VIII

ASTR1108 COSMOLOGY
EXERCISE SHEET 8

1. Universe curvature dominated $\Rightarrow$ Friedmann eq^n is $\frac{\dot{a}^2}{a^2} = \frac{-kc^2}{a^2} = \frac{1}{a^2}$ for $kc^2 = -1$.

solving $\Rightarrow$ $a \propto t$.

$aT = \text{constant} \Rightarrow T \propto \frac{1}{t}$

Normalize from standard expression $\frac{T}{2\times10^{10}K} \triangleq \left(\frac{t}{sec}\right)^{-\frac{1}{2}}$ for $T \triangleq 3\times10^{25}K$

$\Rightarrow t \triangleq 10^{-30}s$

$\Rightarrow \quad \frac{T}{3\times10^{25}K} = \frac{10^{-30}\,sec}{t}$

For $T = 3K \Rightarrow t \triangleq 10^{3}s$

2. $n_{\gamma,0} \triangleq$ number density of photons today $\triangleq 3.7\times10^{8}\,m^{-3}$

$\rho_{\nu,0} = m_\nu n_{\nu,0}$

$\Omega_{\nu,0} = \frac{\rho_{\nu,0}}{\rho_{crit,0}} = \frac{m_\nu n_{\nu,0}}{\rho_{crit,0}} = \frac{m_\nu n_{\nu,0}}{1.88\times10^{-26}h^2}\ kg\,m^{-3}$

$\Rightarrow h^2\Omega_{\nu,0} = \frac{m_\nu \times 3.7\times10^{8}}{1.88\times10^{-26}}\ kg^{-1}$

$\Rightarrow m_\nu = (h^2\Omega_{\nu,0}) \times 5\times10^{-35}\,kg$

$\Rightarrow m_\nu c^2 = (h^2\Omega_{\nu,0}) \times 0.5\times10^{-18}\,J \triangleq 30\,h^2\Omega_{\nu,0}\ eV$

Since $\Omega_{total} \leq 1 \Rightarrow \Omega_{\nu,0} \leq 1$. Since $h<1 \Rightarrow m_\nu c^2 \leq 30\,eV$

For $m_\nu c^2 \triangleq 30\,eV$, becomes non-relativistic when $k_B T \triangleq m_\nu c^2 \triangleq 30\,eV$

$\Rightarrow T = \frac{0.5\times10^{-18}\,J}{k_B} \triangleq \frac{0.5\times10^{-18}}{1.4\times10^{-23}} \triangleq 3\times10^{5}\,K$

$\frac{t}{sec} = \left(\frac{T}{K}\right)^{-2} 20 \Rightarrow t \triangleq 10^{89}s \triangleq 100\ J^{-1}$