperatures T_d and T_a **[2]**

     iv)   Show that in terms of the temperatures the engine efficiency is

$$\eta_E = 1 - \frac{1}{\gamma} \frac{T_d - T_a}{T_c - T_b}$$ **[2]**

Question continued overleaf   **Turn Over**

B2 continued

c)

i) We may define for the engine cycle of part b) an expansion ratio $\gamma_e = \dfrac{V_a}{V_c}$ and a compression ratio $\gamma_c = \dfrac{V_a}{V_b}$.

By considering the adiabats $c \rightarrow d$ and $a \rightarrow b$ show that the engine efficiency may be written in terms of the expansion and compression ratios as

$$\eta_E = 1 - \frac{1}{\gamma} \left[\frac{\gamma_e^{-\gamma} - \gamma_c^{-\gamma}}{\gamma_e^{-1} - \gamma_c^{-1}} \right]$$ **[7]**

ii) If an engine is designed to operate using a gas of rigid diatomic molecules with $V_a = 5000$ cm³, $V_b = 500$ cm³ and $V_c = 3000$ cm³, calculate the engine efficiency.

[3]

iii) If heat $Q_1 = 50$J is absorbed in one cycle and the engine is required to deliver 1kW of power how many cycles per second must the engine perform? **[3]**

Turn Over

B3)   **a)**   The Helmholtz free energy, F, for a fluid is defined as

$$F = U - TS$$

   i)   Use this definition and the thermodynamic identity to find the natural variables for F.   **[2]**

   ii)   Show that $S = -\left(\dfrac{\partial F}{\partial T}\right)_V$ and that $P = -\left(\dfrac{\partial F}{\partial V}\right)_T$.   **[2]**

   iii)   Use the information from parts i) and ii) to establish the Maxwell relation

$$\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$$

                                                                 [2]

   b)

   i)   If a particular gas has a Helmoltz function given by

$$F = -\frac{n^2 a}{V} - RT\,ln(V - nb) + j(T)$$

   where V is the volume of the gas, b is a constant and $j(T)$ is a function of temperature T only. Use information from part a) to demonstrate that this is a van der Waals gas with equation of state

$$\left(P + \frac{n^2 a}{V^2}\right)(V - nb) = nRT$$ **[4]**

   ii)   If this gas is expanded isothermally from V_i to V_f give an expression for the change in entropy of the gas, ΔS.   **[3]**

   iii)   If the temperature of the gas was to be raised from T_i to T_f at constant volume and the entropy change ΔS was measured to be $\Delta S = \dfrac{nk_B}{T_0}\left(T_f - T_i\right)$ where T_0 is some constant temperature, what is the function of temperature $j(T)$ in the expression for F?   **[3]**

   c)

   i)   Write down an expression for the heat capacity, C_V, of a P-V system *in terms of partial differentials involving only state variables* and hence show that $C_V = \dfrac{3}{2}nR$ for a monatomic ideal gas.   **[2]**

Question continued overleaf

Turn Over

B3 continued

ii) Using the thermodynamic identity for a *P-V* system show that

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial S}{\partial V}\right)_T - P \qquad \textbf{[2]}$$

iii) Use the result from ii) with any necessary result from a) and the equation of state of the van der Waals gas,

$$\left(P + \frac{n^2 a}{V^2}\right)(V - nb) = nRT$$

to show that for such a gas

$$\left(\frac{\partial U}{\partial V}\right)_T = \frac{n^2 a}{V^2} \qquad \textbf{[5]}$$

iv) The heat capacity at constant volume of a van der Waals gas is the same as that for an ideal gas. Use this fact and the result from iii) to show that the internal energy of a van der Waals gas is given by

$$U = \frac{3}{2}nRT - \frac{n^2 a}{V} + const \qquad \textbf{[5]}$$

Turn Over

B4)

a) For a gas of molecules of mass m at temperature T the Maxwell speed distribution function is given by

$$P(v) = \left(\frac{m}{2\pi k_B T}\right)^{3/2} 4\pi v^2 \exp\left[-\frac{mv^2}{2k_B T}\right].$$

i) The average speed of a molecule also known as the mean speed is related to the speed distribution function by

$$\bar{v} = \int_0^\infty vP(v)dv.$$

Evaluate this integral by parts and show that

$$\bar{v} = \sqrt{\frac{8k_B T}{\pi m}}.$$ **[7]**

Hint: **Make the substitution $v^2 \to x$ in the integral and write $a = \dfrac{m}{2k_B T}$**

The number of atoms colliding with or crossing unit area per unit time or flux, Φ, is given by $\Phi = \frac{1}{4} n\bar{v}$ where $n = \dfrac{N}{V}$ is the number density of atoms with N being the total number of atoms in a volume V.

ii) Using the ideal gas equation of state to calculate n, find the number density, n, of Oxygen molecules in the air at room temperature (= 300 K) and atmospheric pressure where O_2 makes up 20% of the molecules in the atmosphere.   **[4]**

iii) Given the mass of an O_2 molecule is 32 amu, find the number of O_2 molecules hitting 1 cm^2 of your lung per second.   **[4]**

b)

If a first chamber containing a mixture of two species of gas molecule is connected to a second chamber, under vacuum, by a hole whose diameter is small compared to the mean free path then the number of molecules escaping from the first to the second chamber via the hole will depend on the species. The process is known as effusion.

B4 continued

i) Using some of the information given in part a) show that the ratio of the number of species 1 molecule to species 2 that have escaped, $\dfrac{N_1^e}{N_2^e}$, is given by

$$\frac{N_1^e}{N_2^e} = \frac{n_1}{n_2} \sqrt{\frac{m_2}{m_1}}$$ **[4]**

ii) Suppose we have a mixture of 10% ^{3}He and 90% ^{4}He in the first chamber. What is the ratio of these isotopes that collects in the second chamber? **[4]**

iii) To produce a mixture with 30% ^{3}He we would need to repeat this procedure on the newly produced mixture and continue this recycling approximately how many times? **[7]**

END OF PAPER