UNIVERSITY COLLEGE LONDON
DEPARTMENT OF PHYSICS AND ASTRONOMY

2B21 MATHEMATICAL METHODS IN PHYSICS AND ASTRONOMY

Problem Sheet M9 (2003–2004)

Solutions to be handed in on Tuesday 9 December 2003

1. Using the expressions given in the hand-out sheets for the vector operators in spherical polar coordinates, evaluate $\nabla\psi$, where ψ is the scalar field

$$\psi = x^2 + y^2 - 2z^2 \,,$$

working in both Cartesian and spherical polar coordinates and showing that they are equal. **[8 marks]**

2. Working in Cartesian coordinates, verify the identity

$$\nabla \times (\underline{A} \times \underline{B}) = \underline{A}(\nabla \cdot \underline{B}) - (\underline{A} \cdot \nabla)\underline{B} + (\underline{B} \cdot \nabla)\underline{A} - \underline{B}(\nabla \cdot \underline{A}) \,.$$

for the vector fields

$$\underline{A} = x\,\hat{\underline{e}}_x + y\,\hat{\underline{e}}_y + z\,\hat{\underline{e}}_z \,,$$
$$\underline{B} = -y\,\hat{\underline{e}}_x + x\,\hat{\underline{e}}_y \,.$$

[10 marks]

3. In spherical polar coordinates, the above vectors become

$$\underline{A} = r\,\hat{\underline{e}}_r \,,$$

$$\underline{B} = r\sin\theta\,\hat{\underline{e}}_\phi \,.$$

Using the vector operators in spherical polar coordinates, verify the identity in question 2 in this system. **[12 marks]**

Note that in this coordinate system the directions of the basis vectors depend upon the values of θ and ϕ.