COSMOLOGY ASTM108, EXERCISE VII

2. If $T_{\nu,0} = 1.96$ K

Each neutrino has energy density $\rho_\nu = \frac{7}{16} \alpha T_{\nu,0}^4$. Since there are 6 types (including antiparticles ⇒

$$\rho_{\nu,0} = 6 \times \frac{7}{16} \times \left(\frac{T_{\nu,0}}{T_{\gamma,0}}\right)^4 \rho_{\gamma,0}$$

where $T_{\gamma,0}$ = temperature of radiation today & $\rho_{\gamma,0}$ = energy density of rad^n today.

For $T_{\gamma,0} = 2.728$ K ⇒ $\rho_{\nu,0} = \frac{21}{8}\left(\frac{1.96}{2.728}\right)^4 \rho_{\gamma,0} \simeq 0.7\, \rho_{\gamma,0}$ ie energy density of neutrinos ≃ 70% that of cosmic microwave background radiation

3. Start with relation aT = constant = $a_0 T_0$
Size of universe today ≃ 10^{28} cm
Temperature of universe today ≃ 3 K

$$\Rightarrow \underline{aT = 3\times10^{28} \text{ cm K}}$$

a) Size of head ≃ 10 cm ⇒ $T_{head} \simeq 3\times10^{27}$ K

$T_{head} > T_{eq}$ ⇒ age given by $\frac{t}{sec} \simeq \left(\frac{T}{1.5\times10^{10}\,K}\right)^{-2}$

T_{eq}: temperature @ radiation-matter equality

⇒ $t_{head} \simeq 3\times10^{-35}$ s

ie roughly the time inflation occurred.

b) Room temperature $T_{room} \simeq$ ~~15 K~~ 300 K $\ll T_{eq}$ ⇒ employ Eq. (8.36) from notes

$$\frac{t}{sec} \simeq \left(\frac{T}{9.4\times10^{11}\,K}\right)^{-\frac{3}{2}}$$ for $h \simeq O(1)$ ⇒ $t \simeq 2\times10^{16}$ secs ≃ 5×10^{8} yrs

$$a_{room} \simeq \frac{a_0 T_0}{T_{room}} \simeq \frac{3\times10^{28}}{15} \text{ cm} \simeq 2\times10^{27} \text{ cm}$$

For $T_{room} \simeq 300$ K ⇒ $t \simeq 2\times10^{14}$ s.

c) Particle energy ≃ $E \simeq k_B T \simeq 100$ GeV ⇒ $T \simeq \frac{100 \text{ GeV}}{k_B}$

$k_B \simeq 8.6\times10^{-5}$ eV K^{-1} ⇒ $T \simeq \frac{100\times10^9}{8.6\times10^{-5}}$ K ≃ 10^{15} K

Since 10^{15} K $\gg T_{eq}$, age is $\frac{t}{sec} \simeq \left(\frac{T}{1.5\times10^{10}\,K}\right)^{-2} \simeq \left(\frac{10^{15}}{1.5\times10^{10}}\right)^{-2}$ ⇒ $t \simeq 2\times10^{-10}$ s

5. Nucleosynthesis starts when $k_BT \simeq 0.1\,\text{MeV}$. Age of universe @ that time, $t_{nuc} \simeq 400$ secs

From lectures, ratio of $\left(\frac{N_n}{N_p}\right)$ @ 0.8 MeV when n^0 & p^+ fall out of equilibrium is

$N_n/N_p = 0.2$, By 400s, $\frac{N_n}{N_p} = \left(\frac{N_n}{N_p}\right)_{0.8\,\text{MeV}} \exp\left[-\ln 2 \times \frac{t_{nuc}}{t_{1/2}}\right]$

For $t_{1/2} = 94\,s \Rightarrow \frac{N_n}{N_p} \simeq \frac{1}{5}\exp\left[-\frac{\ln 2 \times 400}{94}\right] = 0.2 \times e^{-2.95} \simeq 0.01$

$Y_4 \equiv \frac{4N_4}{N_n+N_p}$ (see lecture notes for notation) $\Rightarrow Y_4 = \frac{4\left(\frac{N_n}{2}\right)}{N_n+N_p} = \frac{2}{1+N_p/N_n} \simeq \frac{2}{1+100} \simeq 0.02$

6. a) Yes: For $k>0$, $\Omega>1$ & $\Omega \to \infty$ in finite time (for a universe dominated by pressureless matter ~~&~~ and/or radiation) because $\Omega \propto 1/H^2$. $H \to 0$ corresponds to the point of maximum expansion

b) NO! If $\Omega<1$ then $\Omega<1$ always because $\Omega - 1 = \frac{kc^2}{a^2H^2}$ & the sign of $\Omega-1$ is determined by the sign of k & this is ~~a~~ a constant.

7. $\rho_{mono} \propto \frac{1}{a^3} \Rightarrow \frac{\rho_{mono}}{\rho_{rad}} \propto a \Rightarrow \frac{\Omega_{mono}}{\Omega_{rad}} \propto a \propto t^{1/2}$

$\Rightarrow$ Normalizing @ time of formation $\Rightarrow \frac{\Omega_{mono}}{\Omega_{rad}} = \left(\frac{\Omega_{mono}}{\Omega_{rad}}\right)_{form}\left(\frac{t}{t_{form}}\right)^{\frac{1}{2}}$

Form @ temperature $T_{form} \simeq 3\times10^{28}\,K \Rightarrow t_{form} \simeq \left(\frac{3\times10^{28}}{1.5\times10^{10}}\right)^{-2}$ secs $\simeq 3\times10^{-37}\,s$

Monopoles begin to dominate when ρ_{mono} reaches ρ_{rad} ie $\Omega_{mono} = \Omega_{rad}$

Given that $\left(\frac{\Omega_{mono}}{\Omega_{rad}}\right)_{form} = 10^{-10}$ [since @ t_{form} $\rho_{rad} \gg \rho_{mono} \Rightarrow \Omega_{rad} \triangleq \Omega_{total} = 1$]

$\Rightarrow t_{dom} = t_{form} \times \left(\frac{\Omega_{mono}}{\Omega_{rad}}\right)_{form}^{-2} \simeq 3\times10^{-37}\times10^{20} \simeq 3\times10^{-17}\,s$

This is much earlier than epoch of nucleosynthesis $t_{nuc} \simeq 1$ sec
$\Rightarrow$ monopoles dominate before nucleosynthesis & this would imply that nucleosynthesis could not proceed, in violation of what we observed. This is the monopole problem.

$\Rightarrow T_{dom} \simeq 1.5\times10^{10}\left(3\times10^{-17}\right)^{-\frac{1}{2}}\,K$
$\simeq 3\times10^{18}\,K$