UNIVERSITY COLLEGE LONDON
DEPARTMENT OF PHYSICS AND ASTRONOMY

2B21 MATHEMATICAL METHODS IN PHYSICS AND ASTRONOMY

Problem Sheet M10 (2003–2004)

Solutions to be put in my pigeon hole by Tuesday 12 January 2004

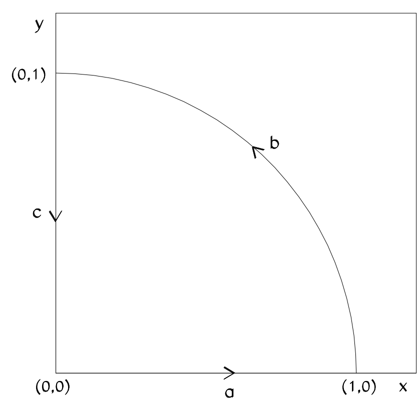
1. In plane polar coordinates, where the Cartesian components are given by $x = r\cos\theta$ and $y = r\sin\theta$, show that the unit vector in the θ direction is

$$\underline{\hat{e}}_\theta = -\sin\theta\,\underline{\hat{e}}_x + \cos\theta\,\underline{\hat{e}}_y\,.$$ **[2 marks]**

Calculate the line integral $I = \oint_\gamma \underline{W}\cdot\underline{ds}$ of the vector

$$\underline{W} = (x+y)\,\underline{\hat{e}}_x + xy^2\,\underline{\hat{e}}_y + x^2\,\underline{\hat{e}}_z$$

anticlockwise around the figure shown in the plane $z = 0$. This consists (a) of the axis $y = 0$, (b) a quarter-circle of radius 1, with its centre at the origin, and (c) the axis $x = 0$. **[10 marks]**



Evaluate $curl\,\underline{W} = \nabla\times\underline{W}$ and hence verify Stokes' theorem by integrating $curl\,\underline{W}$ over the area of the quarter-circle in the x-y plane. **[6 marks]**

2. An eighth of a sphere $x^2+y^2+z^2$ lies in the first octant, $x\geq 0$, $y\geq 0$, $z\geq 0$. Calculate the net flux of the vector

$$\underline{F} = z\,\underline{\hat{e}}_x + y\,\underline{\hat{e}}_y + x\,\underline{\hat{e}}_z$$

through the three straight and one curved surfaces. **[10 marks]**

Verify the result using the divergence theorem. **[4 marks]**