BSc/MSci EXAMINATION

PHY-213 Space Time and Gravity

Time Allowed: 2 hours 15 minutes

Date:

Time:

Instructions: **Answer ALL questions in section A. Answer ONLY TWO questions from section B. Section A carries 40 marks, each question in section B carries 30 marks. An indicative marking-scheme is shown in square brackets [ ] after each part of a question.**

COMPLETE ALL ROUGH WORKINGS IN THE ANSWER BOOK AND CROSS THROUGH ANY WORK WHICH IS NOT TO BE ASSESSED.

NUMERIC CALCULATORS ARE PERMITTED IN THIS EXAMINATION.

Course Organiser: Dr. D S Berman

Deputy Course Organiser: Prof. W J Spence

Data

Gravitational constant	G	6.67×10^{-11}	$\mathrm{Nm}^2\mathrm{kg}^{-2}$
Speed of light	c	3.0×10^{8}	$m\mathrm{s}^{-1}$
Mass of Sun	M_{sun}	1.99×10^{30}	kg
Radius of Sun	R_{sun}	6.96×10^{8}	m
Distance of Earth from Sun	1 Au	1.50×10^{11}	m

YOU ARE NOT PERMITTED TO START READING THIS QUESTION PAPER UNTIL INSTRUCTED TO DO SO BY AN INVIGILATOR

Questions

Section A

A1. State the principle of equivalence. [6]

A2. Describe how a covariant tensor denoted A_μ transforms under a coordinate transformation. [6]

A3. On the surface of a sphere, whose radius is R:

i) Do lines that begin in *parallel* converge, diverge or remain parallel? [1]

ii) Is the ratio of the circumference to the diameter of a circle on the sphere bigger than or less than 2π? [1]

iii) Do the angles of a triangle on the sphere add up to more than, less than or are equal to 180 degrees? [1]

iv) Does the curvature get larger or smaller as R increases? [1]

v) Describe a geodesic on the surface of the sphere. [1]

vi) Does a cylinder have positive, negative or zero cuvature? [1]

A4. Write down Einstein's equation for the vacuum defining any symbols used. [4]

A5. Consider a blackhole.

Describe what happens to the light cone as one goes from infinity, crosses the blackhole horizon and then continues to the centre of the blackhole. Where does the curvature become infinite? [7]

A6. Name three tests of general relativity. [6]

A7. What is the cosmological principle? [5]

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Section B

B1. The line element of a two dimensional hyperbolic space is given by:

$$ds^2 = \frac{1}{y^2}(dx^2 + dy^2)$$

i) What is the metric, $g_{\mu\nu}$ and its inverse $g^{\mu\nu}$. [2]

ii) Calculate all the Christoffel Symbols for this space? [14]

You may use:

$$\Gamma^{\alpha}{}_{\beta\gamma} = \frac{1}{2} g^{\alpha\tau} (\partial_\beta g_{\tau\gamma} + \partial_\gamma g_{\tau\beta} - \partial_\tau g_{\beta\gamma}) \, .$$

iii) A straight line in flat space may be defined by the equation:

$$U^\nu \frac{\partial}{\partial x^\nu} U^\mu = 0$$

Where U^μ is the tangent vector given by:

$$U^\mu = \frac{dx^\mu}{ds} \, .$$

Derive the geodesic equation by making the equation for a straight line covariant. [10]

iv) Write out the geodesic equations for the hyperbolic space described above. [4]

B2. This question is for the spacetime exterior to a spherically symmetric mass distribution.

i) Write out the metric. [4]

ii) What does the metric become in the weak field limit. [4]

iii) What is the effective velocity of light due to the curvature of spacetime (in the weak field limit) and what astronomical effect does this lead to? [10]

iv) Derive the formula for gravitation redshift (in the weak field limit). [8]

v) A hydrogen atom emits a photon with wavelength $\lambda = 410.2nm$ from the surface of the sun. What is the wavelength seen by an observer on the earth? [4]

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B3. **i)** Explain the relationship between matter and the geometry of spacetime giving as much detail as possible. In particular, include a description of how the curvature of spacetime is related to the metric of spacetime. [14]

ii) Beginning with:

$$D_\rho R^\epsilon{}_{\mu\nu\sigma} + D_\nu R^\epsilon{}_{\mu\sigma\rho} + D_\sigma R^\epsilon{}_{\mu\rho\nu} = 0\,.$$

Show that

$$D_\mu (2R^{\mu\nu} - g^{\mu\nu} R) = 0\,.$$

[10]

iii) How does the above identity imply the conservation of energy? [6]

B4. The flat FRW universe is described by the following line element:

$$ds^2 = -dt^2 + R(t)^2 (d\sigma^2 + \sigma^2 d\Omega^2_{(2)})$$

i) Explain Hubble's law and derive the relationship between Hubble's constant and the function $R(t)$ in the FRW universe described above. [14]

ii) Write down the stress energy tensor for the matter dominated unverse and for the radiation dominated universe in a frame static with respect to cosmological time. [4]

iii) Use the conservation of mass to find an equation relating ρ and $R(t)$ in a matter dominated universe. [4]

iv) The first Einstein equation for the FRW matter dominated spactime is:

$$8\pi G\rho = 3\left(\frac{\dot{R}(t)}{R(t)}\right)^2$$

Use this and your answer to part (iii) to find $R(t)$. [8]

End of Examination Paper
Dr D S Berman