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TASI lectures on the Holographic Principle

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Abstract

These TASI lectures review the Holographic principle. The first lecture describes the puzzle of black hole information loss that led to the idea of Black Hole Complementarity and subsequently to the Holographic Principle itself. The second lecture discusses the holographic entropy bound in general space-times. The final two lectures are devoted to the ADS/CFT duality as a special case of the principle. The presentation is self contained and emphasizes the physical principles. Very little technical knowledge of string theory or supergravity is assumed.

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1 Black Hole Complementarity

New scientific ideas are usually characterized by simple organizing principles that can be expressed in a phrase or two. The invariance of the speed of light, the equivalence principle the uncertainty principle and survival of the fittest are famous examples. Is there a comparable simple summary of the new principles which our science is now uncovering? Some people think

it is supersymmetry, others think it is duality. But the real world is not supersymmetric, nor is it known to have dual descriptions in any deep sense. My own view is that the lasting idea will be the *holographic principle* [1][2], the assertion that the number of possible states of a region of space is the same as that of a system of binary degrees of freedom distributed on the boundary of the region. The number of such degrees of freedom is not indefinitely large but is bounded by the area of the region in Planck units. These lectures are about the motivations and evidence for this principle.

The holographic principle grew out of the deep insights of Bekenstein [3] and Hawking [4] in the 70's. In particular Hawking raised a very profound question concerning the consistency of gravitation and the usual operational principles of quantum mechanics [5]. To state the paradox clearly it is useful to think of a black hole as an intermediate state in a scattering process. Particles, perhaps in the form of stars, galaxies or just ordinary quanta come together in an initial state $|in\rangle$. A black hole forms and evaporates leaving outgoing photons, gravitons neutrinos and other quanta. No energy is lost in the process so there are no unaccounted for degrees of freedom in the final state. According to the usual rules, such a process is described by a unitary scattering matrix S.

$$|out\rangle = S|in\rangle \tag{1.1}$$

Since S is unitary we can also write

$$|in\rangle = S^\dagger|out\rangle \tag{1.2}$$

In other words it must be possible to recover the initial quantum state from the final state in a unique way. However, Hawking gave arguments, that appeared to many as completely persuasive, that information is irretrievably lost when matter falls behind the horizon of the black hole. Thus, from an operational point of view, the rules of quantum mechanics as set out by Dirac would have to be modified as collision energies approach and exceed the Planck energy. In particular the conventional S matrix would not exist.

Not everyone believed Hawking's arguments [6] [7]. *Black hole complementarity* [8] and the holographic principle [1] [2] are counter-proposals that preserve intact the general principles of quantum mechanics but question some of the naive beliefs about locality and the objectivity and invariance of space-time events.

The Schwarzschild Black Hole

To understand the issues we will need to review the geometry of black holes. There are many kinds of black holes in string theory but we will confine our attention to the usual $3+1$ dimensional Schwarzschild case.

The ordinary Schwarzschild black hole is described by the metric

$$ds^2 = \left(1 - \frac{2MG}{r}\right) dt^2 - \left(1 - \frac{2MG}{r}\right)^{-1} dr^2 - r^2 d\Omega^2 \tag{1.3}$$

M, G and $d\Omega^2$ are the black hole mass, the gravitational constant and the metric of a unit 2-sphere. The curvature singularity at $r = 0$ will not concern us but the coordinate singularity at the Schwarzschild radius $r = 2MG$ defines the all important horizon. Despite its singular importance, the horizon is not a mathematical singularity of the geometry, at least in the usual sense. To see that let us concentrate on the "near horizon limit". We consider a small angular region near a point on the horizon. Define

$$y = r - 2MG \tag{1.4}$$

For $y << 2MG$ the metric has the form

$$ds^2 = \frac{y}{2MG}dt^2 - \frac{2MG}{y}dy^2 - dx^i dx^i \tag{1.5}$$

where dx^i is an element of length in the two dimensional plane tangent to the horizon. Now define

$$\begin{aligned} \rho &= \sqrt{8MGy} \\ \omega &= \frac{t}{4MG} \end{aligned} \tag{1.6}$$

and the metric takes the form

$$ds^2 = \rho^2 d\omega^2 - d\rho^2 - dx^i dx^i \tag{1.7}$$

Expression (1.7) is the metric of ordinary Minkowski space in hyperbolic polar coordinates. If we define

$$\begin{aligned} X^+ &= \rho e^{\omega} \\ X^- &= -\rho e^{-\omega} \end{aligned} \tag{1.8}$$

the metric becomes

$$ds^2 = dX^+ dX^- - dx^i dx^i \tag{1.9}$$

which is the standard light cone form of the Minkowski metric. ¿From this fact it is apparent that the horizon is not singular.

The relation between the flat minkowski coordinates $X^{\pm}$ and the Schwarzschild coordinates r, t is shown in figure(1) for the region outside the horizon. The entire horizon $r = 2MG$ is mapped to the point (2D-surface) $X^+ = X^- = 0$. The extended horizon is defined by the 3 dimensional surface $X^- = 0$. Notice that a signal originating from a point behind the horizon, $X^- > 0$ can never escape to the outside, $X^- < 0$. For the region $X^+ > 0$, the extended horizon coincides with the asymptotic limiting value of Schwarzschild time $t = \infty$. Although the flat Minkowski coordinates only describe the near horizon region, a generalization to Kruskal-Szekeres (KS) coordinates covers the whole

black hole space-time. Suppressing the angular coordinates Ω the KS metric has the form

$$ds^2 = F(X^+X^-)dX^+dX^- \tag{1.10}$$

where $F \to 1$ for $X^+X^- \to 0$ and

$$F \to \frac{16M^2G^2}{\rho^2} \tag{1.11}$$

for $X^+X^- \to \infty$. Equation (1.11) insures that the metric far from the black hole tends to flat space

$$ds^2 \to dt^2 - dr^2 - r^2 d\Omega^2 \tag{1.12}$$

In KS coordinates the singularity at $r = 0$ is defined by the *space-like* surface

$$X^+X^- = M^2G^2 \tag{1.13}$$

In figure (2) the geometry of the black hole is shown for the region $X^+ > 0$.

Now consider a particle trajectory which begins outside the black hole, falls through the horizon and eventually hits the singularity as shown in figure (3). In Schwarzschild coordinates the particle does not cross the horizon until infinite time has elapsed. Thus from the viewpoint of an observer outside the black hole, the particle asymptotically approaches the horizon, but never crosses it. Indeed, all the matter which collapsed to form the black hole never crosses the horizon in finite Schwarzschild time. Classically it forms progressively thinner layers which asymptotically approach the horizon.

On the other hand, from the point of view of a freely falling observer accompanying the infalling particle the horizon is crossed after a finite time. In fact from figure 3 it is obvious that nothing special happens to the infalling matter at the horizon. This discrepancy is the first instance of an under-appreciated complementarity or relativity between the descriptions of matter by external and infalling observers.

Penrose Diagrams

Penrose diagrams provide an intuitively clear way to visualize the global geometry of black holes. They are especially useful for spherically symmetric geometries. The Penrose diagram describes the r, t plane. Here are the rules for a Penrose diagram.

1. Use coordinates which map the entire geometry to a finite portion of the plane.

2. The coordinates should be chosen so that radial light rays correspond to line oriented at ± 45 degrees to the vertical.

As an example the Penrose diagram for flat space is shown in figure (4). The vertical axis is the spatial origin at $r = 0$ and the point labeled $r = \infty$ represents the asymptotic endpoints of space-like lines. The points $t = \pm\infty$

are the points where time-like trajectories begin and end. Light rays enter from past null infinity, $\Im^-$ and exit at future null infinity, $\Im^+$.

The Penrose diagram for the Schwarzschild geometry is shown in figure (5). As we will see the regions III and IV are unphysical. Region I is the outside of the black hole and like flat Minkowski space it has space-like, time-like and null infinities. Obviously future directed time-like or light-like trajectory that begins in region II will collide with the singularity. Thus region II is identified as being behind the horizon. The extended horizon (from now on called the horizon) is the light-like line $t = \infty$.

A real black hole must be formed in a collapse. Thus in the remote past there is no black hole and the geometry should resemble the lower portion of figure (4). At late times the black hole has formed and the geometry should resemble figure (5). Thus the Penrose diagram for the collapse looks is shown in figure (6).

Black Hole Thermodynamics

It is well known that black holes are thermodynamic objects [3] [4] [9]. In addition to their energy, M they have a temperature and entropy. To understand this we need to study the behavior of quantum fields in the near horizon geometry. We will see later that quantum field theory can not really be an adequate description of a world including gravity but it is a starting point which will allow us to abstract some important lessons.

As we have seen, the near horizon geometry is just Minkowski space described in hyperbolic polar coordinates. In particular the portion of the near horizon region ($X^+X^- < 0$) outside the black hole is called Rindler space.

The usual time coordinate of Minkowski space is $x^0 = \frac{X^+ + X^-}{2}$ and conjugate to it is the momentum component p_0. However, p_0 is not the energy or Hamiltonian appropriate to the study of black holes by distant observers. For such observers the natural time is the Schwarzschild time $t = 4MG\omega$. The conjugate Hamiltonian which represents the energy or Mass of the black hole is

$$H_t = \frac{1}{4MG} H_\omega = \frac{i}{4MG} \partial_\omega \tag{1.14}$$

where H_ω is a dimensionless Hamiltonian conjugate to the dimensionless Rindler time ω.

An observer outside the horizon has no access to the degrees of freedom behind the horizon. For this reason all observations can be described in terms of a density matrix $\Re$ obtained by tracing over the degrees of freedom behind the horizon [9]. To derive the form of the density matrix for external observations we begin with the Minkowski space vacuum. The coordinates of Minkowski space are

$$x^0 = (X^+ + X^-)/2$$

$$x^3 = (X^+ - X^-)/2 \tag{1.15}$$

and the horizon coordinates x^i. The instant of Rindler time $\omega = 0$ coincides with the half-surface

$$\begin{aligned} x^0 &= 0 \\ x^3 &> 0 \end{aligned} \tag{1.16}$$

The other half of the surface $x^3 < 0$ is behind the horizon and is to be traced over.

Let us consider a set of quantum fields labeled ϕ. To specify the field configuration at $x^0 = 0$ we need to give the values of ϕ on both half-surfaces. Let ϕ_I and ϕ_F represent the field configurations for $x^3 > 0$ and $x^3 < 0$ respectively. A quantum state is represented by a wave functional

$$\Psi(\phi) = \Psi(\phi_I, \phi_F) \tag{1.17}$$

We use the standard Euclidean Feynman path integral formula to compute Ψ.

$$\Psi(\phi_I, \phi_F) = \int d\phi \exp -S \tag{1.18}$$

where the path integral is over all fields in the future half space $ix^0 > 0$ with boundary condition $\phi = (\phi_I, \phi_F)$ at $x^0 = 0$.

The trick to compute the density matrix $\Re$ is to divide the upper half plane $ix^0 > 0$ into infinitesimal angular wedges as in figure (7). The path integral can then be evaluated in terms of a generator of angular rotations. This generator is nothing but iH_ω. Thus the expression for the Minkowski vacuum is

$$\Psi(\phi_F, \phi_I) = \langle \phi_F | \exp(-H_\omega \pi) | \phi_I \rangle \tag{1.19}$$

In other words the Minkowski vacuum wave functional is a transition amplitude for elapsed Euclidean time π.

Now consider the density matrix given by

$$\Re = \int d\phi_F \Psi^*(\phi_F, \phi_I') \Psi(\phi_F, \phi_I) \tag{1.20}$$

Using eq.(1.19)and the completeness of the states $\langle \phi_F |$ gives

$$\Re = \langle \phi_I' | \exp(-2\pi H_\omega) | \phi_I \rangle \tag{1.21}$$

or more concisely

$$\Re = \exp(-H_\omega / T_\omega) \tag{1.22}$$

with $T_\omega = 1/2\pi$.

Equation (1.22) is has the remarkable property of being a thermal density matrix for temperature T_ω. Notice that the derivation is exact and in no way relies on the free field approximation. It is valid for any quantum field theory for any strength of coupling.

The temperature $T_\omega = 1/2\pi$ does not have the usual dimensions of energy. This is because the Rindler time and therefore the Rindler Hamiltonian is dimensionless. To convert to a proper temperature with dimensions of energy we consider the proper time interval corresponding to an interval $d\omega$. From eq.(1.7)

$$ds = \rho d\omega \tag{1.23}$$

Thus an observer at distance ρ from the horizon converts from dimensionless quantities using the conversion factor ρ. The proper temperature at distance ρ is given by

$$T(\rho) = \frac{1}{\rho} T_\omega = \frac{1}{2\pi\rho} \tag{1.24}$$

In this way we arrive at the important conclusion that an observer outside a black hole but in the near horizon region will detect a temperature that varies as the inverse distance from the horizon [9].

Next consider the temperature as measured by a distant observer asymptotically far from the black hole. The proper time variable for such an observer is the Schwarzschild time $t = 4MG\omega$. Thus such distant observers measure temperature

$$T_H = \frac{T_\omega}{4\pi MG} \tag{1.25}$$

This is the Hawking temperature [4] of the black hole. It represents the true thermodynamic temperature of an isolated black hole.

The thermodynamic relation between temperature and mass (energy) allow us to compute an entropy for the black hole. Using

$$dM = TdS \tag{1.26}$$

we find

$$S = 4\pi MG \tag{1.27}$$

or in terms of the horizon area A

$$S = \frac{A}{4G} \tag{1.28}$$

Equation (1.28) is far more general than the derivation given here. It applies to every kind of black hole, be it rotating, charged or in arbitrary dimensions. In the general $(d+1)$ dimensional case the concept of two dimensional area only needs to be replaced by the $(d-1)$ dimensional measure of the horizon which we continue to call area.

The Thermal Atmosphere

Because the region above the horizon has a non-vanishing temperature, it has a kind of thermal atmosphere [10] consisting of thermally excited quanta. In regions where the field theory is weakly coupled the thermal atmosphere consists of ordinary black body radiation. Some of these quanta have sufficient energy to escape the gravitational pull of the black hole and appear as Hawking radiation. However, for a large black hole, this process is very slow. The equilibrium approximation for the thermal atmosphere of the near horizon region is a very good one.

The thermal atmosphere contributes to the entropy of the black hole [11]. Let us consider the ordinary quantum fields of the standard model or its suitable generalization. For simplicity lets ignore the interactions as well as masses. The entropy stored in the shell between ρ and $\rho + d\rho$ for free massless fields is given by

$$\frac{dS}{d\rho d^2 x^i} = cT^3 \tag{1.29}$$

where c constant proportional to the effective number of massless fields at that temperature. Using $T = 1/2\pi\rho$ we find

$$S \sim A \int \frac{d\rho}{\rho^3} \tag{1.30}$$

Evidently if this formula made sense all the way to $\rho = 0$ the entropy of the black hole would be infinite. But since we know that the entropy is $A/4G$ the field theory description must break down at some small ρ_0. In this case the entropy in the thermal atmosphere of ordinary quanta will be

$$S \sim Ac/\rho_0^2 \tag{1.31}$$

Since the total black hole entropy is $A/4G$ the contribution from the thermal atmosphere must be less than this. Accordingly [11] ρ_0 can not be smaller than $\sim G^{1/2}$.

Perhaps a more illuminating way to express this is to say that the number of effective degrees of freedom must tend to zero as the Planck temperature is approached [12]. In conventional quantum field theory the number of effective degrees of freedom is a non-decreasing function of temperature. The finiteness of black hole entropy is the first evidence that quantum field theory overestimates the number of independent degrees of freedom.

It is not too surprising that quantum field theory has too many degrees of freedom at short distances to describe a world with gravity. The non-renormalizability of quantum gravity has led to many suggestions of a Planck scale cutoff over the years. Roughly speaking, the idea was that there should be about 1 binary degree of freedom per Planck volume. What we will see in the following is that this idea still vastly overestimates the number of degrees

of freedom. The correct reduction in the number of degrees of freedom is that there is no more than $1/R$ degrees of freedom per Planck volume where R is infrared cutoff radius, that is, the size of the spatial region being studied.

The Quantum Xerox Principle

The Holographic Principle represents a radical departure from the principles of local quantum field theory. In order to understand why we are driven to it we need to follow Hawking's original arguments about the loss of quantum coherence in black hole processes. The argument as I will present it is based on a principle that I call the *quantum Xerox principle*. It prohibits the existence of a machine which can duplicate the information in a quantum system and in so doing, produce two copies of the original information. To illustrate an example, consider a two-state system with states $|u\rangle$ and $|d\rangle$. We will call the system a q-bit. The general state of the q-bit is

$$|\psi\rangle = a|u\rangle + b|d\rangle \tag{1.32}$$

Now assume we had a machine which could clone the q-bit and duplicate a second q-bit in the same state. We can express this by

$$|\psi\rangle \rightarrow |\psi\rangle|\psi\rangle \tag{1.33}$$

For example

$$\begin{array}{lcl} |u\rangle & \rightarrow & |u\rangle|u\rangle \\ |d\rangle & \rightarrow & |d\rangle|d\rangle \end{array} \tag{1.34}$$

Suppose a q-bit in the quantum state $|u\rangle + |d\rangle$ is fed into the machine. The output of the machine is

$$\begin{array}{rcl} \{|u\rangle + |d\rangle\} \otimes \{|u\rangle + |d\rangle\} = |u\rangle & \otimes & |u\rangle \\ +|d\rangle & \otimes & |d\rangle \\ +|d\rangle & \otimes & |u\rangle \\ +|u\rangle & \otimes & |d\rangle \end{array} \tag{1.35}$$

However this is inconsistent with the most basic principle of quantum mechanics, the linearity of the evolution of state vectors. Linearity together with eq. (1.34) requires

$$|u\rangle + |d\rangle \rightarrow |u\rangle \otimes |u\rangle + |d\rangle \otimes |d\rangle \tag{1.36}$$

In this way we see that the principles of quantum mechanics forbid the duplication of quantum information. What has all this to do with black holes?

Consider the following thought experiment [13]. A black hole is formed as in figure (6). Before the black hole has a chance to evaporate a q-bit is thrown in. According to the observer who falls with the q-bit, the information at

a later time will be localized behind the horizon at point (a) in figure (8). On the other hand an observer outside the horizon eventually sees all of the energy returned in the form of Hawking radiation. In order that the usual laws of quantum mechanics are satisfied for the outside observer, the q-bit of information must be found in the state of the outgoing evaporation products localized at point (b) in figure (8). Since there can not be two copies of the same information it would seem that either the infalling observer or the outside observer must experience a violation of the known laws of nature. Either the horizon is not such a benign place as we thought ($\sim$ Minkowski space) and infalling matter is severely disrupted or else the outside observer experiences a loss of information in contradiction with quantum principles!

The principle of Black Hole Complementarity flatly denies that either of these undesirable things happens. According to this principle no real observer ever detects a violation of the usual laws of nature. External observations are assumed to be consistent with a description in which infalling information is absorbed, thermalized near the hot horizon and returned in the form of subtle correlations in the Hawking radiation. Furthermore, infalling observers detect nothing unusual at the almost flat horizon and only experience violent effects as the singularity is approached. Reconciliation of these two facts will require that we radically modify our naive ideas of locality so that the space-time location of an event loses its invariant significance and becomes a relative concept.

As we have seen, quantum mechanics forbids information cloning. Let us take that to mean that no real observer is ever allowed to detect duplicate information. The outside observer has no problem with this since she can never detect signals from behind the horizon. However, it is more subtle to argue that observers behind the horizon can never detect duplicate information. Here is how it might happen:

An observer, O, stationed outside the horizon in figure (9) collects information stored in the Hawking radiation. After some time she has collected the information stored in the infalling q-bit. At that time, she jumps into the black hole, carrying the information to point (c) behind the horizon . Now there are two copies of the q-bit behind the horizon, one at (a) and one at (c). A signal from (a) to (c) can reveal that information has been duplicated. In fact we will argue that there is a *quantum Xerox censorship* mechanism which always prevents this from happening. To understand it we need one more concept.

Information Retention Time

Consider a conventional complex system such as a piece of coal. Suppose the coal begins in its ground state and is heated by shining a laser beam on it. As its temperature rises it begins to glow and emit thermal radiation. Assume the laser beam is modulated so that it can convey information and

that it sends in a bit.

Let S be the maximum entropy that the coal is heated to before being allowed to cool back to its ground state. By the time it does cool, all the information in the laser beam has been returned in the almost thermal radiation. An interesting question is how many photons are involved in carrying out the single bit. The answer has been given in a paper by Don Page [14]. The number of photons that have to be measured in order to collect a single bit is of order $S/2$. This is roughly half the photons that will be emitted. Another way to s say it is that no information can be retrieved until the coal has cooled to the point where its entropy is about half its maximum value.

Given the luminosity, this restriction on collecting information from thermal radiation can be translated to a time scale for the coal to retain the original bit. This time is called the information retention time. How long is it for a massive black hole? The answer can easily be deduced from the known luminosity of black holes. In $(3+1)$ dimensions one finds

$$t_R \sim G^2 M^3 \tag{1.37}$$

For times much shorter than t_R we can expect that information which has been absorbed by the thermal horizon to be inaccessible.

Quantum Xerox Censorship

Let us return to the thought experiment in figure (9) designed to detect information duplication behind the horizon. The resolution of the dilemma is as follows. The point (c) must occur before the trajectory of O intersects the singularity. On the other hand O may not cross the horizon until the information retention time has elapsed. The implication of these two constraints is most easily seen using KS coordinates

$$\begin{aligned} X^+ &= \rho e^{\omega} \\ X^- &= -\rho e^{-\omega} \\ \omega &= \frac{t}{4MG} \end{aligned} \tag{1.38}$$

An observer outside the horizon must wait a time $t \sim M^3 G^2$ to collect a bit from the Hawking radiation. Thus she may not jump into the black hole until $(X^+ \sim e^{M^2 G})$. On the other hand the singularity is at $X^+ X^- = M^2 G^2$. This means that O will hit the singularity at a point satisfying

$$X^- < \exp -M^2 G \tag{1.39}$$

Thus for the original infalling system to send a signal which will reach O before she hits the singularity, the message must be sent within a time interval δt of the same order of magnitude, an incredibly short time.

Classically there is no obstruction to sending as much information as you like in as small a time as you like using as little energy as you like. Quantum

physics changes this. A bit of information requires at least one quantum to transmit it. The uncertainty principle requires that the quantum have energy of order $(\delta t)^{-1}$. Thus the message requires a photon of energy

$$E_{signal} \sim \exp M^2 G \tag{1.40}$$

This is completely inconsistent with the assumption that the entire black hole, including the q-bit had energy M. If the observer at (a) had that much energy available, the black hole would have been much heavier and its horizon would have been at a very different place. Thus we see that quantum mechanics and gravity conspire to prevent O from detecting duplicate information.

We can now see that there is something wrong with the usual ideas of local quantum field theory in black hole backgrounds. The points (a) and (b) can be widely separated by a large space-like separation. Quantum field theory would say that the fields at these two points are independent commuting variables and it would predict correlations between them. But as we have seen, these correlations are unmeasurable by any real observer subject to the usual limitations of relativity and quantum mechanics. If you share the belief that a theory should not predict things which are in principle unobservable then you must conclude that local quantum field theory in a black hole background is the wrong starting point.

Baryon Violation and Black Hole Horizons

It is generally conceded that there are no additive conserved quantities in a consistent quantum theory that includes gravity except for those that couple to long range fields. If nothing else, black hole evaporation will lead to violations of global conservation laws such as baryon conservation. An interesting question is where in the black hole geometry does the violation take place? Does it happen at or near the almost flat horizon or only at the violently curved singularity [15] or, is it more subtle as suggested by black hole complementarity [13]?

For definiteness lets assume that baryon violation takes place in a conventional Grand Unification scheme such as $SU(5)$. Begin with a system of baryons and an observer all falling freely through the horizon of a very large black hole. Since the near horizon limit is nearly flat it is certain that the freely falling observer will detect negligible baryon violating effects in this region. However as time elapses the system will enter regions in which the curvature becomes of order the GUT scale. At that point there is every reason to think that baryon violating effects will be observed if the observer is in any shape to observe them.

The observer outside the black hole has a very different story to tell. According to him, the baryonic system entered the near horizon region where it was subjected to increasing proper temperature. When the temperature

becomes of order M_{gut} the baryons are exposed to a flux of high energy particles in the thermal atmosphere and baryon violating processes must occur. Who is correct?

In order to answer this question consider the propagation of a quark through empty space. Virtual baryon violating processes of the kind shown in the Feynman Diagram in figure (10) are continuously taking place. In other words the quark spends part of the time in a virtual state with the wrong baryon number even in empty flat space. What percentage of the time is the baryon number wrong? One might think the answer is incredibly small given the stability of the proton. But it is not. An explicit calculation gives a probability of order g^2 where g is the gauge coupling constant. Thus the quark has the wrong baryon number about 1 percent of the time. The reason we don't see this as baryon violation is that the lifetime of the intermediate states is of order the gut scale. The baryon number is constantly undergoing very rapid quantum fluctuations. The usual approximately conserved quantum number is the time averaged baryon number normalized to 1 for the nucleon. Now consider a quark falling through the horizon as in figure (11). It is evident from the figure that there is a significant probability that when the quark passes the horizon at $t = \infty$ it has the wrong baryon number. From the viewpoint of the infalling observer doing ordinary low energy experiments on the baryon the fluctuation is too fast to see. However, from the outside the rapid fluctuations slow down and the quark is caught frozen with the wrong baryon number. Of course the this description fails to take gravitation into account but it nevertheless shows that understanding the apparent contradictory descriptions involves analyzing the behavior of matter at extremely short time scales and high frequencies.

Another thought experiment can illuminate the interplay between gravity and quantum mechanics. Suppose an observer O falls through the horizon just before the baryon as in figure (12). This observer sends out a signal (photon) which interacts with the infalling baryon and measures its baryon number. The signal is then received by a distant observer. Let us suppose that the experiment is arranged so that the signal-photon encounters the baryon in a region where the temperature is at least M_{gut}. In the rest frame of the infalling quark, it has a time of order M_{gut}^{-1} before it crosses the horizon. Thus the photon must be concentrated in a wave packet of size less that or equal to M_{gut}^{-1}. Its energy must be so high that it will resolve the baryon violating virtual state and will therefore have a finite probability of reporting baryon violation at the horizon. Complementarity works!

String Theory at High Frequency

Ordinary quantum field theory can not resolve the paradoxes of black holes. We have already seen that Q.F.T. drastically overestimates the number of ultraviolet degrees of freedom in the near horizon region and leads to a di-

vergent entropy in the thermal atmosphere. String theory is widely believed to be a consistent quantum mechanical framework that includes gravitation. If so it must differ from Q.F.T. in very non-trivial ways at short times.

Although we are far from achieving a definitive understanding of black hole complementarity in string theory, there are some simple and suggestive ways to see that string theory is very different from Q.F.T. at high frequency [16].

Let us consider a string falling through a horizon. For our purposes we can approximate the horizon by the light-like surface $X^- = 0$. To study the string as it falls we use light cone coordinates. It is conventional to use X^+ for the light cone time variable. We are going to be unconventional and use X^-. Thus we choose the string theory gauge

$$\tau = X^- \tag{1.41}$$

The string starts out at negative X^- and reaches the horizon at $X^- = 0$.

Suppose the string falls through the horizon near $X^+ = 1$. Using

$$\begin{aligned} X^- &= -\rho e^{-\omega} \\ X^+ &= \rho e^{\omega} = 1 \end{aligned} \tag{1.42}$$

we find that near the string

$$X^- = \exp(-2\omega) = -\exp(-t/2MG) \tag{1.43}$$

The unusual properties of strings can already be seen at the level of free string theory. In light cone gauge a free string is described by a set of transverse coordinates $x^m(\sigma)$ where $0 \leq \sigma < 2\pi$. The coordinates are expressed in terms of harmonic oscillator variables $\alpha(n)$ and $\tilde{\alpha}(n)$. In string units

$$x(\sigma) = x_{cm} + \sum_n \frac{\alpha(n)}{n} e^{in(\tau-\sigma)} + \frac{\tilde{\alpha}(n)}{n} e^{in(\tau+\sigma)} \tag{1.44}$$

The question that will interest us has to do with the spatial size of the string. For simplicity we will consider the ground state of the string which classically has zero size. We usually envision the quantum fluctuations to spread the string over a size of order l_s, the string scale. However explicit calculation gives a very different result. The spatial size R will be defined in an obvious way.

$$R^2 = \langle 0|(x - x_{cm})^2|0\rangle \tag{1.45}$$

Using the standard commutation rules for the $\alpha's$ we find

$$R^2 = \sum_n \frac{1}{n} = \log(\infty) \tag{1.46}$$

Evidently the spatial size of the string is dependent of the frequency cutoff. If the frequency cutoff for a given observation is n_{max} then the apparent size of the string is

$$R^2 = \log n_{max} \tag{1.47}$$

We see a small string only if we average over sufficient time (τ) to eliminate the very high frequencies. This lesson is an important one and it will be repeated later in the form of the *ultraviolet infrared connection* in lecture III.

Consider the outside observer's description of the infalling string as it approaches the horizon. At any given point the string has a light cone time $|\tau|$ before it crosses the horizon at $\tau = 0$. Thus it makes no sense for the outside observer to average modes of frequency smaller than $|\tau|^{-1}$. In other words the frequency cutoff appropriate for an outside observer increases as the horizon is approached. Using eq.(1.47) and setting $n_{max} = |\tau|^{-1}$ we find

$$R^2 = \log \tau = t/2MG \tag{1.48}$$

Free string theory predicts that as a string falls toward the horizon it grows and appears to become an increasing tangled mass of string but only to the external observer. The infalling observer, depending on how she interacts with the string has a fixed time resolution and sees no growth.

The Space Time Uncertainty Relation

Even more revealing are the fluctuations of the longitudinal [17] coordinate X^+ (usually called X^-). First consider a classical point particle. It crosses the horizon, ($X^- = 0$), at a finite value of X^+. At that point the radial space-like distance from the horizon vanishes.

$$\rho^2 = -X^+X^- = 0 \tag{1.49}$$

Now consider the falling string. The coordinate $X^+(\sigma)$ is not an independent variable in string theory. To find out how it behaves we use the constraint equation

$$\partial_\sigma X^+ = \partial_\sigma x^i \partial_\tau x^i \tag{1.50}$$

The fact that the string does not require an independent degree of freedom for fluctuations in the X^- direction was one of the early indications of the large reduction in the number of degrees of freedom expected in a holographic theory. Using eq.(1.50) we can express $X^+(\sigma)$ in terms of harmonic oscillators. An explicit calculation gives

$$\begin{aligned} (\Delta X^+)^2 &\equiv \langle 0|(X^+ - X^+_{cm})^2|0\rangle \\ &= l_s^2 \sum_n \frac{1}{n^3} \\ &= l_s^2 n_{max}^2 \end{aligned} \tag{1.51}$$

This is a special case of a fundamental new uncertainty relation [17] [18] which occurs throughout string theory and which we will return to. To write it in a more suggestive form we write $n_{max} = (\Delta\tau)^{-1}$ or equivalently $n_{max} = (\delta X^-)^{-1}$. Equation (1.51) then takes the symmetrical form

$$\Delta X^+ \Delta X^- = l_s^2 \tag{1.52}$$

This is the *string uncertainty principle.* It implies that there is a fundamental unit of area in the X^+, X^- plane. It is reminiscent of uncertainty principles which occur in non-commutative geometry but it is not put in by hand.

To appreciate the implications of the space time uncertainty relation, let us consider an infalling massless string whose center of mass moves along the trajectory $X^+ = 1$. As X^- tends to zero the fluctuation in X^+, as seen by an outside observer, increases like l_s^2/X^-. Thus the stringy matter will be spread over region $X^+X^- \leq l_s^2$. From the point of view of Schwarzschild coordinates, instead of asymptotically approaching the horizon, the stringy matter can not be localized more precisely than to say that it is within a proper distance l_s from the horizon.

What we are seeing is a new relativity principle. According to the usual relativity principles, two observers in relative motion will disagree about the length of rods and the rate of clocks. But there is an invariant concept, the event, which occurs at a well defined space-time location. Even this is eliminated by black hole complementarity. External and freely falling observers will radically disagree about where and when events such as baryon violation take place or where the energy and momentum of a string is located. As we have seen, quantum mechanics and relativity conspire to insure that no observer ever sees a violation of the laws of quantum mechanics.

We have also seen that the origin of this relativity of descriptions is the behavior of the very high frequency fluctuations which are invisible to the freely falling observer but which dominate the description of the outside observer.

How can it be that the usual ideas of local quantum field theory fail so badly? In the remaining lectures we will see that conventional ideas of locality badly overestimate the number of independent degrees of freedom of a system. The key to black hole complementarity is the vast reduction implied by the holographic principle.

2 Entropy Bounds

Maximum Entropy

The Holographic Principle is about the counting of quantum states of a system. We begin by considering a large region of space Γ. For simplicity we

take the region to be a sphere. Now consider the space of states that describe arbitrary systems that can fit into Γ such that the region outside Γ is empty space. Our goal is to determine the dimensionality of that state-space. Lets consider some preliminary examples. Suppose we are dealing with a lattice of spins. Let the lattice spacing be a and the volume of Γ be V. The number of spins is V/a^3 and the number of orthogonal states supported in Γ is

$$N_{states} = 2^{\frac{V}{a^3}} \tag{2.1}$$

A second example is a continuum quantum field theory. In this case the number of quantum states will diverge for obvious reasons. We can limit the states, for example by requiring the energy density to be no larger than some bound ρ_{max}. In this case the states can be counted using some concepts from thermodynamics. One begins by computing the thermodynamic entropy density s as a function of the energy density ρ. The total entropy is

$$S = s(\rho)V \tag{2.2}$$

The total number of states is of order

$$N_{states} \sim \exp S = \exp s(\rho_{max})V \tag{2.3}$$

In each case the number of distinct states is exponential in the volume V. This is a very general property of conventional local systems and represents the fact that the number of independent degrees of freedom is additive in the volume.

In counting the states of a system the entropy plays a central role. In general entropy is not really a property of a given system but also involves ones state of knowledge of the system. To define entropy we begin with some restrictions that express what we know, for example, the energy within certain limits, the angular momentum and whatever else we may know. The entropy is by definition the logarithm of the number of quantum states that satisfy the given restrictions.

There is another concept that we will call the *maximum entropy*. This is a property of the system. It is the logarithm of the total number of states. In other words it is the entropy given that we know nothing about the state of the system. For the spin system the maximum entropy is

$$S_{max} = \frac{V}{a^3} \log 2 \tag{2.4}$$

This is typical of the maximum entropy. Whenever it exists it is proportional to the volume. More precisely it is proportional to the number of simple degrees of freedom that it takes to describe the system.

Let us now consider a system that includes gravity. Again we focus on a spherical region of space Γ with a boundary $\partial\Gamma$. The area of the boundary

is A. Suppose we have a thermodynamic system with entropy S that is completely contained within Γ. The total mass of this system can not exceed the mass of a black hole of area A or else it will be bigger than the region.

Now imagine collapsing a spherically symmetric shell of matter with just the right amount of energy so that together with the original mass it forms a black hole which just fills the region. In other words the area of the horizon of the black hole is A. This is shown in figure (13). The result of this process is a system of known entropy, $S = A/4G$. But now we can use the second law of thermodynamics to tell us that the original entropy inside Γ had to be less than or equal to $A/4G$. In other words the maximum entropy of a region of space is proportional to its area measured in Planck units. Such bounds are called *holographic*.

Entropy on Light-Like Surfaces

We will see that it is most natural to define holographic entropy bounds on light-like surfaces [2] as opposed to space-like surfaces. Under certain circumstances the bounds can be translated to space-like surfaces but not always.

Let us start with an example in asymptotically flat space-time. We assume that flat Minkowski coordinates X^+, X^-, x^i can be defined at asymptotic distances. In this lecture we will revert to the usual convention in which X^+ is used as a light cone time variable. We will now define a "light sheet". Consider the set of all light rays which lie in the surface $X^+ = X_0^+$ in the limit $X^- \to +\infty$. In ordinary flat space this congruence of rays define a flat 3-dimensional light-like surface. In general they define a light like surface called a light sheet. The light sheet will typically have singular caustic lines but can be defined in a unique way [19]. When we vary X_0^+ the light sheets fill all space-time except for those points that lie behind black hole horizons.

Now consider a space-time point p. We will assign it light-cone coordinates as follows. If it lies on the light sheet X_0^+ we assign it the value $X^+ = X_0^+$. Also if it lies on the light ray which asymptotically has transverse coordinate x_0^i we assign it $x^i = x_0^i$. The value of X^- that we assign will not matter. The two dimensional x^i plane is called the Screen.

Next assume a black hole passes through the light sheet X_0^+. The stretched horizon [1] of the black hole describes a two dimensional surface in the 3 dimensional light sheet as shown in figure (14). Each point on the stretched horizon has unique coordinates X^+, x^i. More generally if there are several black holes passing through the light sheet we can map each of their stretched horizons to screen in a single valued manner.

Since the entropy of the black hole is equal to $1/4G$ times the area of the horizon we can define an entropy density of $1/4G$ on the stretched horizon.

[1] The stretched horizon is a time-like surface just outside the mathematical light-like surface. Its precise definition is not important here.

The mapping to the screen then defines an entropy density in the x^i plane, $\sigma(x)$. It is a remarkable fact that $\sigma(x)$ is always less than or equal to $1/4G$.

To prove that $\sigma(x) \leq \frac{1}{4G}$ we make use of the focusing theorem of general relativity. The focusing theorem depends on the positivity of energy and is based on the tendency for light to bend around regions of non-zero energy. Consider bundle of light rays with cross sectional area α. The light rays are parameterized by an affine parameter λ. The focusing theorem says that

$$\frac{d^2\alpha}{d\lambda^2} \leq 0 \tag{2.5}$$

Consider a bundle of light rays in the light sheet which begin on the stretched horizon and go off to $X^- = \infty$. Since the light rays defining the light sheet are parallel in the asymptotic region $d\alpha/d\lambda \to 0$. The focusing theorem tells us that as we work back toward the horizon, the area of the bundle decreases. It follows that the image of a patch of horizon on the screen is larger than the patch itself. The holographic bound immediately follows.

$$\sigma(x) \leq \frac{1}{4G} \tag{2.6}$$

This is a surprising conclusion. No matter how we distribute the black holes in 3 dimensional space, the image of the entropy on the screen always satisfies the entropy bound (2.6). An example which helps clarify how this happens involves two black holes. Suppose we try to hide one of them behind the other along the X^- axis, thus doubling the entropy density in the x plane. The bending and focusing of light always acts as in figure (15) to prevent $\sigma(x)$ from exceeding the bound. These considerations lead us to the more general conjecture that for any system, when it is mapped to the screen the entropy density obeys the bound (2.6).

Robertson Walker Geometry

This kind of bound has been generalized to *flat* Robertson Walker geometries by Fischler and Susskind [20] and to more general geometries by Bousso [21] [22]. First review the RW case. We will consider the general case of $d+1$ dimensions. The metric has the form

$$ds^2 = dt^2 - a(t)^2 dx^m dx^m \tag{2.7}$$

where the index m runs over the d spatial directions. The function $a(t)$ is assumed to grow as a power of t.

$$a(t) = a_0 t^p \tag{2.8}$$

Lets also make the usual simplifying cosmological assumptions of homogeneity. In particular we assume that the spatial entropy density (per unit d

volume) is homogeneous. Later, following Bousso, we will relax these assumptions.

At time t we consider a spherical region Γ of volume V and area A. The boundary $(d-1)$-sphere, $\partial\Gamma$, will play the same role as the screen in the previous discussion. The light-sheet is now defined by the backward light cone formed by light rays that propagate from $\partial\Gamma$ into the past.

As in the previous case the holographic bound applies to the entropy passing through the light sheet. The bound states that the total entropy passing through the light sheet does not exceed $A/4G$. The key to a proof is again the focusing theorem. We observe that at the screen the area of the outgoing bundle of light rays is increasing as we go to later times. In other words the light sheet has positive expansion into the future and negative expansion into the past. The focusing theorem again tells us that if we map the entropy of black holes passing through the light sheet to the screen, the resulting density satisfies the holographic bound.

It is now easy to see why we concentrate on light sheets instead of space like surfaces. Obviously if the spatial entropy density is uniform and we choose Γ big enough, the entropy will exceed the area. However if Γ is larger than the particle horizon at time t the light sheet is not a cone but rather a truncated cone which is cut off by the big bang at $t = 0$. Thus a portion of the entropy present at time t never passed through the light sheet. If we only count that portion of the entropy which did pass through the light sheet it will scale like the area A. We will return to the question of space-like bounds after discussing Bousso's generalization [21] of the FS bound.

Bousso's Generalization

Consider an arbitrary cosmology. Take a space-like region Γ bounded by the space-like boundary $\partial\Gamma$. Following Bousso [21], at any point on the boundary we can construct four light rays that are perpendicular to the boundary. We will call these the four *branches*. Two branches go toward the future. One of them is composed of outgoing rays and the other is ingoing. Similarly two branches go to the past. On any of these branches a light ray, together with its neighbors define a positive or negative expansion as we move away from the boundary. In ordinary flat space-time if $\partial\Gamma$ is convex the outgoing (ingoing) rays have positive (negative) expansion. However in non-static universes other combinations are possible. For example in a rapidly contracting universe the outgoing future branch may have negative expansion.

If we consider general boundaries the sign of the expansion of a given branch may vary as we move over the surface. For simplicity we restrict attention to those regions for which a given branch has a unique sign. We can now state Bousso's rule:

From the boundary $\partial\Gamma$ construct all light sheets which have negative

expansion as we move away. These light sheets may terminate at the tip of a cone or a caustic or even a boundary of the geometry. Bousso's bound states that the entropy passing through these light sheets is less that $A/4G$ where A is the boundary of $\partial\Gamma$.

To help visualize how Bousso's construction works we will consider spherically symmetric geometries and use Penrose diagrams to describe them. The Penrose diagram represents the radial and time directions. Each point of such a diagram really stands for a 2-sphere (more generally a $(d-1)$-sphere). The four branches at a given point on the Penrose diagram are represented by a pair of 45 degree lines passing through that point. However we are only interested in the branches of negative expansion. For example in figure(16) we illustrate flat space-time and the negative expansion branches of a typical local 2-sphere.

In general as we move around in the Penrose diagram the particular branches which have negative expansion may change. For example if the cosmology initially expands and then collapses, the outgoing future branch will go from positive to negative expansion. Bousso introduced a notation to indicate this. The Penrose diagram is divided into a number of regions depending on which branches have negative expansion. In each region the negative expansion branches are indicated by their directions at a typical point. Thus in figure(17) we draw the Penrose- Bousso (PB) diagram for a positive curvature, matter dominated universe that begins with a bang and ends with a crunch. It consists of four distinct regions.

In region I of figure (17) the expansion of the universe causes both past branches to have negative expansion. Thus we draw light surfaces into the past. These light surfaces terminate on the initial boundary of the geometry and are similar to the truncated cones that we discussed in the flat RW case. The holographic bound in this case says that the entropy passing through either backward light surface is bounded by the area of the 2-sphere at point p. Bousso's rule tells us nothing in this case about the entropy on space like surfaces bounded by p.

Now move on to region II. The relevant light sheets in this region begin on the 2-sphere q and both terminate at the spatial origin. These are untruncated cones and the entropy on both of them is holographically bounded. There is something new in this case. We find that the entropy is bounded on a future light sheet. Now consider a space like surface bounded by q and extending to the spatial origin. It is evident that any matter which passes through the space-like surface must also pass through the future light sheet. By the second law of thermodynamics the entropy on the space-like surface can not exceed the entropy on the future light sheet. Thus in this case the entropy in a space-like region can be holographically bounded. Thus, one condition for a space-like bound is that the entropy is bounded by a corresponding future light sheet. With this in mind we return to region I. For

region I there is no future bound and therefore the entropy is not bounded on space-like regions with boundary p.

In region III the entropy bounds are both on future light sheets. Nevertheless there is no space-like bound. The reason is that not all matter which passes through space-like surfaces is forced to pass through the future light sheets.

Region IV is identical to region II with the spatial origin being replaced by the diametrically opposed antipode. Thus we see that there are light-like bounds in all four regions but only in II and IV are there holographic bounds on space-like regions.

Another example of interest is inflationary cosmology. The PB diagram for de-Sitter space is shown in figure (18a). This time region I has both light sheets pointing to the future. This is due to the fact that de-Sitter space is initially contracting. In order to describe inflationary cosmology we must terminate the de sitter space at some late time and attach it to a conventional RW space. This is shown in figure (18b). The dotted line where the two geometries are joined is the reheating surface where the entropy of the universe is created.

Let us focus on the point p in figure (18b). It is easy to see that in an ordinary inflationary cosmology p can be chosen so that the entropy on the space-like surface $p - q$ is bigger than the area of p. However Bousso's rule applied to point (p) only bounds the entropy on the past light sheet. In this case most of the newly formed entropy on the reheating surface is not counted since it never passed through the past light sheet. Typical inflationary cosmologies can be studied to see that the past light sheet bound is not violated.

As a final example we consider anti-de Sitter (AdS) space. The PB diagram consists of an infinite strip bounded on the left by the spatial origin and of the right by the AdS boundary. The PB diagram consists of a single region in which both negative expansion light sheets point toward the origin. Let us consider a static surface of large area A far from the spatial origin. The surface is denoted by the dotted vertical line L in figure (19). We will think of L as an infrared cutoff.

Consider an arbitrary point p on L. Evidently Bousso's rules bound the entropy on past and future light sheets bounded by p. Therefore the entropy on any space-like surface bounded by p and including the origin is also holographically bounded. In other words the entire region to the left of L can be foliated with space-like surfaces such that the maximum entropy on each surface is $A/4G$.

AdS space is an example of a special class of geometries which have time-like killing vectors and which can be foliated by surfaces that satisfy the Holographic bound. These two properties imply a very far reaching conclusion. All physics taking place in such backgrounds (in the interior of the

infrared cutoff L) must be described in terms of a Hamiltonian that acts in a Hilbert space of dimensionality

$$N_{states} = \exp(A/4G) \tag{2.9}$$

The holographic description of AdS space is the subject of the next lecture.

3 The AdS/CFT Correspondence and the Holographic Principle

AdS Space

As we saw in Lecture II, AdS space enjoys certain properties which make it a natural candidate for a holographic Hamiltonian description. In this lecture we will review the holographic description of $AdS(5) \otimes S(5)$ [23] [24][25]. Maldacena, in his lectures to this school has explained how this space arises in type IIb string theory, either as the near horizon geometry of a stack of D3-branes or as a solution of ten dimensional supergravity. We will begin with a brief review of AdS geometry.

For our purposes 5 dimensional AdS space may be considered to be a solid 4 dimensional spatial ball times the infinite time axis. The geometry can be described by dimensionless coordinates t, r, Ω where t is time, r is the radial coordinate ($0 \leq r < 1$) and Ω parametrizes the unit 3-sphere. The geometry has uniform curvature R^{-2} where R is the radius of curvature. The metric we will use is

$$ds^2 = \frac{R^2}{(1-r^2)^2}\left\{(1+r^2)^2 dt^2 - 4dr^2 - 4r^2 d\Omega^2\right\} \tag{3.1}$$

There is another form of the metric which is in common use,

$$ds^2 = \frac{R^2}{y^2}\left\{dt^2 - dx^i dx^i - dy^2\right\} \tag{3.2}$$

where i runs from 1 to 3.

The metric (3.2) is related to (3.1) in two different ways. First of all it is an approximation to (3.1) in the vicinity of a point on the boundary at $r = 1$. The 3 sphere is replaced by the flat tangent plane parameterized by x^i and the radial coordinate is replaced by y with $y = (1 - r)$.

The second way that (3.1) and (3.2) are related is that (3.2) is the exact metric of an incomplete patch of AdS space. A time-like geodesic can get to $y = \infty$ in a finite proper time so that the space in eq. (3.2) is not geodesically complete. As discussed in the lectures of Maldacena the metric (3.2) describes the near horizon geometry of a stack of D3-branes located

at the horizon $y = \infty$. The metric (3.2) may be expressed in terms of the coordinate $z = 1/y$.

$$ds^2 = R^2 \left\{ z^2(dt^2 - dx^i dx^i) - \frac{1}{z^2} dz^2 \right\} \tag{3.3}$$

In this form the horizon is at $z = 0$ and the boundary is at $z = \infty$.

To construct the space $AdS(5) \otimes S(5)$ all we have to do is define 5 more coordinates ω_5 describing the unit 5 sphere and add a term to the metric

$$ds_5^2 = R^2 d\omega_5^2 \tag{3.4}$$

Although the boundary of AdS is an infinite proper distance from any point in the interior of the ball, light can travel to the boundary and back in a finite time. For example, it takes a total amount of (dimensionless) time $t = \pi$ for light to make a round trip from the origin at $r = 0$ to the boundary at $r = 1$ and back. For all practical purposes AdS space behaves like a finite cavity with reflecting walls. The size of the cavity is of order R. In what follows we will think of the cavity size R as being much larger than any microscopic scale such as the Planck or string scale.

Holography in AdS Space

In order to have a benchmark for the counting of degrees of freedom in $AdS(5) \otimes S(5)$ imagine constructing a cutoff field theory in the interior of the ball. A conventional cutoff would involve a microscopic length scale such as the 10 dimensional Planck length l_p. One way to do this would be to introduce a spatial lattice in nine dimensional space . It is not generally possible to make a regular lattice but a random lattice with an average spacing l_p is possible. We can then define a simple theory such as a Hamiltonian lattice theory on the space. In order to count degrees of freedom we also need to regulate area of the boundary of AdS which is infinite. The way to do that was hinted at in lecture II. We introduce a surface L at $r = 1 - \delta$. The total 9 dimensional spatial volume in the interior of L is easily computed using the metric (3.1).

$$V(\delta) \sim \frac{R^9}{\delta^3} \tag{3.5}$$

and the number of lattice sites and therefore the number of degrees of freedom is

$$\frac{V}{l_p^9} \sim \frac{1}{\delta^3} \frac{R^9}{l_p^9} \tag{3.6}$$

In such a theory we also will find that the maximum entropy is of the same order of magnitude.

On the other hand the holographic bound discussed in lecture II requires the maximum entropy and the number of degrees of freedom to be of order

$$S_{max} \sim \frac{A}{l_p^8} \tag{3.7}$$

where A is the 8 dimensional area of the boundary L. This is also easily computed. We find

$$S_{max} \sim \frac{1}{\delta^3}\frac{R^8}{l_p^8} \tag{3.8}$$

In other words when R/l_p becomes large the holographic description requires a reduction in the number of independent degrees of freedom by a factor l_p/R. To say it slightly differently, the holographic principle implies a complete description of all physics in the bulk of a very large AdS space in terms of only l_p/R degrees of freedom per spatial Planck volume.

The AdS/CFT Correspondence

The correspondence between string theory in $AdS(5) \otimes S(5)$ and Super Yang Mills (SYM) theory on the boundary has been discussed in other lectures in this school and we will only review some of the salient features. The correspondence states that there is a complete equivalence between superstring theory in the bulk of $AdS(5)\otimes S(5)$ and maximally supersymmetric (16 real supercharges), $3+1$ dimensional, $SU(N)$, SYM theory on the boundary of the AdS space [23][24][25]. In these lectures SYM theory will always refer to this particular version of supersymmetric gauge theory.

It is well known that SYM is conformally invariant and therefore has no dimensional parameters. It will be convenient to define the theory to live on the boundary parametrized by the dimensionless coordinates t, Ω or t, x. The corresponding momenta are also dimensionless. In fact we will use the convention that all SYM quantities are dimensionless. On the other hand the bulk gravity theory quantities such as mass, length and temperature carry their usual dimensions. To convert from SYM to bulk variables the conversion factor is R. Thus if E_{sym} and M represent the energy in the SYM and bulk theories

$$E_{sym} = MR$$

Similarly bulk time intervals are given by multiplying the t interval by R.

One might think that the boundary of $AdS(5)\otimes S(5)$ is (8+1) dimensional but there is an important sense in which it is $3+1$ dimensional. To see this let us Weyl rescale the metric by a factor $\frac{R^2}{(1-r^2)^2}$ so that the rescaled metric at the boundary is finite. The new metric is

$$dS^2 = \left\{(1+r^2)^2 dt^2 - 4dr^2 - 4r^2 d\Omega^2\right\} + \left\{(1-r^2)^2 d\omega_5^2\right\} \tag{3.9}$$

Notice that the size of the 5-sphere shrinks to zero as the boundary at $r=1$ is approached. The boundary of the geometry is therefore $3+1$ dimensional.

Let us return to the correspondence between the bulk and bounday theories. The ten dimensional bulk theory has two dimensionless parameters. These are:

1. The radius of curvature of the AdS space measured in string units R/l_s
2. The dimensionless string coupling constant g.

The string coupling constant and length scale are are related to the ten dimensional Planck length and Newton constant by

$$l_p^8 = g^2 l_s^8 = G \tag{3.10}$$

On the other side of the correspondence, the gauge theory also has two constants. They are

1. The rank of the gauge group N
2. The gauge coupling g_{ym}

The relation between the string and gauge parameters was given by Maldacena [23]. It is

$$\begin{aligned} \frac{R}{l_s} &= (N g_{ym}^2)^{\frac{1}{4}} \\ g &= g_{ym}^2 \end{aligned} \tag{3.11}$$

We can also write ten dimensional Newton constant in the form

$$G = R^8/N^2 \tag{3.12}$$

There are two distinct limits that are especially interesting, depending on one's motivation. The AdS/CFT correspondence has been widely studied as a tool for learning about the behavior of gauge theories in the strongly coupled 't Hooft limit. From the gauge theory point of view the 't Hooft is defined by

$$\begin{aligned} g_{ym} &\rightarrow 0 \\ N &\rightarrow \infty \\ g_{ym}^2 N &= constant \end{aligned} \tag{3.13}$$

From the bulk string point of view the limit is

$$\begin{aligned} g &\rightarrow 0 \\ \frac{R}{l_s} &= constant \end{aligned} \tag{3.14}$$

Thus the strongly coupled 't Hooft limit is also the classical string theory limit in a fixed and large AdS space. This limit is dominated by classical supergravity theory.

The interesting limit from the viewpoint of the holographic principle is a different one. We will be interested in the behavior of the theory as the AdS radius increases but with the parameters that govern the microscopic physics in the bulk kept fixed. This means we want the limit

$$g = constant$$

$$R/l_s \rightarrow \infty \tag{3.15}$$

On the gauge theory side this is

$$\begin{aligned} g_{ym} &= constant \\ N &\rightarrow \infty \end{aligned} \tag{3.16}$$

Our goal will be to show that the number of quantum degrees of freedom in the gauge theory description satisfies the holographic behavior in eq. (3.8).

The Infrared Ultraviolet Connection

In either of the metrics (3.1) or (3.2) the proper area of any finite coordinate patch tends to ∞ as the boundary of AdS is approached. Thus we expect that the number of degrees of freedom associated with such a patch should diverge. This is consistent with the fact that a continuum quantum field theory such as SYM has an infinity of modes in any finite three dimensional patch. In order to do a more refined counting [26] we need to regulate both the area of the AdS boundary and the number of ultraviolet degrees of freedom in the SYM. As we will see, these apparently different regulators are really two sides of the same coin. We have already discussed infrared (IR) regulating the area of AdS by introducing a surrogate boundary L at $r = 1 - \delta$ or similarly at $y = \delta$.

That the the IR regulator of the bulk theory is equivalent to an ultraviolet (UV) regulator in the SYM theory is called the IR/UV connection [26]. It can be motivated in a number of ways. In this lecture we give an argument based on the quantum fluctuations of the positions of the D3-branes which are nominally located at the origin of the coordinate z in eq. (3.3). The location of a point on a 3 brane is defined by six coordinates z, ω_5. We may also choose the six coordinates to be cartesian coordinates $(z^1, ..., z^6)$. The original coordinate z is defined by

$$z^2 = (z^1)^2 + ... + (z^6)^2 \tag{3.17}$$

The coordinates z^m are represented in the SYM theory by six scalar fields on the world volume of the branes. If the six scalar fields ϕ^n are canonically normalized then the precise connection between the $z's$ and $\phi's$ is

$$z = \frac{g_{ym} l_s^2}{R^2} \phi \tag{3.18}$$

Strictly speaking eq.(3.18) does not make sense because the fields ϕ are $N \times N$ matrices. The situation is the same as in matrix theory where we identify the N eigenvalues of the matrices in eq.(3.18) to be the coordinates z^m of the N D3-branes. As in matrix theory the geometry is noncommutative and only

configurations in which the six matrix valued fields commute have a classical interpretation. However the radial coordinate $z = \sqrt{z^m z^m}$ can be defined by

$$z^2 = \left(\frac{g_{ym} l_s^2}{R^2}\right)^2 \frac{1}{N} Tr\phi^2 \tag{3.19}$$

A question which is often asked is; Where are the D3-branes located in the AdS space? The usual answer is that they are at the horizon $z = 0$. However our experiences in lecture I with similar questions should warn us that the answer may be more subtle. In lecture I (see the discussion from eq(1.45) to eq.(1.52)) a question was asked about the location of a string. What we found is that the answer depends on what frequency range it is probed with. High frequency or short time probes see the string widely spread in space while low frequency probes see a well localized string.

To answer the corresponding question about D3-branes we need to study the quantum fluctuations of their position. The fields ϕ are scalar quantum fields whose scaling dimensions are known to be exactly $(length)^{-1}$. From this it follows that any of the N^2 components of ϕ satisfies

$$\langle \phi_{ab}^2 \rangle \sim \delta^{-2} \tag{3.20}$$

where δ is the ultraviolet regulator of the field theory. It follows from eq(3.20) that the average value of z satisfies

$$< z >^2 \sim \left(\frac{g_{ym} l_s^2}{R^2}\right)^2 \frac{N}{\delta^2} \tag{3.21}$$

or, using eq's(3.12)

$$< z >^2 \sim \delta^{-2} \tag{3.22}$$

In terms of the coordinate y which vanishes at the boundary of AdS

$$< y >^2 \sim \delta^2 \tag{3.23}$$

Evidently low frequency probes see the branes at $z = 0$ but as the frequency of the probe increases the brane appears to move toward the boundary at $z = \infty$. The precise connection between the UV SYM cutoff and the bulk-theory IR cutoff is given by eq.(3.23).

Counting Degrees of Freedom

Let us now turn to the problem of counting the number of degrees of freedom needed to describe the region $y > \delta$ [26]. The UV/IR connection implies that this region can be described in terms of an ultraviolet regulated theory with a cutoff length δ. Consider a patch of the boundary with unit coordinate area. Within that patch there are $1/\delta^3$ cutoff cells of size δ.

Within each such cell the fields are constant in a cutoff theory. Thus each cell has of order N^2 degrees of freedom corresponding to the $N \otimes N$ components of the adjoint representation of $U(N)$. Thus the number of degrees of freedom on the unit area is

$$N_{dof} = \frac{N^2}{\delta^3} \tag{3.24}$$

On the other hand the 8-dimensional area of the regulated patch is

$$A = \frac{R^3}{\delta^3} \times R^5 = \frac{R^8}{\delta^3} \tag{3.25}$$

and the number of degrees of freedom per unit area is

$$\frac{N_{dof}}{A} \sim \frac{N^2}{R^8} \tag{3.26}$$

Finally we may use eq.(3.12)

$$\frac{N_{dof}}{A} \sim \frac{1}{G} \tag{3.27}$$

This is exactly what is required by the holographic principle.

AdS Black Holes

The apparently irreconcilable demands of black hole thermodynamics and the principles of quantum mechanics have led us to a very strange view of the world as a hologram. Now we will return, full circle, to see how the holographic description of $AdS(5) \otimes S(5)$ provides a description of black holes. What would be most interesting would be to give a holographic description of 10-dimensional black hole formation and evaporation in an $AdS(5) \otimes S(5)$ space which is much larger than the black hole. Unfortunately we will see that this is far beyond our present ability. There are however, black hole solutions in $AdS(5) \otimes S(5)$ which are within our current understanding. These are the black holes which have Schwarzschild radii as large or larger than the radius of curvature R. Such black holes are stable against decay and do not evaporate. In fact these black holes homogeneously fill the 5-sphere. They are solutions of the dimensionally reduced 5-dimensional Einstein equations with a negative cosmological constant. The thermodynamics can be derived from the black hole solutions by first computing the area of the horizon and then using the Bekenstein Hawking formula .

One finds that the entropy is related to their mass by

$$S = c \left(M^3 R^{11} G^{-1} \right)^{\frac{1}{4}} \tag{3.28}$$

Where G is the ten dimensional Newton constant and c is a numerical constant. Using the thermodynamic relation $dM = TdS$ we can compute the

relation between mass and temperature.

$$M = c\frac{R^{11}T^4}{G} \tag{3.29}$$

or in terms of dimensionless SYM quantities

$$\begin{aligned} E_{sym} &= c\frac{R^8}{G}T_{sym}^4 \\ &= cN^2T_{sym}^4 \end{aligned} \tag{3.30}$$

Eq.(3.30) has a surprisingly simple interpretation. Recall that in $3+1$ dimensions the Stephan-Boltzmann law for the energy density of radiation is

$$E = T^4V \tag{3.31}$$

where V is the volume. In the present case the relevant volume is the dimensionless 3-area of the unit boundary sphere. Furthermore there are $\sim N^2$ quantum fields in the $U(N)$ gauge theory so that apart from a numerical constant eq.(3.30) is nothing but the Stephan-Bolzmann law for black body radiation. Evidently the holographic description of the AdS black holes is a simple as it could be; a black body thermal gas of N^2 species of quanta propagating on the boundary hologram.

The Horizon

The high frequency quantum fluctuation of the location of the D3-branes are invisible to a low frequency probe. Roughly speaking this is insured by the renormalization group as applied to the SYM description of the branes. The renormalization group is what insures that our bodies are not severely damaged by constant exposure to high frequency vacuum fluctuations. We are not protected in the same way from classical fluctuations. An example is the thermal fluctuations of fields at high temperature. All probes sense thermal fluctuations of the brane locations. Let us return to eq.(3.20) but now, instead of using eq.(3.21) we use the thermal field fluctuations of ϕ. For each of the N^2 components the thermal fluctuations have the form

$$< \phi^2 >= T_{sym}^2 \tag{3.32}$$

and we find eqs.(3.22) and (3.23) replaced by

$$\begin{aligned} < z >^2 &\sim T_{sym}^2 \\ < y >^2 &\sim T_{sym}^{-2} \end{aligned} \tag{3.33}$$

It is clear that the thermal fluctuations will be strongly felt out to a coordinate distance $z = T_{sym}$. In terms of r the corresponding position is

$$1 - r \sim 1/T_{sym} \tag{3.34}$$

In fact this coincides with the location of the horizon of the AdS black hole.

A more precise definition of the horizon was given by Kabat and Lifschytz [27]. In the D-brane description the zero temperature stack of branes can be thought of as an extreme black brane with the horizon at $z = 0$. We would like to find something special about the corresponding point $\phi = 0$ in the SYM description. Let us displace one of the branes of the stack to a classical location z. At zero temperature supersymmetry insures the stability of this configuration. From the gauge theory point of view we have shifted a scalar field and broken the gauge symmetry to $U(1) \otimes U(N-1)$. The effect is to give the "W-bosons" a mass $g\phi$. From the brane point of view we have given a mass to the strings which extend between the displaced brane at z and the others at $z = 0$. Now we see what is special about $z = 0$. If we place a brane probe at a distance from the horizon there are massive modes of the brane. These modes become massless at the horizon. Presumably if we went even further these modes would become tachyonic and lead to an instability involving the irreversible production of strings connecting the probe and stack.

Kabat and Lifschytz [27] conjecture that this is the general feature of horizons in both the AdS/CFT theory and Matrix theory. In the AdS case we begin with a spontaneously broken SYM at finite temperature. It is well known that the mass of the W boson is corrected by finite temperature effects. Kabat and Lifschytz argue that at finite temperature the tachyonic instability occurs at a non-zero value of ϕ. This value corresponds to the position of the horizon.

The string theory correspondence gives a fairly convincing picture of the thermal effects on the W mass [27]. Let the probe brane be at z. The thermal effects are represented by a black hole or black brane with a horizon at z_H. We assume $z > z_H$. Now the string connecting the probe to the stack is terminated at the black hole horizon and its mass is

$$M = (z - z_H)/l_s \tag{3.35}$$

As $z \to z_H$ the string becomes massless and then tachyonic.

4 The Flat Space Limit

The Flat Space Limit

Gauge theory, gravity correspondences are especially interesting because they provide nonperturbative definitions of some quantum-gravity systems. The first example was matrix theory which uses SYM theory to define 11 dimensional supergravity in the DLCQ framework. To effectively decompactify the light cone direction we must pass to the large N limit keeping the gauge coupling fixed.

It has also been proposed that the AdS/CFT correspondence can be used to give a non-perturbative definition of type IIb string theory [28]. For this purpose we regard AdS space in the form of eq.(3.1) as a finite cavity with reflecting walls. It provides an ideal "box" for the purpose of infrared regulating a theory. Although the actual metric distance from any point in the bulk geometry to the boundary is infinite, it nevertheless closely resembles an ordinary finite box of size R. For example the time for light to propagate from $r = 0$ to the boundary and back is finite πR. Another indication of the finiteness of the box is that the energy eigenvalues of a particle moving in the metric (3.1) are discrete with the scale of energy being $1/R$.

To define the infinite volume limit we want to let $R \to \infty$ while keeping fixed the microscopic parameters of the theory such as g and l_s. We also want to keep fixed the energy and length scales in string units. Let us see what this means in terms of SYM quantities. From eq's(3.11) we see that we must allow $N \to \infty$ while keeping g_{ym} fixed just as in matrix theory. Furthermore the SYM energy is related to the mass M by $E_{sym} = MR = Ml_s(Ng_{ym}^2)^{\frac{1}{4}}$ Accordingly, to keep M fixed we must allow E_{sym} to grow like $N^{\frac{1}{4}}$ while time intervals must scale like $t \to N^{-\frac{1}{4}}$ Matrix theory also requires a scaling of energy with N but it is different. Instead of eq.(4.1) matrix theory involves energy of order $1/N$.

The next question is what quantities make sense in the limit

$$\begin{array}{ccc} N & \to & \infty \\ g_{ym} & = & constant \\ E_{sym} & \to N^{\frac{1}{4}} & \end{array} \tag{4.1}$$

The answer must be that any quantity that has a well defined flat space limit in ten dimensional IIb string theory should correspond to a quantity with a good limit under (4.1). The most obvious quantities are the spectrum and scattering matrix of stable particles. The only such particles are the massless supergravity multiplet. This includes Kaluza-Klein particles with non-zero momentum on the 5-sphere. From the point of view of the 5 dimensional AdS space these objects have non zero mass but they are stable. The 5-dimensional AdS mass of a particle with momentum k on the 5-sphere is

$$M = |k| \tag{4.2}$$

or in terms of the $S(5)$ angular momentum J

$$M = J/R \tag{4.3}$$

The existence and stability of these ten-dimensionally massless particles has been established beyond doubt from properties of the SYM theory (See Maldacena's lectures). The existence and properties of an S-matrix have also

been studied [29][28] but much less can be rigorously established. The idea for constructing scattering amplitudes is to use appropriate local gauge-invariant operators in the boundary theory as sources of the bulk particles. The particles can be aimed from the boundary toward the origin ($r = 0$) of the cavity and by carefully controlling the sources they can be made to interact in a small enough region that the curvature of the space is irrelevant. All kinds of interesting phenomena could occur during the collision. This includes the formation and evaporation of 10 dimensional black holes. You can look up the details of this kind of construction in the papers by Polchinski and Susskind [28] [29]. In this lecture we will concentrate on a couple of the poorly understood issues connected with the holographic description of in the interior of AdS.

High Energy Gravitons Deep in the Bulk

The first issue has to do with the description of high energy particles far from the boundary. Let us consider a massless graviton emitted from the boundary with vanishing $S(5)$ momentum. The creation operator for emitting the graviton is made out of the energy-momentum tensor of the boundary theory by integrating T_{ij} with a test function whose frequency spectrum is concentrated around some value ω.

$$\omega = pR = pl_s(g_{ym}^2 N)^{\frac{1}{4}} \tag{4.4}$$

Acting with the resulting operator creates a graviton of bulk momentum p propagating from the boundary toward the origin.

Once the particle has entered the bulk and passed the surrogate boundary at $y = \delta$, the holographic principle requires that it has a description in the regulated SYM theory with momentum cutoff $1/\delta$. Let us first consider the case of low graviton momentum by which we mean $pR = \omega < 1/\delta$. In this case the source function is slowly varying on the cutoff scale and the ordinary renormalization group strategy applies. Integrating out the modes beyond the cutoff results in a renormalized theory. Because the SYM theory is scale invariant, the cutoff theory has the same form as the original theory and the graviton is description is the same as in the continuum theory.

However, the renormalization group does not apply to situations in which the field theoretic source functions vary more rapidly than the cutoff scale. Thus if ($p > \delta/R$) there is no guarantee that the cutoff theory can describe the graviton correctly. The problem is that the holographic principle demands that we be able to describe all the physical states in the region $y > \delta$ by states of the cutoff theory even if they contain high energy gravitons.

To phrase the paradox differently, note that a massless particle with momentum p moving in the y direction can be localized in the x plane with an uncertainty

$$R\Delta x \sim \frac{1}{p} \tag{4.5}$$

Thus it should be possible to distinguish two such particles if their separation x is of order $1/pR$ or bigger. On the other hand the largest momenta in the cutoff SYM theory is $1/\delta << pR$. How is it possible to construct such well localized objects out of the low momentum modes of the SYM fields? We will argue that the only possible answer is that the high energy graviton is created by operators that involve many SYM quanta. In other words the effective operator which creates the high energy graviton in the cutoff theory must be high order in the fundamental SYM fields.

The order can be estimated by taking the total dimensionless energy ω of the graviton and dividing up among gauge quanta of energy $1/\delta$.

$$n = \omega\delta = pR\delta \tag{4.6}$$

To illustrate the point consider an n-particle wave function (as long as $n << N$ the SYM quanta can be treated as non-identical Boltzmann particles). As an example we choose a product wave function

$$\psi(x_1, x_2, ..., x_n) = \psi(x_1)\psi(x_2)....\psi(x_n) \tag{4.7}$$

with

$$\psi(x) = \exp - \left(\frac{|x|}{\delta}\right) \tag{4.8}$$

Note that wave functions of this type are composed of momenta of order $1/\delta$ and make sense in the cutoff theory.

Suppose we have two such states which are identical except one of them is displaced a distance a in the x direction. The inner product of these states is given by

$$\left\{\int \psi^*(x)\psi(x-a)\right\}^n \sim \exp -na/\delta \tag{4.9}$$

The function $exp{-}na/\delta$ in eq.(4.9) is narrowly peaked on the cutoff scale if n is large. In other words these states are distinguishable when they are displaced by distance δ/n even though the largest individual momentum is only $1/\delta$.

Thus we see that fine details can be distinguished in the coarse grained theory but only if the gravitons and other bulk particles are identified as an increasingly large number of gauge quanta as the UV cutoff of the SYM is lowered and/or the momentum is increased. This is very similar to matrix theory in which a graviton of momentum P_- is represented by a number of partons which grow with P_-.

Kaluza Klein Modes

So far we have considered particles which are massless in the 5 dimensional sense. Now let us consider a graviton with non-vanishing 5-momentum k.

We want to hold k fixed as we let $R \to \infty$. The 5 dimensional mass is k. Let us also assume p, the momentum in the y direction is also kept fixed. The dimensionless SYM energy of the state is

$$\omega = R\sqrt{k^2 + p^2} \tag{4.10}$$

Once again it is known how to create such particles by introducing a source at the boundary. The source in this case is a local gauge invariant SYM operator of the form

$$S_n = Tr(\phi)^n \tag{4.11}$$

This expression stands for an nth order monomial in the scalar SYM fields ϕ. The integer n is equal to the $S(5)$ angular momentum kR.

$$n = kR \tag{4.12}$$

To construct a creation operator for a particle of momentum p, k we integrate S_n with a test function of frequency ω given in eq.(4.10).

The puzzling feature of this prescription is that it injects the particle into the system with a local boundary operator. But a massive particle with energy $\sqrt{k^2 + p^2}$ can never get near the boundary. This can be seen from the motion of a massive classical particle in AdS space. If a particle of mass M moves along the y axis with total bulk energy $E = \omega/R$ then the closest it comes to the boundary is

$$y^* = M/E \tag{4.13}$$

where y^* is the classical turning point of the trajectory. It is also true that the solution of the classical wave equation for such a particle has its largest value at this point. For $y < y*$ the wave function quickly goes to zero. Somehow the local boundary field S_n must be creating bulk particles far from the boundary.

This behavior can be qualitatively be understood in an elementary way from the SYM theory. The operator S_n in eq.(4.11) describes the creation of n quanta. Suppose that the SYM energy ω is divided among the quanta so that each carries ω/n. Equivalently the quanta have wave length n/ω. According to the UV/IR connection quanta of this wave length correspond to bulk phenomena at $y = n/\omega$. Using eq's.(4.10) and (4.12) we see that this corresponds to the position $y*$. In this way we see that the local operator constructed from S_n by projecting out given frequency components actually corresponds to a bulk particle at its classical turning point.

Before concluding this final lecture the are some negative features of holographic descriptions which need to be mentioned. These negative features become apparent when we begin to ask how ordinary phenomena near the origin of a very large AdS space are described in SYM theory [30] [31]. Suppose we have some object which may be macroscopic in size but which is

very much smaller than the radius of curvature R. According to the UV/IR connection if the object is near the origin only the longest wavelength modes of the SYM fields should be important for their description. On the 3-sphere this means the almost homogeneous modes. The number of such homogeneous modes is of order N^2 and these must be the degrees of freedom which describe entire physics within a region of size R near the origin. In other words all the physics within a region small enough to be considered flat must be described by the matrix degrees of freedom of the SYM and not by the spatial variations of the fields. There is nothing wrong with this except that we have no idea how to translate ordinary physics into the holographic description. For example we would have no idea how to determine if a given SYM state were describing a small ten dimensional black hole, a rock or an elephant of the same mass.

I would like to suggest that there is a way to do physics which is complementary to the holographic way but in which bulk phenomena are much easier to recognize. I would expect that this new way would be in terms of local bulk fields which would either include the gravitational field or would allow its construction in some simple way. What would be unusual about this theory is that it would be extremely rich in gauge redundancies, so rich in fact that when the gauge is completely fixed and the non-redundant degrees of freedom are counted their number would be proportional to the area in Planck units. By some particular gauge fixing this would be made manifest. But after insuring ourselves that the counting is holographic other gauge choices might be much better for recognizing ordinary local physics. The kind of theory I have in mind is some generalization of Chern Simons theory which does have the property that the real states live on the boundary. Unfortunately this is just a speculation at the moment.

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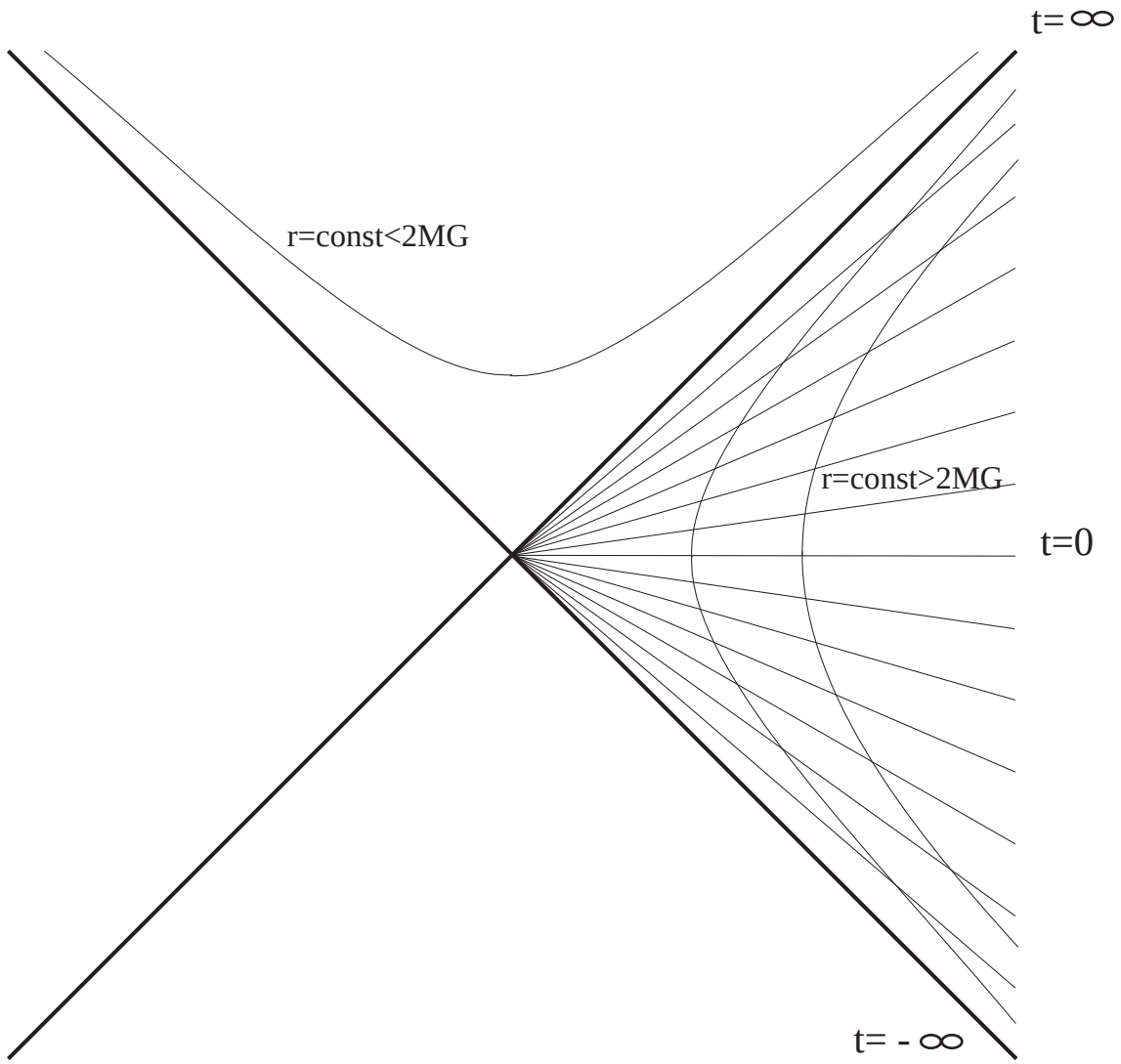


Fig. 1

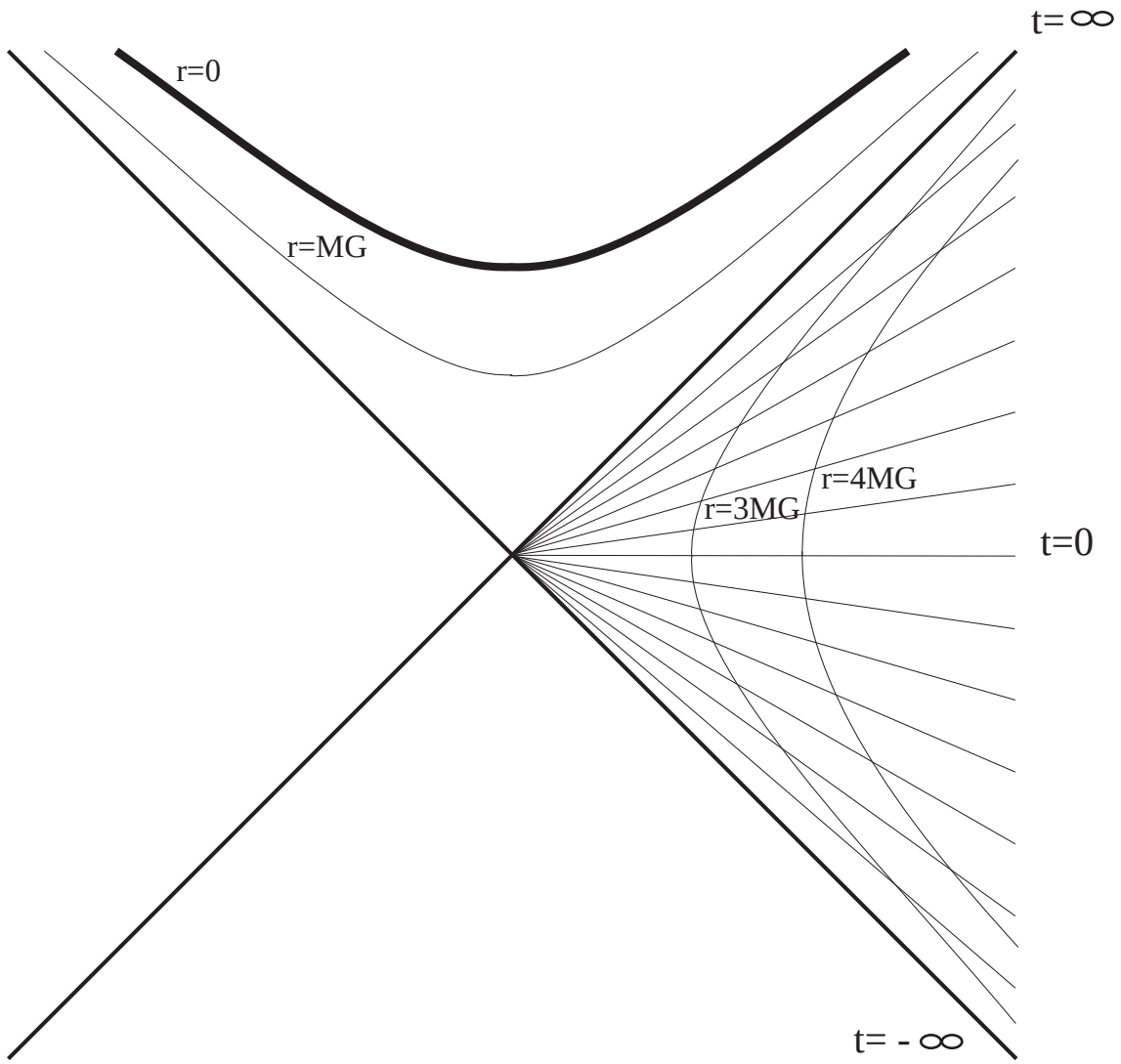


Fig. 2

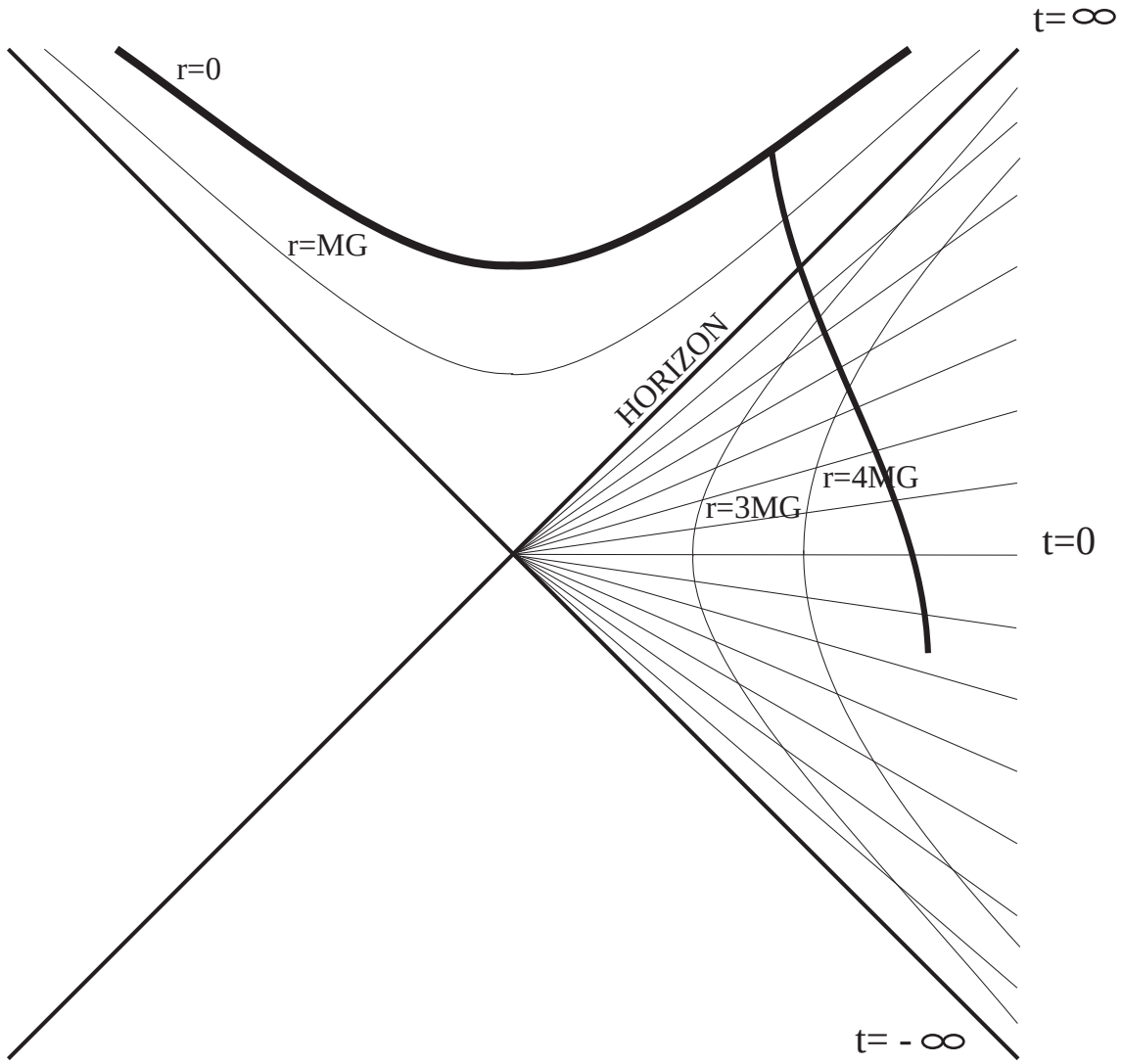
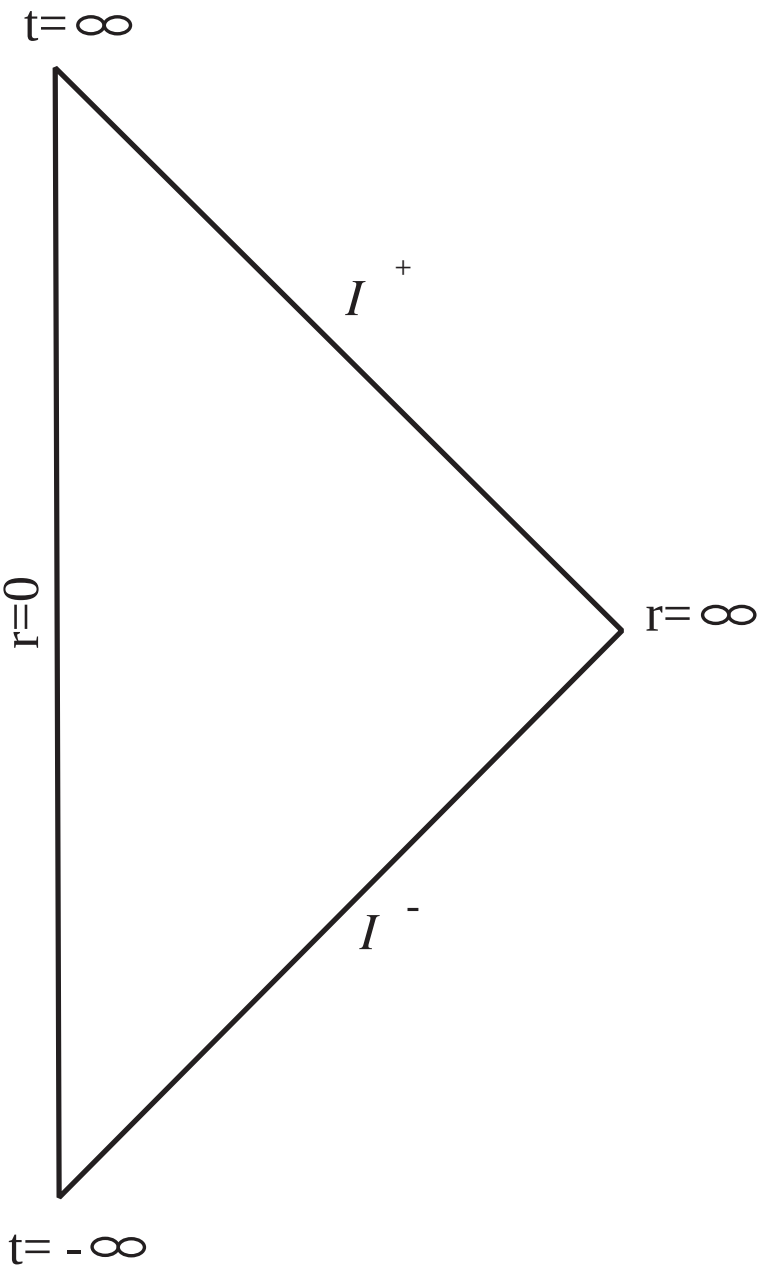


Fig. 3



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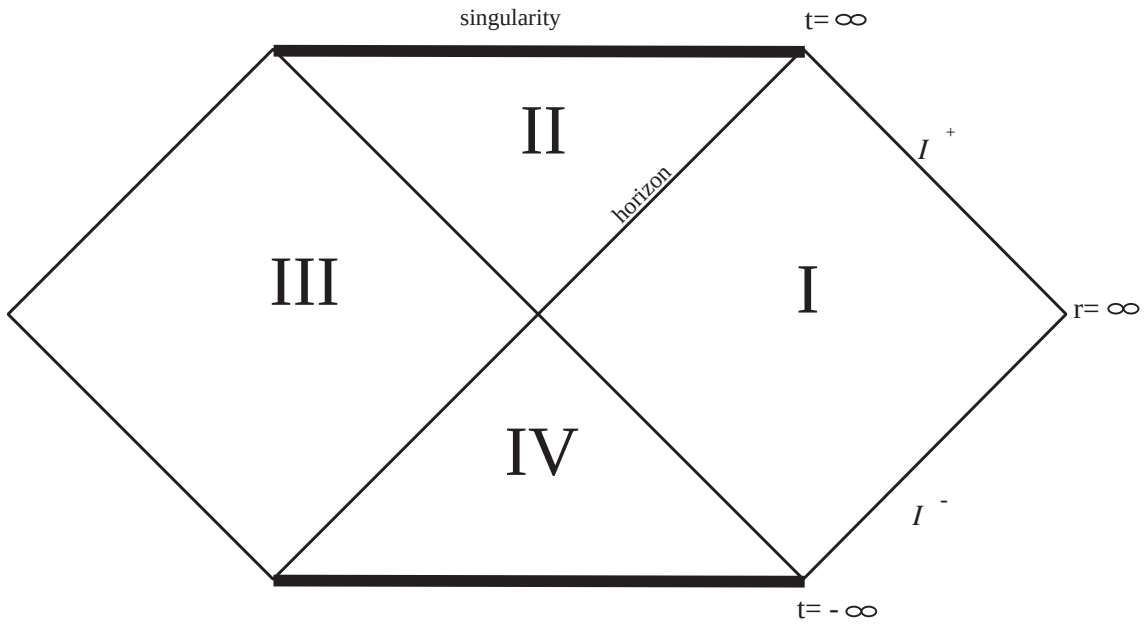

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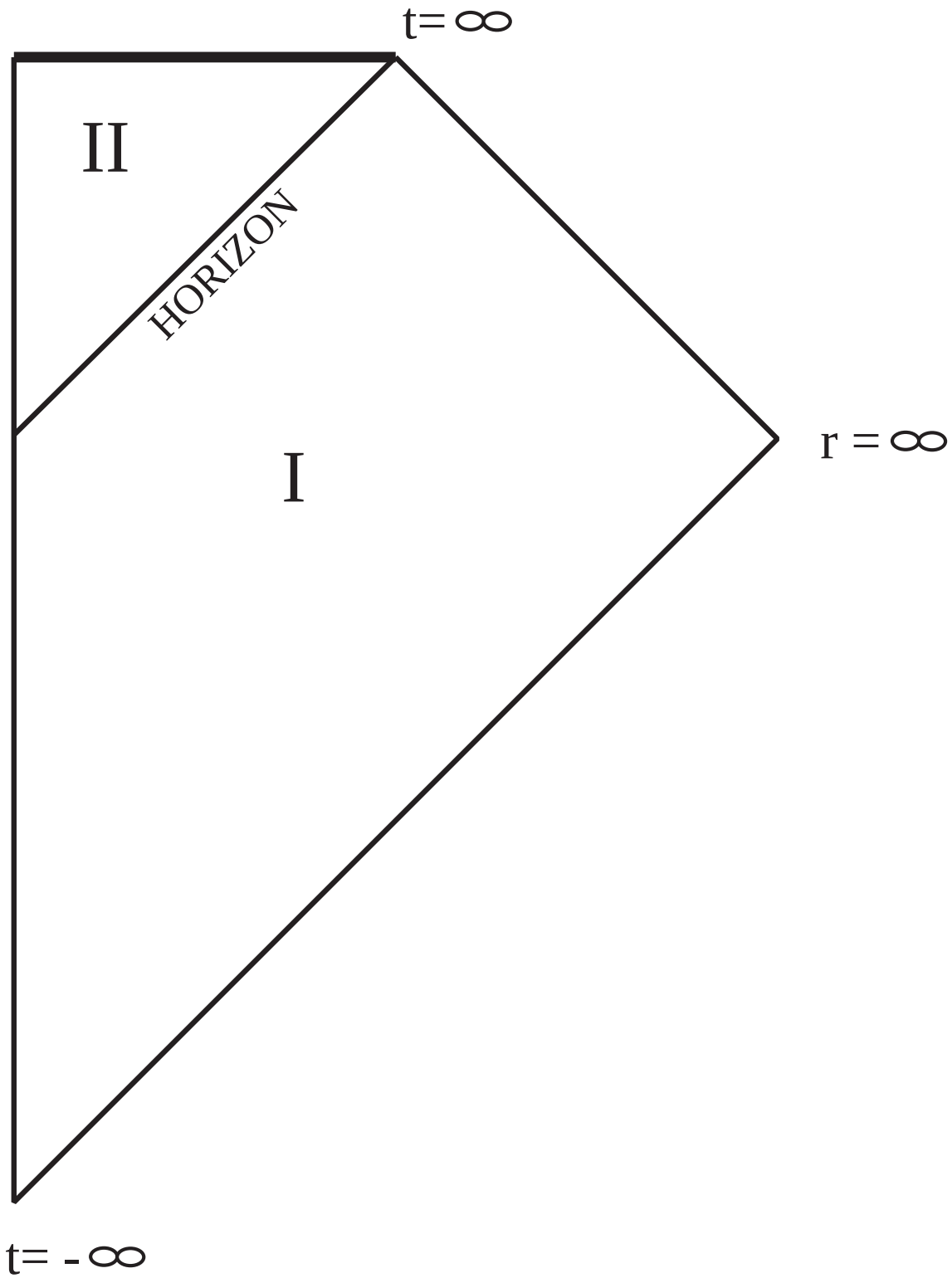


Fig. 6

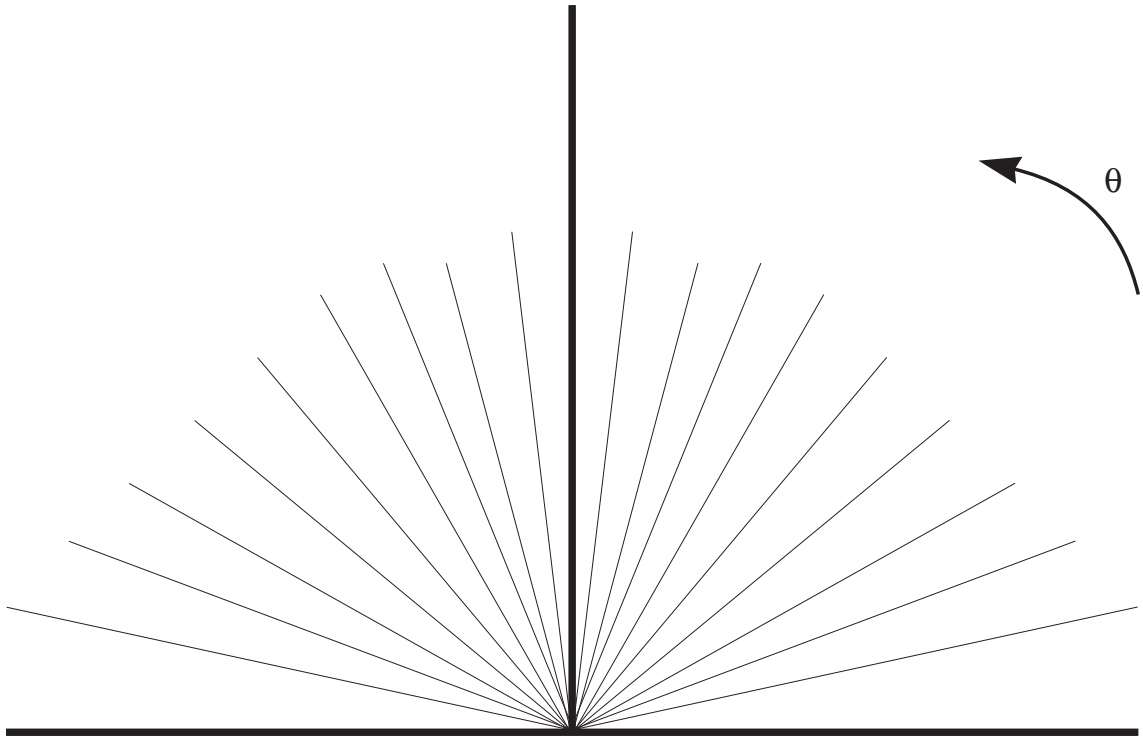


Fig. 7

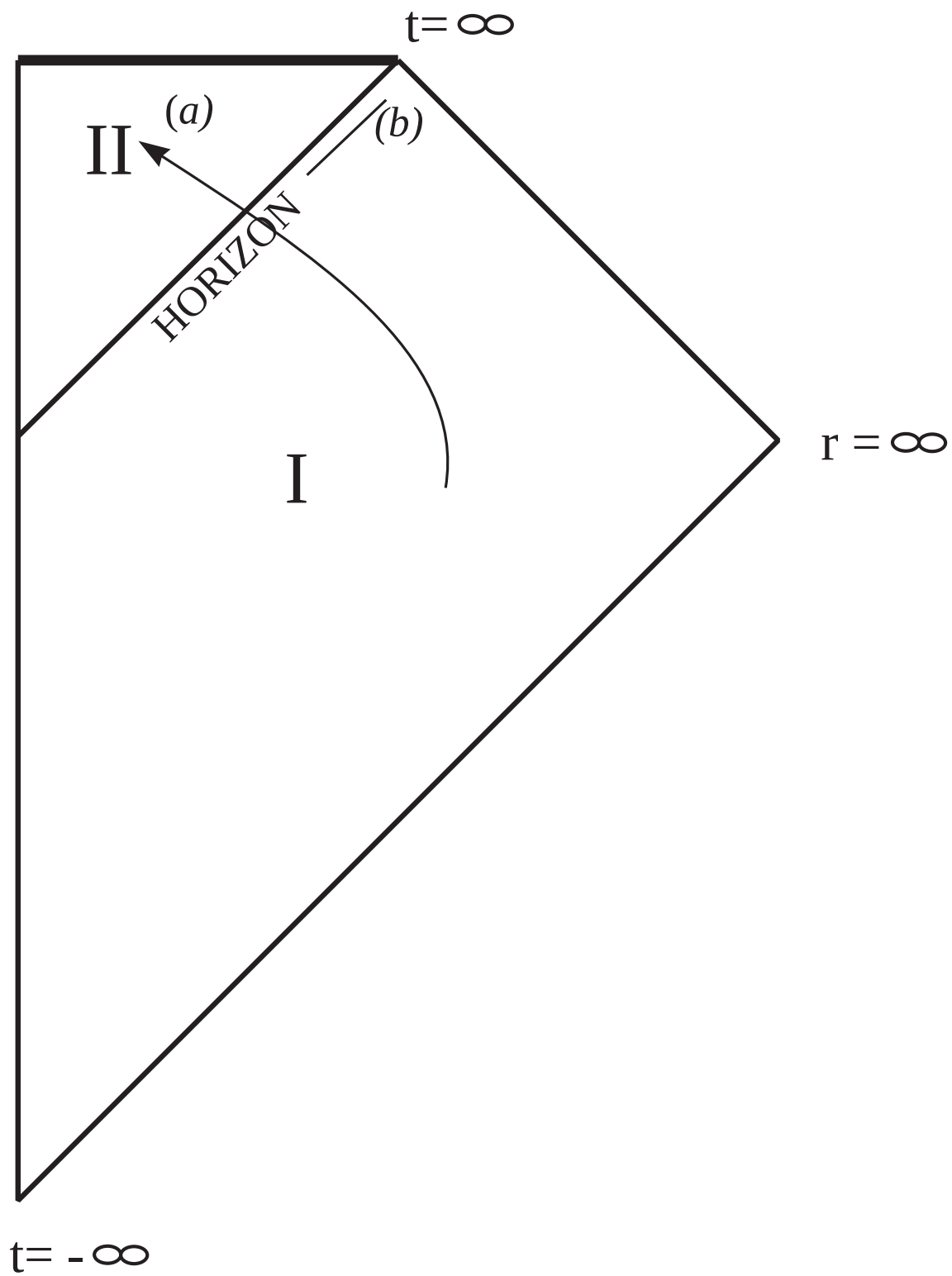


Fig. 8

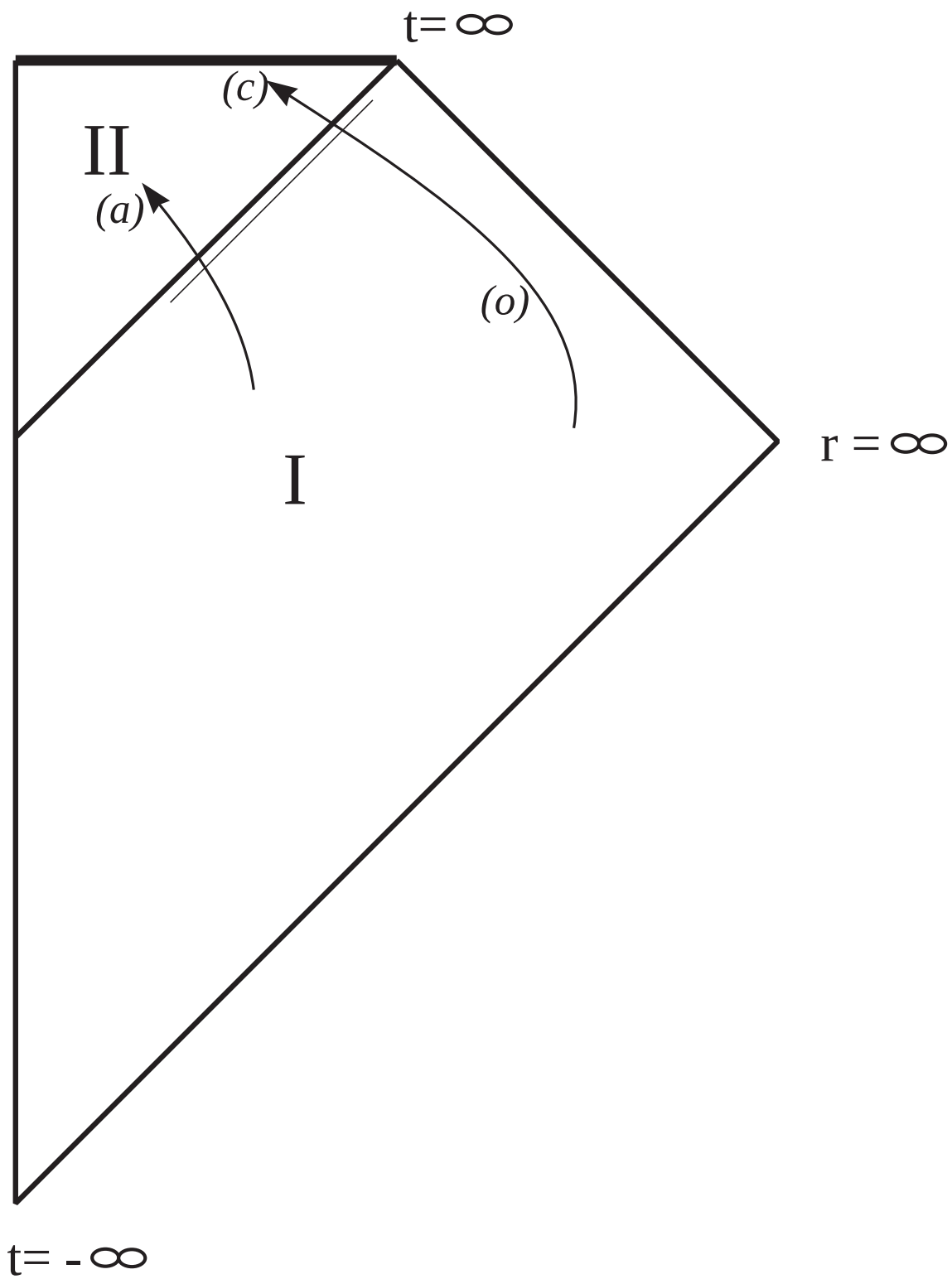


Fig. 9

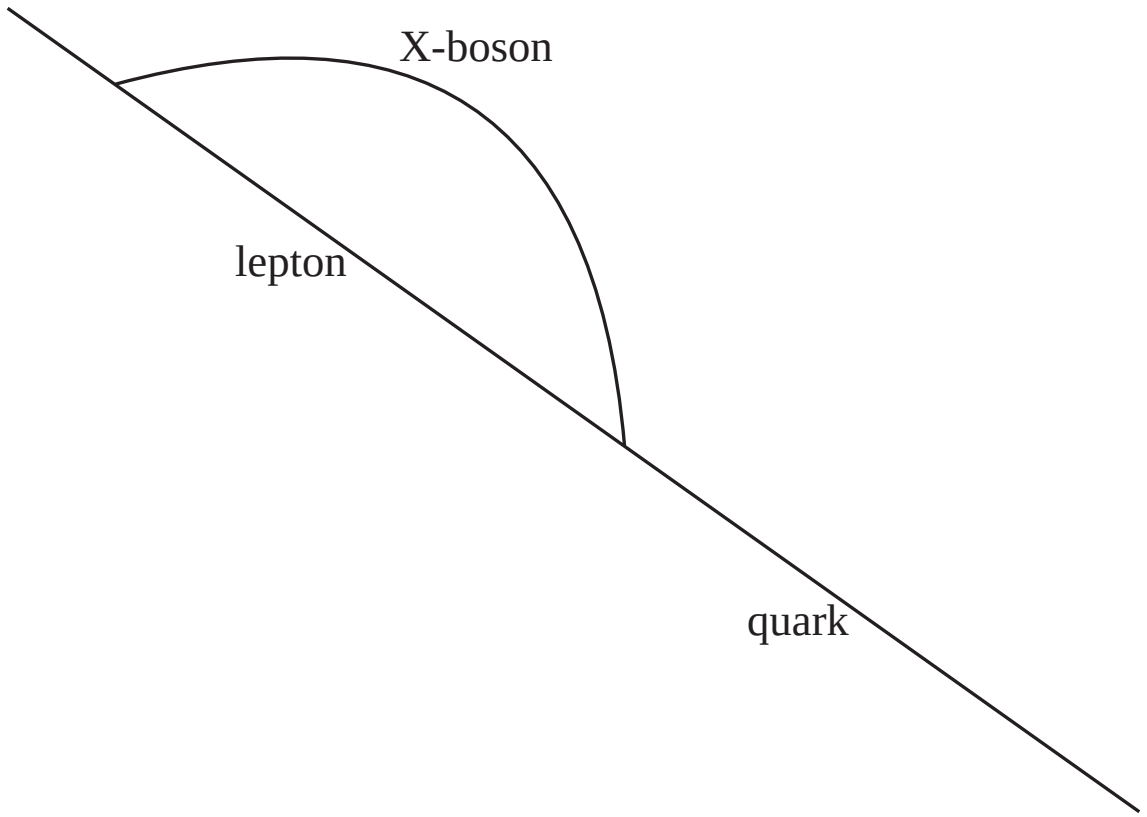


Fig. 10

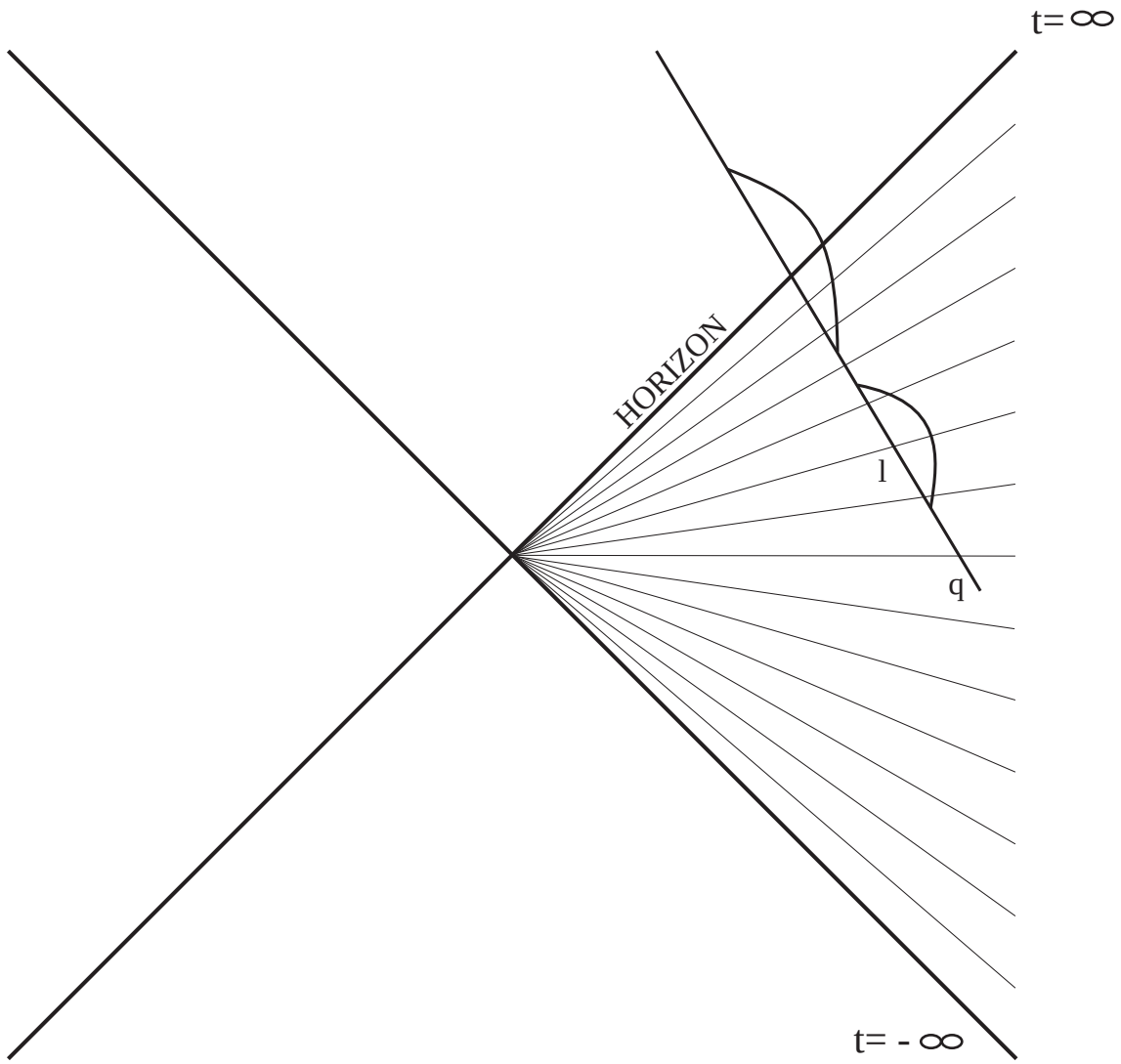


Fig. 11

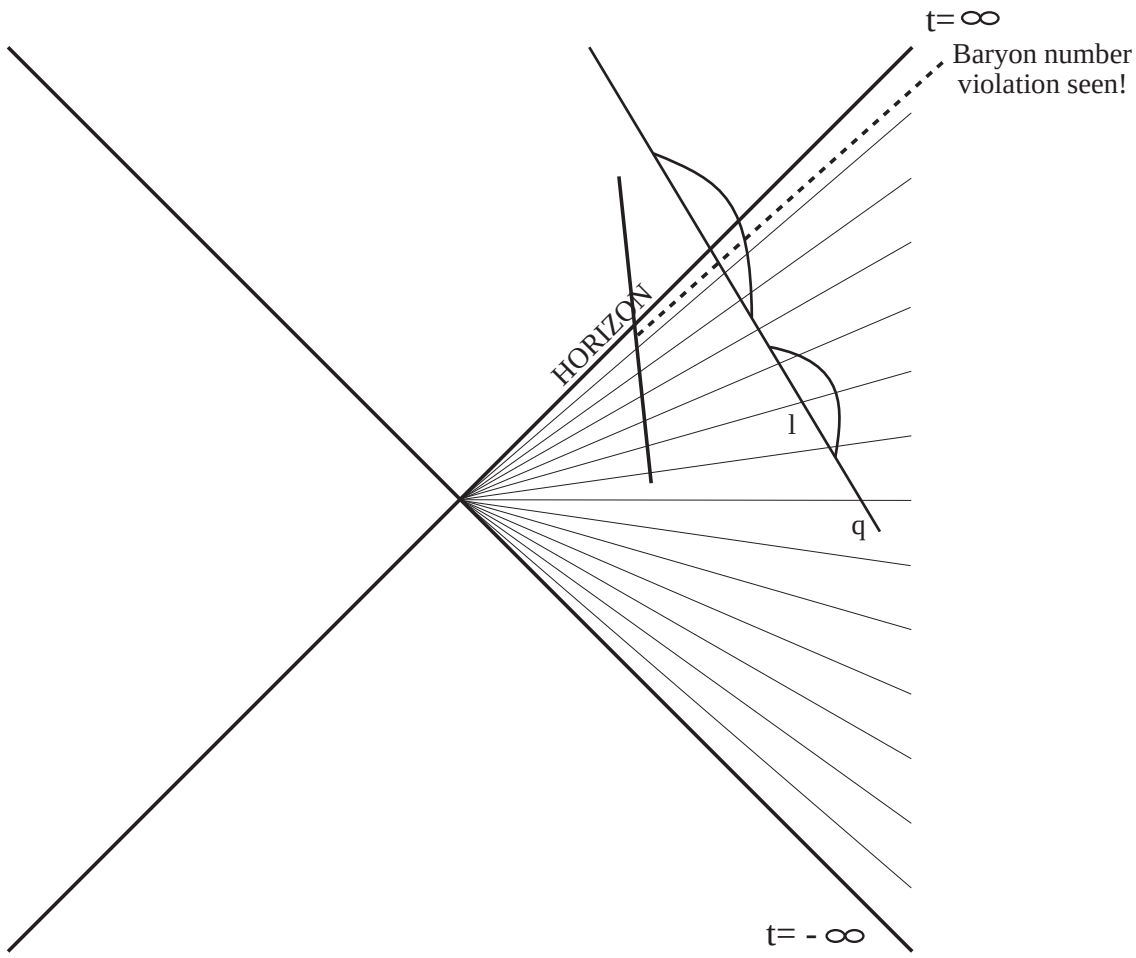


Fig. 12

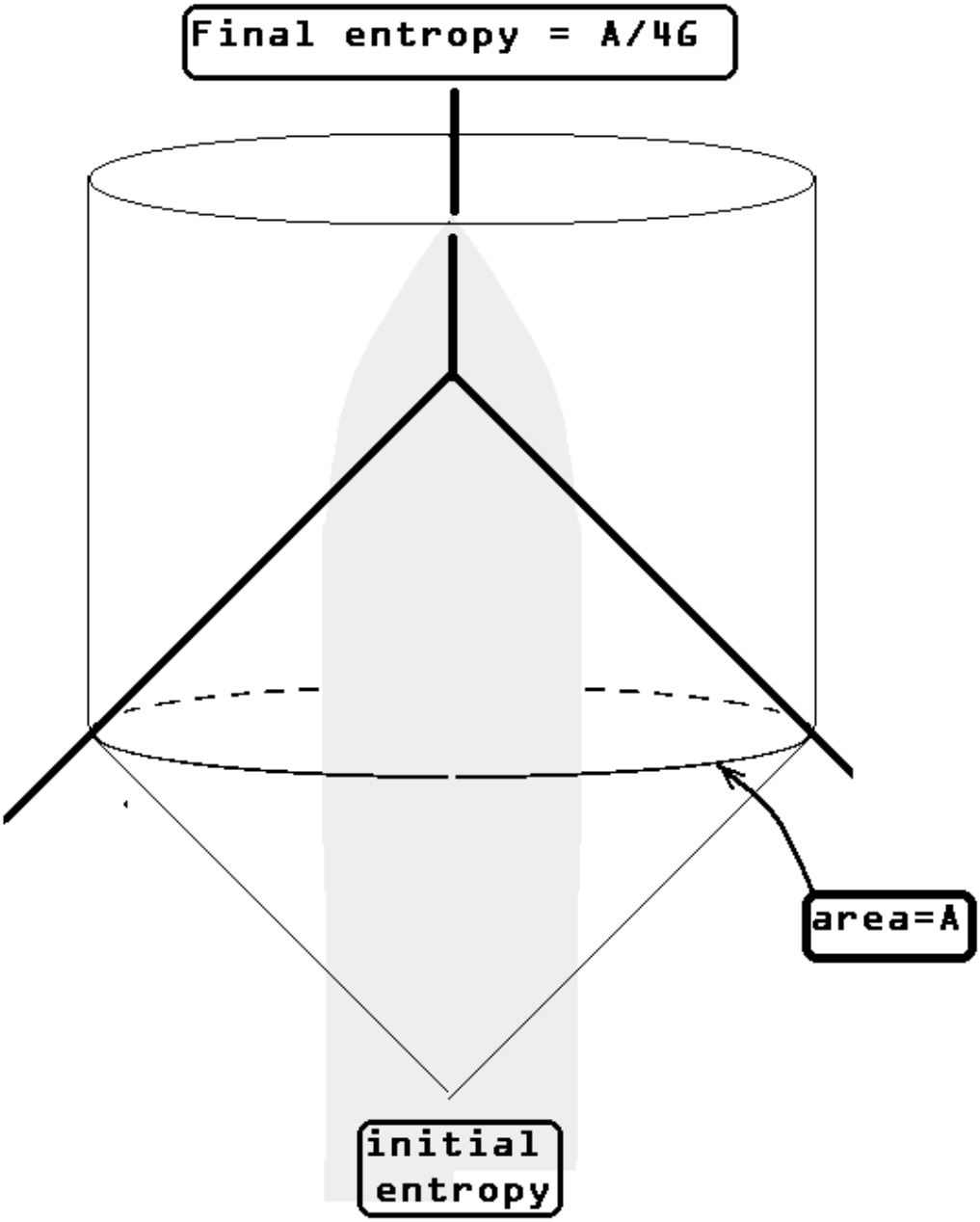


Figure (13)

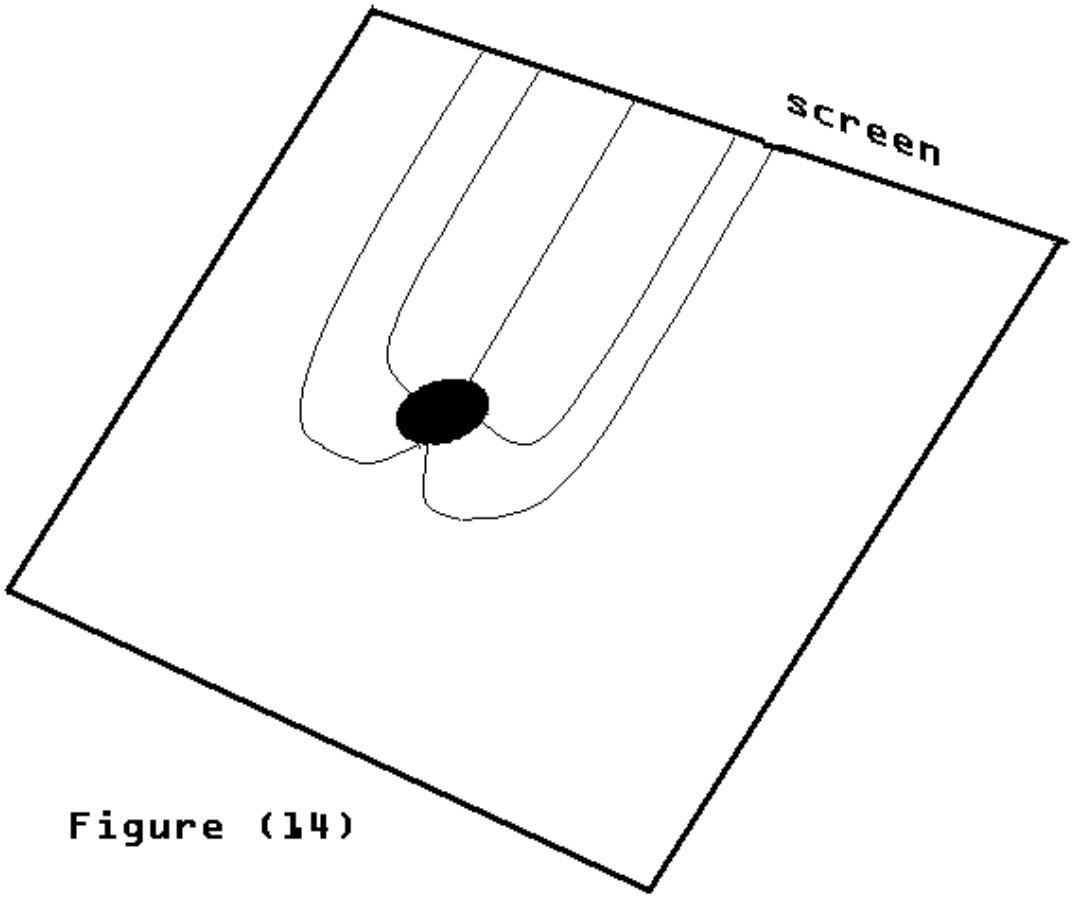


Figure (14)

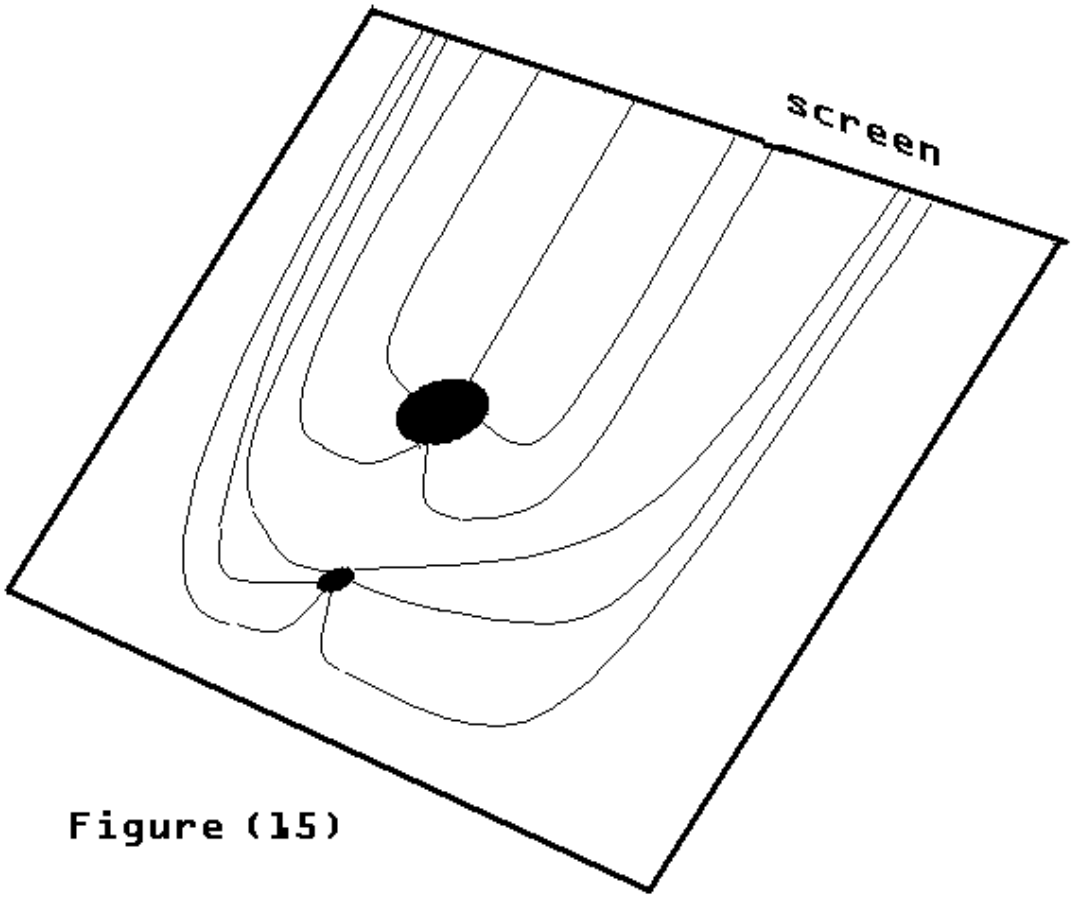
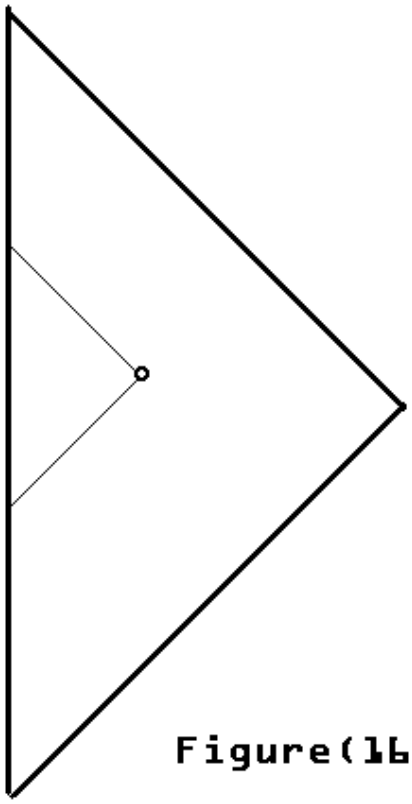


Figure (15)



Figure(16)

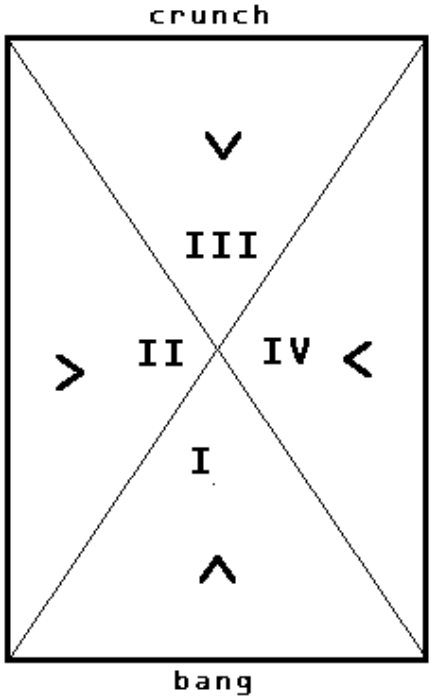
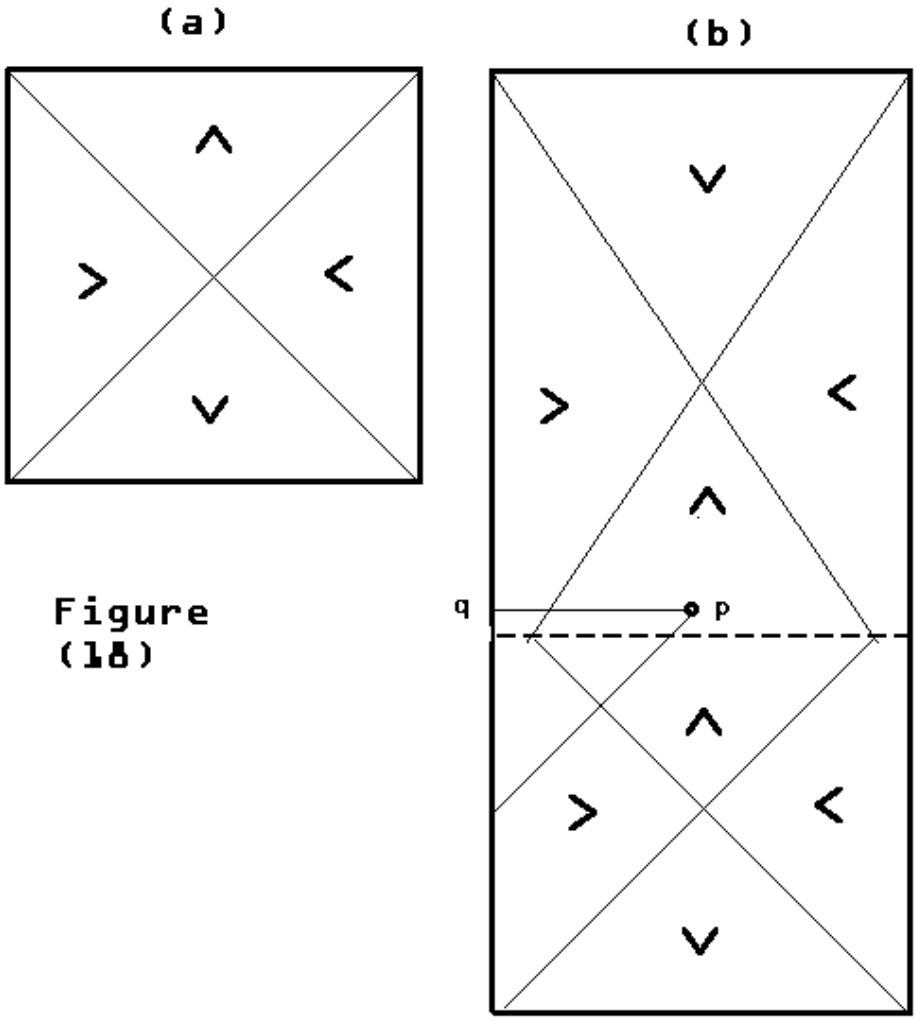


Figure (17)



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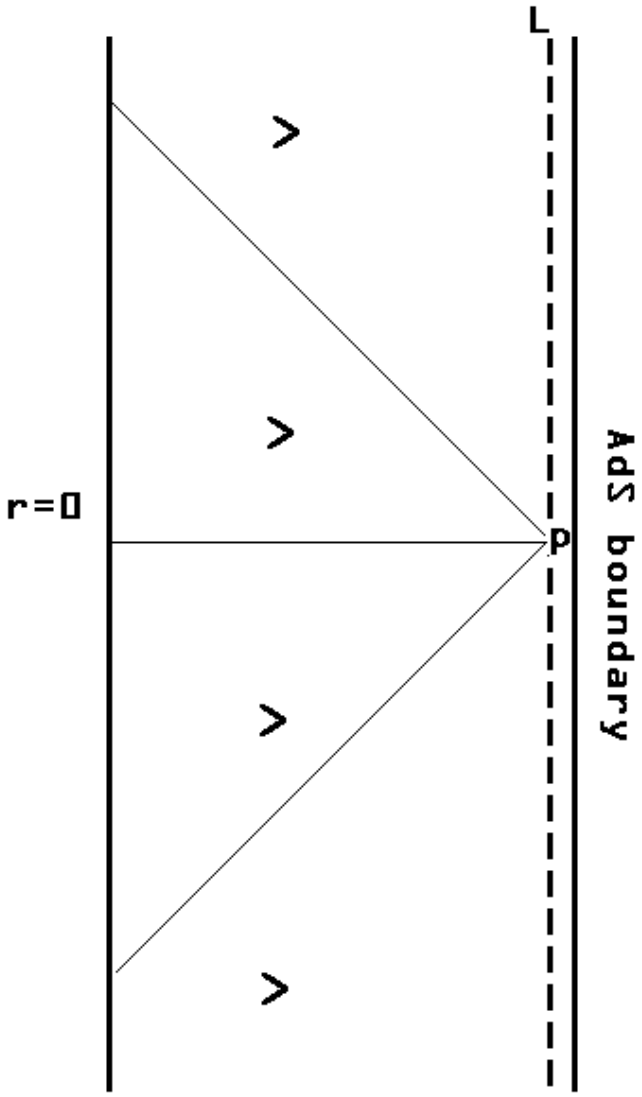

Figure (19)