

UNIVERSITY COLLEGE LONDON
DEPARTMENT OF PHYSICS AND ASTRONOMY
2B21 MATHEMATICAL METHODS IN PHYSICS AND ASTRONOMY

Problem Sheet M1 (2003–2004)

Solutions to be handed in on Tuesday 14 October 2003

1. Solve the simultaneous equations $y_1 = x_1 + 2x_2$ and $y_2 = 3x_1 + 4x_2$ for x_1 and x_2 . Putting both sets of equations in matrix form,

$$\underline{y} = \underline{A}\underline{x} \quad \text{and} \quad \underline{x} = \underline{B}\underline{y},$$

write down the 2×2 matrices \underline{A} and \underline{B} . Show that $\underline{A}\underline{B} = \underline{I} = \underline{B}\underline{A}$, where \underline{I} is the 2×2 unit matrix. [8 marks]

2. Evaluate the 4×4 determinant

$$\Delta = \begin{vmatrix} 2 & 1 & 0 & 3 \\ 1 & 0 & 3 & 2 \\ 0 & 3 & 2 & 1 \\ 3 & 2 & 1 & 0 \end{vmatrix}$$

by either expanding by the first row or by taking linear combinations to reduce the size of the determinant. [5 marks]

3. For the 3×3 matrices

$$\underline{A} = \begin{pmatrix} 1 & -1 & 1 \\ -3 & 2 & -1 \\ -2 & 1 & 0 \end{pmatrix} \quad \text{and} \quad \underline{B} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 2 & 3 \end{pmatrix},$$

evaluate the products $\underline{C} = \underline{A}\underline{B}$ and $\underline{D} = \underline{B}\underline{A}$. [4 marks]

Show that, although $\underline{C} \neq \underline{D}$, the determinants of \underline{C} and \underline{D} are both equal to the product of the determinants of \underline{A} and \underline{B} . [3 marks]

Show also that the sums of the diagonal elements of \underline{C} and \underline{D} are the same. [2 marks]

Some evidence of working is required. Examination calculators will NOT be able to handle determinants and matrices!