Queen Mary
UNIVERSITY OF LONDON

M.Sc. EXAMINATION BY COURSE UNITS

ASTM108 Cosmology

25 May 2007
Time 14:30-17:30

You may attempt as many questions as you wish and all questions carry equal marks. Except for the award of a bare pass, only the best FOUR questions answered will be counted.

The question paper must not be removed by candidates from the examination room. You must not start reading the question paper until instructed to do so.

Calculators ARE permitted in this examination, but no programming, graph plotting or algebraic facility may be used. The unauthorised use of material stored in a pre-programmable memory constitutes an examination offence. Please state on your answer book the name and type of machine used.

The following constants may be assumed:

Speed of light, $c = 3.0 \times 10^8\,\mathrm{m\,s^{-1}}$

Gravitational constant, $G = 6.67 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$

Boltzmann's constant, $k_B = 1.38 \times 10^{-23}\,\mathrm{JK^{-1}}$

Proton mass–energy, $m_p c^2 = 938.3\,\mathrm{MeV}$

Neutron mass–energy, $m_n c^2 = 939.6\,\mathrm{MeV}$

Mega Parsec, $1\,\mathrm{Mpc} = 3.09 \times 10^{22}\,\mathrm{m}$

Hubble time, $H_0^{-1} = 9.8 \times 10^9 h^{-1}\,\mathrm{yr} = 3.09 \times 10^{17} h^{-1}\,\mathrm{s}$

The Conversion Factor, $1\,\mathrm{eV} = 1.602 \times 10^{-19}\,\mathrm{J}$

The following formulae may be assumed:

Friedmann Equation

$$H^2 = \frac{8\pi G}{3}\rho + \frac{8\pi G}{3}\Lambda - \frac{kc^2}{a^2}$$

where $H = \dot{a}/a$ is the Hubble parameter, a is the scale factor of the universe, ρ is the mass density, Λ is the cosmological constant, k is a constant and overdots denote time derivatives.

Conservation Equation

$$\dot{\rho} + 3H\left(\rho + \frac{p}{c^2}\right) = 0$$

where p represents the pressure of the matter in the universe.

1. (a) *[5 marks]* Explain what is meant by the isotropy and homogeneity of the universe. Over what distance scales does the Cosmological Principle apply?

(b) *[5 marks]* Given that the observable universe contains approximately 10^{78} baryons (protons and neutrons), estimate the density of these particles in the universe at the present time in units of $\mathrm{kg\,m^{-3}}$. [You may assume that the observable universe is a sphere of radius 5 Gpc].

(c) *[4 marks]* For the best-fit observed value of the Hubble constant, estimate the present value of the critical density of the universe.

(d) *[5 marks]* By comparing your answers to parts (b) and (c) deduce the ultimate fate of our universe if it contains only baryons. Sketch the corresponding behaviour of the scale factor with time. [You should explain your reasoning.]

(e) *[6 marks]* Summarize, briefly, the observational evidence that there is some form of non-baryonic dark matter in the universe.

2. (a) *[2 marks]* Define the deceleration parameter, q, in terms of the scale factor and its first two time derivatives.

(b) *[6 marks]* Assume the universe contains pressureless matter and a cosmological constant. By differentiating the Friedmann equation with respect to time, show that

$$q = \frac{\Omega_{\mathrm{m}}}{2} - \Omega_{\Lambda},$$

where $\Omega_{\mathrm{m}} = \rho/\rho_{\mathrm{crit}}$ and $\Omega_{\Lambda} = \Lambda/\rho_{\mathrm{crit}}$ represent the densities of the matter and cosmological constant, respectively, relative to the critical density, ρ_{crit}.

(c) *[5 marks]* If $\Omega_{\Lambda} = 0$, explain in outline how deviations from Hubble's law at sufficiently high redshifts can in principle be used to determine the present density of the universe.

(d) *[6 marks]* Summarize, briefly, the key cosmological observations that indicate the total density of the universe is close to the critical density, $\Omega_0 = 1$, and that $q_0 = -0.55$, i.e., that the universe is presently undergoing accelerated expansion.

[This question continues overleaf . . .]

(e) *[6 marks]* Give two reasons why the universe can not always have been undergoing accelerated expansion.

3. (a) *[2 marks]* Define redshift, z, in terms of the scale factor.

(b) *[4 marks]* Starting from the definition of the Hubble parameter, show that the age of the universe at time t can be expressed in the form

$$t = \int_0^a \frac{da}{aH}.$$

Hence, show that the age of the universe is related to redshift by

$$t(z) = -\int_\infty^z dz \, \frac{1}{(1+z)H(z)}.$$

(c) *[5 marks]* Assume that the universe is spatially flat and contains only pressureless matter with a density varying as $\rho \propto 1/a^3$. Derive the dependence of the Hubble parameter on redshift and hence deduce that the age of the universe is related to redshift by

$$t(z) = \frac{2}{3H_0} \frac{1}{(1+z)^{3/2}}.$$

(d) *[5 marks]* Write a short note on the physical processes that occurred during the decoupling era.

(e) *[5 marks]* Given that the universe was about a thousand times hotter at the decoupling era than it is at the present time, use your answer to part (c) to estimate a numerical value for the age of the universe at that time.

(f) *[4 marks]* How would the age of the universe for a given value of the Hubble parameter be affected if the universe was negatively curved instead of being spatially flat?

4. (a) *[4 marks]* The Friedmann equation can be expressed in the form

$$\Omega - 1 = \frac{kc^2}{a^2 H^2},$$

where Ω is the total density of the universe relative to the critical density. Suppose that the value of Ω at some early time t_e is given by $\Omega(t_e) = 2$. Explain why we should not expect to observe $\Omega_0 < 1$ at the present time.

(b) *[6 marks]* Assume that the scale factor has varied as $a \propto t^{2/3}$ for most of its history. If $\Omega_0 = 1.04$ today, calculate the value of Ω at the time $t_e = 10^{-34}$ sec.

[This question continues overleaf ...]

(c) *[5 marks]* Why is the value you obtain in part (b) regarded as a problem for the standard big bang scenario?

(d) *[5 marks]* Explain how a period of inflation, where the scale factor grows exponentially with time such that $a \propto e^{H_I t}$ where H_I is a constant, can solve the flatness problem.

(e) *[5 marks]* Suppose inflation started at a time $t_b = 10^{-36}$ sec when $\Omega(t_b) = 2$ and ended at a time $t_e = 10^{-34}$ sec. Estimate the value of H_I in units of sec^{-1} that is required to solve the flatness problem.

5. (a) *[4 marks]* The temperature, T, of a universe dominated by relativistic particles is related to its age, t, by the relation

$$T \approx \frac{2 \times 10^{10}}{\sqrt{t}},$$

where T is measured in degrees Kelvin and t is measured in seconds.

State how the energy density of relativistic particles varies with temperature and deduce how the density varies with time. Write down the equation of state for relativistic particles.

(b) *[5 marks]* Using your answer to part (a), together with the conservation equation given in the rubric, show that the scale factor for a universe dominated by relativistic particles varies as $a \propto t^{1/2}$. Over what timescales during the history of the universe do you expect such behaviour to apply? [You should quote some order of magnitude estimates for the relevant timescales involved].

(c) *[5 marks]* Given that the typical energy of a relativistic particle in a thermal distribution is $3k_BT$, estimate the age of the universe when the protons became non-relativistic.

(d) *[5 marks]* Write a short note describing the physical processes that occurred during primordial nucleosynthesis.

(e) *[6 marks]* Suppose that when the temperature of the universe was 2×10^{23} K, a small fraction of matter in the universe formed into non-relativistic, stable particles with an initial density 10^{-8} times that of the relativistic particles. Estimate the age of the universe when these particles come to dominate the density.

6. (a) *[5 marks]* Why is the existence of the Cosmic Microwave Background regarded as key evidence in favour of the hot big bang scenario?

(b) *[4 marks]* The horizon distance (the maximum distance a light pulse could have travelled since the origin of the universe at $t = 0$) is given by

$$d_H(t) = a(t) \int_0^t \frac{c}{a(t)} dt.$$

[This question continues overleaf . . .]

Assuming a spatially flat universe with vanishing cosmological constant, derive an expression for the horizon distance at the present time in terms of the age of the universe, t_0.

(c) *[5 marks]* Explain what is meant by the 'rescaled horizon distance'. Derive an expression for the rescaled horizon distance that corresponds to the horizon distance at the epoch of decoupling, t_{dec}. Use numerical estimates for t_{dec} and t_0 to determine the factor by which such a rescaled horizon distance is less than the horizon distance at the present time.

(d) *[5 marks]* Using your answer to part (c), explain why the observed isotropy of the Cosmic Microwave Background is regarded as a problem for the hot big bang scenario?

(e) *[6 marks]* Discuss, briefly, the three primary sources of anisotropy in the temperature of the Cosmic Microwave Background.

[End of examination paper]