Appendix A

SI units

A.1 Base units

Astronomers have been rather bad at using the SI system; they prefer their own 'custom' units ($M_\odot$, pc, etc.), or adopt cgs units (centimetre, gram, second) in preference to mks units (metre, kilogram, second) from which the SI system derives. Nevertheless, students, in particular, really *should* strive to use SI, which (among other advantages) greatly simplifies treatments of electricity.[1]

The irreducible base units of the SI system [and their cgs counterparts, where different] are:

Quantity	*SI Unit*		*cgs equivalent*	
length	metre	m	[centimetre	cm = 10^{-2} m]
mass	kilogram	kg	[gram	g = 10^{-3} kg]
time	second	s		
electric current	ampere	A	[Biot	bi = 10^{-1} A]
amount of substance	mole	mol		
luminous intensity	candela	cd		
thermodynamic temp.	kelvin	K	[degree Celsius	°C = K − 273.15]

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[1] In SI, electric current is defined in terms of the directly measurable magnetic *force* it exerts, and charge is then defined as current multiplied with time.
In cgs 'electrostatic units', the unit of charge (or statcoulomb), is defined by the quantity of charge which gives a force constant of 1 in Coulomb's law. That is, for two point charges, each with charge 1 statcoulomb, separated by 1 centimetre, the electrostatic force between them is one dyne. This also has the effect of making electric charge dimensionless (and not requiring a fundamental unit).

A.2 Derived units

'Derived quantities' can be defined in terms of the seven base quantities. There are 20 derived quantities which are not dimensionless and which, for convenience, have named units; these are tabulated overleaf.

The units of angle and solid angle are, formally, simply the number 1 (being ratios of dimensionally identical quantities). Nonetheless, these two further derived quantities have named units, as the lack of units could easily be confusing. They are:

- radian (rad): the unit of angle is the angle subtended at the centre of a circle by an arc of the circumference equal in length to the radius of the circle (so there are 2π radians in a circle).
- steradian (sr): the unit of solid angle is the solid angle subtended at the centre of a sphere of radius r by a portion of the surface of the sphere having an area r^2 (so there are 4π steradians on a sphere).

Many other derived quantities in more or less common use don't have special names for their units; some are given in the following tables. In a few cases, they have named units in the cgs system.A number of other convenient units are not directly derived from the SI base units, but can nonetheless be expressed in terms of those units, and are recognized by the guardians of the SI system. Important examples for astrophysics include:

- The day (d = 86 400 s), hour (h = 3600 s), and minute (m=60 s).
  (The year is not an admitted unit, though for rough calculations it's usually adequate to assume 1 yr $\simeq$ 365.25 d.)
- the degree ($^\circ = {}^{2\pi}/_{360}$ rad), arcminute ($' = {}^{2\pi}/_{21\,600}$ rad), and arcsecond ($'' = {}^{2\pi}/_{1.296\times10^6}$ rad)
- the atomic mass unit (amu = $1.66053886 \times 10^{-27}$ kg)
- the electron volt (eV = $1.60217646 \times 10^{-19}$ J)
- the ångström (Å = 10^{-10} m = 0.1 nm)
- the astronomical unit (AU = 1.49598×10^{11} m) and
  the parsec (pc = $3.08568025 \times 10^{16}$ m).

Named Derived SI units

Quantity	*Unit*			*Base units*	*Equivalent cgs unit*		
angular measure	radian	rad	1				
solid angle	steradian	sr	1				
frequency	hertz	Hz	s^{-1}	s^{-1}			
force, weight	newton	N	$m\ kg\ s^{-2}$	$m\ kg\ s^{-2}$	dyne	dyn	10^{-5} N
pressure, stress	pascal	Pa	$N\ m^{-2}$	$m^{-1}\ kg\ s^{-2}$			
energy, work, heat	joule	J	N m	$m^2\ kg\ s^{-2}$	erg	erg	10^{-7} J
power, radiant flux	watt	W	$J\ s^{-1}$	$m^2\ kg\ s^{-3}$			
electric charge or electric flux	coulomb	C	A s	A s			
voltage, electrical potential difference, electromotive force	volt	V	$W\ A^{-1} = J\ C^{-1}$	$m^2\ kg\ s^{-3}\ A^{-1}$			
electric capacitance	farad	F	$C\ V^{-1}$	$m^{-2}\ kg^{-1}\ s^4\ A^2$			
electric resistance, impedance, reactance	ohm	Ω	$V\ A^{-1}$	$m^2\ kg\ s^{-3}\ A^{-2}$			
electrical conductance	siemens	S	Ω^{-1}	$m^{-2}\ kg^{-1}\ s^3\ A^2$			
magnetic flux	weber	Wb	$J\ A^{-1}$	$m^2\ kg\ s^{-2}\ A^{-1}$	maxwell	Mx	10^{-8} Wb
magnetic field	tesla	T	$V\ s\ m^{-2} = Wb\ m^{-2} = N\ A^{-1}\ m^{-1}$	$kg\ s^{-2}\ A^{-1}$	gauss	G	10^{-4} T
inductance	henry	H	$V\ s\ A^{-1} = Wb\ A^{-1}$	$m^2\ kg\ s^{-2}\ A^{-2}$			
Celsius temperature	Celsius	°C	$t(°C) = t(K) - 273.15$	K			
luminous flux	lumen	lm	cd sr	cd			
illuminance	lux	lx	$lm\ m^{-2}$	$m^{-2}\ cd$	phot	ph	10^4 lx
radioactivity (decays per unit time)	becquerel	Bq	Hz	s^{-1}			
absorbed dose (of ionizing radiation)	gray	Gy	$J\ kg^{-1}$	$m^2\ s^{-2}$			
equivalent dose (of ionizing radiation)	sievert	Sv	$J\ kg^{-1}$	$m^2\ s^{-2}$			
catalytic activity	katal	kat	$mol\ s^{-1}$	$mol\ s^{-1}$			

Unnamed Derived SI units

Quantity	*Units*	
area	$\mathrm{m^{2}}$	$\mathrm{m^{2}}$
volume	$\mathrm{m^{3}}$	$\mathrm{m^{3}}$
speed, velocity	$\mathrm{m\ s^{-1}}$	$\mathrm{m\ s^{-1}}$
acceleration	$\mathrm{m\ s^{-2}}$	$\mathrm{m\ s^{-2}}$
jerk	$\mathrm{m\ s^{-3}}$	$\mathrm{m\ s^{-3}}$
angular velocity	$\mathrm{rad\ s^{-1}}$	$\mathrm{s^{-1}}$
momentum, impulse	$\mathrm{N\ s}$	$\mathrm{kg\ m\ s^{-1}}$
angular momentum	$\mathrm{N\ m\ s}$	$\mathrm{kg\ m^{2}\ s^{-1}}$
torque, moment of force	$\mathrm{N\ m}$	$\mathrm{kg\ m^{2}\ s^{-2}}$
wavenumber	$\mathrm{m^{-1}}$	$\mathrm{m^{-1}}$
mass density	$\mathrm{kg\ m^{-3}}$	$\mathrm{kg\ m^{-3}}$
heat capacity, entropy	$\mathrm{J\ K^{-1}}$	$\mathrm{kg\ m^{2}\ s^{-2}\ K^{-1}}$
specific heat capacity, specific entropy	$\mathrm{J\ K^{-1}\ kg^{-1}}$	$\mathrm{m^{2}\ s^{-2}K^{-1}}$
specific energy	$\mathrm{J\ kg^{-1}}$	$\mathrm{m^{2}\ s^{-2}}$
energy density	$\mathrm{J\ m^{-3}}$	$\mathrm{kg\ m^{-1}\ s^{-2}}$
surface tension	$\mathrm{N\ m^{-1} = J\ m^{-2}}$	$\mathrm{kg\ s^{-2}}$
heat flux density, irradiance	$\mathrm{W\ m^{-2}}$	$\mathrm{kg\ s^{-3}}$
thermal conductivity	$\mathrm{W\ m^{-1}\ K^{-1}}$	$\mathrm{kg\ m\ s^{-3}\ K^{-1}}$
diffusion coefficient	$\mathrm{m^{2}\ s^{-1}}$	$\mathrm{m^{2}\ s^{-1}}$
dynamic viscosity[1]	$\mathrm{Pa\ s = N\ s\ m^{-2}}$	$\mathrm{kg\ m^{-1}\ s^{-1}}$
kinematic viscosity[2]	$\mathrm{m^{2}\ s^{-1}}$	$\mathrm{m^{2}\ s^{-1}}$
electric charge density	$\mathrm{C\ m^{-3}}$	$\mathrm{m^{-3}\ A\ s}$
electric current density	$\mathrm{A\ m^{-2}}$	$\mathrm{A\ m^{-2}}$
conductivity	$\mathrm{S\ m^{-1}}$	$\mathrm{kg^{-1}\ m^{-3}\ s^{3}\ A^{2}}$
permittivity	$\mathrm{F\ m^{-1}}$	$\mathrm{kg^{-1}\ m^{-3}\ s^{4}\ A^{2}}$
permeability	$\mathrm{H\ m^{-1}}$	$\mathrm{kg\ m\ s^{-2}\ A^{-2}}$
electric field strength	$\mathrm{V\ m^{-1}}$	$\mathrm{kg\ m\ s^{-3}\ A^{-1}}$
magnetic field strength[3]	$\mathrm{A\ m^{-1}}$	$\mathrm{A\ m^{-1}}$
luminance[4]	$\mathrm{cd\ m^{-2}}$	$\mathrm{cd\ m^{-2}}$
cgs named units:		
[1]poise	$\mathrm{P = 0.1\ Pa\ s}$	
[2]stokes	$\mathrm{St = 10^{-4}\ m^{2}\ s^{-1}}$	
[3]oersted	$\mathrm{Oe} = \frac{1000}{4\pi}\ \mathrm{A\ m^{-1}}$	
[4]stilb	$\mathrm{sb = 10^{4}\ cd\ m^{-2}}$	

A.3 Prefixes

The SI system also specifies that names of multiples and submultiples of units are formed by means of the following prefixes:

Multiplying Factor	*Prefix*	*Symbol*	*Multiplying Factor*	*Prefix*	*Symbol*
10^{24}	yotta	Y	10^{-1}	deci	d
10^{21}	zetta	Z	10^{-2}	centi	c
10^{18}	exa	E	10^{-3}	milli	m
10^{15}	peta	P	10^{-6}	micro	μ
10^{12}	tera	T	10^{-9}	nano	n
10^{9}	giga	G	10^{-12}	pico	p
10^{6}	mega	M	10^{-15}	femto	f
10^{3}	kilo	k	10^{-18}	atto	a
10^{2}	hecto	h	10^{-21}	zepto	z
10^{1}	deca	da	10^{-24}	yocto	y

Multiple prefixes may not be used, even for the kilogram (unique among SI base units in having one of these prefixes as part of its name), for which the prefix names are used with the unit name ‘gram’, and the prefix symbols are used with the unit symbol ‘g’; e.g, 10^{-6} kg = 1 mg (not 1 μkg).

With this exception, any SI prefix may be used with any SI unit (whether base or derived, including the degree Celsius and its symbol °C). Note that use of ‘micron’ for the μm persists very widely (almost universally!) in astrophysics, although the approved SI name is the micrometre.

According to SI rules, these prefixes strictly represent powers of 10, and should not be used to represent powers of 2. Thus one kilobit (1 kbit) is 1000 bit – not 2^{10} bit = 1024 bit. In an attempt to resolve this ambiguity, prefixes for binary multiples have been recommended by the International Electrotechnical Commission for use in information technology (though they’re achieving acceptance only slowly):

Factor	Name	Symbol	Origin			
2^{10}	kibi	Ki	‘kilobinary’,	$(2^{10})^1$	kilo,	$(10^3)^1$
2^{20}	mebi	Mi	‘megabinary’,	$(2^{10})^2$	mega,	$(10^3)^2$
2^{30}	gibi	Gi	‘gigabinary’,	$(2^{10})^3$	giga,	$(10^3)^3$
2^{40}	tebi	Ti	‘terabinary’,	$(2^{10})^4$	tera,	$(10^3)^4$
2^{50}	pebi	Pi	‘petabinary’,	$(2^{10})^5$	peta,	$(10^3)^5$
2^{60}	exbi	Ei	‘exabinary’,	$(2^{10})^6$	exa,	$(10^3)^6$

A.4 Writing style

For those really interested in the details, here are some of the more important elements of recommended writing style:

- Symbols are written in upright Roman type ('m' for metres, 'l' for litres).
- Units are written without a capital (other than where the rules of punctuation require it), as are their corresponding symbols, except for symbols derived from the name of a person; thus "the symbol for the coulomb is 'C' ". However, some American-speaking countries use 'L' for 'litre' (to avoid confusion with numeric '1').
- Names of units take plurals according to the usual rules of grammar; e.g., 20 kilograms, 40 henries. 'Hertz', 'lux', and 'siemens' have the same form in the singular and the plural. Symbols of units are *not* pluralised ('20 kg', not '20 kgs'), thereby avoiding confusion with the second ('s').
- A space should separates a number and its unit ('20 kg', not '20kg'). Exceptions are the symbols for degrees, arcminutes, and arcseconds ($^\circ$, $'$, $''$), which should be contiguous with the number (e.g., $20^\circ\ 15'$).
- Symbols do not have an appended full stop (other than where the rules of punctuation require it; specifically, at the end of a sentence).
- Commas should not be used to break up long runs of digits, though spaces may be used (3.141 592 654, not 3.141,592,654).

Appendix B

Constants

B.1 Physical constants

Speed of light	c	2.99792458×10^{8} m s^{-1}
Universal gravitational constant	G	6.67300×10^{-11} m^{3} kg^{-1} s^{-2} (= N m^{2} kg^{-2})
Planck's constant	h	6.626068×10^{-34} m^{2} kg s^{-1} (=J s)
Boltzmann's constant	k	$1.3806503 \times 10^{-23}$ m^{2} kg s^{-2} K^{-1} (=J K^{-1})
Stefan-Boltzmann constant	σ	5.67040×10^{-8} W m^{-2} K^{-4}
Radiation constant	$a = 4\sigma/c$	7.55×10^{-16} J m^{-3} K^{-4}
Atomic mass unit	amu	$1.66053886 \times 10^{-27}$ kg
Hydrogen mass	$m(\mathrm{H})$	1.00794 amu
Proton mass	m_{P}	$1.67262158 \times 10^{-27}$ kg
Electron mass	m_{e}	$9.10938188 \times 10^{-31}$ kg
Electron charge	e	$1.60217646 \times 10^{-19}$ C
	$\frac{\pi e^2}{m_{\mathrm{e}} c}$	2.654×10^{-6} m^{2} s^{-1}

B.2 Astronomical constants

Astronomical unit	AU	1.49598×10^{11} m
Parsec	pc	$3.08568025 \times 10^{16}$ m

B.2.1 Solar parameters

The 'solar constant' is the (very slightly variable) energy flux from the Sun measured at the mean distance of the Earth; numerically,

$$\text{solar constant, } C_\odot = 1366 \text{ J m}^{-2} \text{ s}^{-1}.$$

The mean Earth–Sun distance is

$$d_\odot \equiv \; 1 \text{ AU} = 1.496 \times 10^{11} \text{ m}$$

whence, since $\mathrm{L}_\odot = 4\pi d_\odot^2 C_\odot$,

$$\mathrm{L}_\odot = 3.827 \times 10^{26} \text{ W}$$

This allows us to define the Sun's effective temperature, from $\mathrm{L}_\odot = 4\pi \mathrm{R}_\odot^2 \sigma T_{\text{eff}}^4$, using

$$\begin{aligned} \mathrm{R}_\odot &= 6.960 \times 10^8 \text{ m;} \\ T_{\text{eff}}(\odot) &= 5770 \text{ K.} \end{aligned}$$

The solar mass is

$$\mathrm{M}_\odot = 1.989 \times 10^{30} \text{ kg}$$

(which follows from equating centrifugal and gravitational accelerations of the Earth in orbit,

$$\frac{\mathrm{M}_\oplus v_\oplus^2}{\mathrm{R}_\oplus} = \frac{G\mathrm{M}_\odot \mathrm{M}_\oplus}{d_\odot^2}$$

whence the mean density is

$$\begin{aligned} \overline{\rho} &= \frac{\mathrm{M}_\odot}{{}^4\!/\!{}_3 \pi \mathrm{R}_\odot^3} \\ &= 1.4 \times 10^3 \text{ kg m}^{-3} \end{aligned}$$

Finally, the mean number density is

$$\overline{n} = \frac{\overline{\rho}}{\mu m(\mathrm{H})} \simeq 1.4 \times 10^{30} \text{ m}^{-3}$$

(using $\mu \simeq 0.61$).