Topic 33 —Optical Instruments II

compound lenses

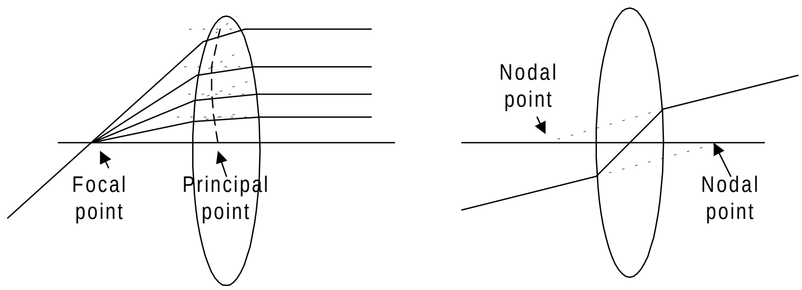

Figure L33.1: The cardinal points of a thick lens system: focal points, principal points, and nodal points.

If we need to describe the image formation by a thick lens, or a lens made up from several components (which will usually be treated as several thin lenses), it is the cardinal points and the rays through the focal points which are the key – as a reminder they are shown in figure L33.1.

general treatment of two-lens system (not for examination

We can derive a general expression for the focal length of a two-lens system, made of two *thin* lenses, using the geometry shown in figure L33.2.

Consider the two-lens system in which the incoming parallel light is focused by lens 1 at the point G. This will form the virtual object for lens 2, and so this object is a distance $f_1 - d$ to the right of lens 2. Then the distance s from lens 2 to the focus F_2 will be given by

$$\frac{1}{s} - \frac{1}{f_1 - d} = \frac{1}{f_2}.$$

If the initial ray enters lens 1 at a height h_1 and exits lens 2 at a height h_2 then the intersection point (giving the second principal point H_2) is found from the similar triangles CF_2V_2 and BF_2H_2

$$\frac{h_2}{s} = \frac{h_1}{f}$$

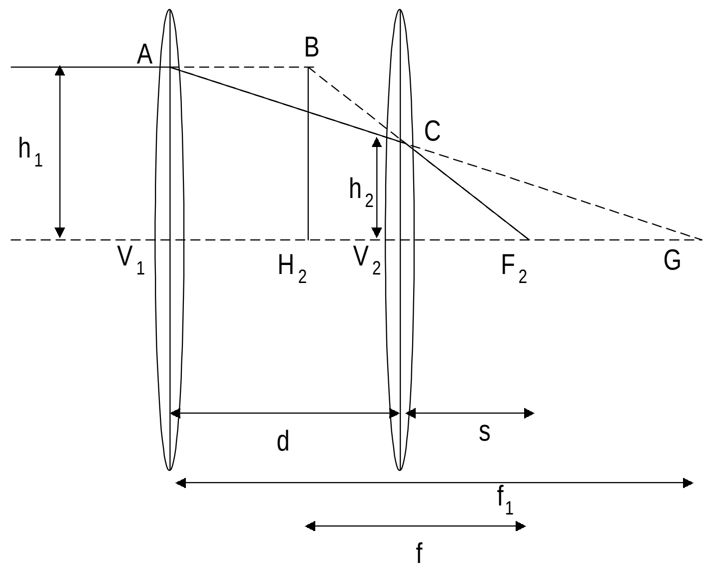

Figure L33.2: The geometry of a two-lens system.

and from CGV_2 and AGV_1

$$\frac{h_2}{f_1 - d} = \frac{h_1}{f}$$

which gives us two expressions for h_2/h_1

$$\frac{h_2}{h_1} = \frac{s}{f} = \frac{f_1 - d}{f_1}$$

giving us a value for s to substitute back

$$\frac{f_1}{f(f_1 - d)} - \frac{1}{f_1 - d} = \frac{1}{f_2}$$

which may be rearranged to give

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}.$$

Similarly, one may go back to find the positions of the principal points. As

$$s = \frac{f}{f_1}(f_1 - d)$$

the position of H_2 relative to the second lens is then

$$s - f = \frac{f}{f_1}(f_1 - d) - f = -\frac{fd}{f_1}.$$

Note the sign, showing that if both lenses are converging then the second principal point lies to the left of the second lens.

Similarly, the first principal point is at $\frac{fd}{f_2}$ relative to the first lens.

L33.1 Two-lens system - example

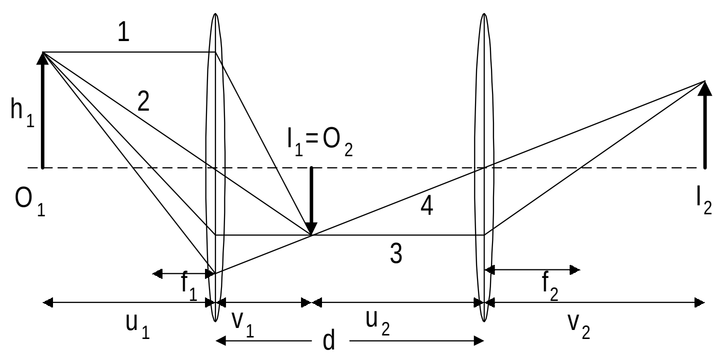

Figure L33.3: An example of a system with two lenses.

In most cases, though, it is easier to work through the analysis of the system from first principles. Consider (figure L33.3) an objective lens of focal length 100 mm, eyepiece of focal length 75 mm, 450 mm apart. Then if the object is 150 mm to the left of lens 1 the image in lens 1 will be given by

$$\frac{1}{v_1} = \frac{1}{f_1} + \frac{1}{u_1} = \frac{1}{100} - \frac{1}{150} = \frac{1}{300}$$

that is 300 mm to the right of lens 1, or 150 mm to the left of lens 2. Thus the final image will be given by

$$\frac{1}{v_2} = \frac{1}{f_2} + \frac{1}{u_2} = \frac{1}{75} - \frac{1}{150} = \frac{1}{150}.$$

The overall magnification is given by

$$M = \frac{v_1}{u_1}\frac{v_2}{u_2} = \frac{300}{-150} \quad \frac{150}{-150} = 2.$$

L33.2 Compound microscope *AF891*

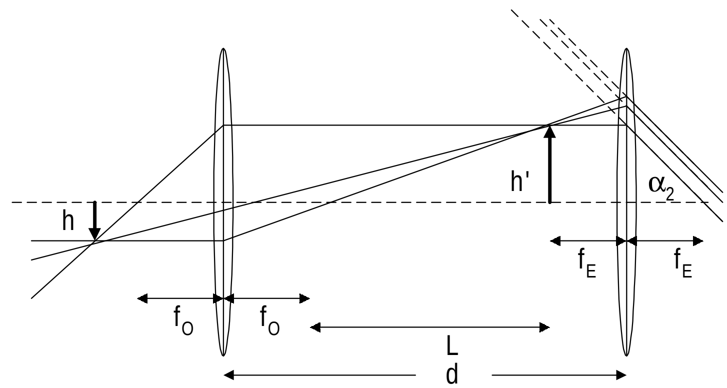

Figure L33.4: The compound microscope.

magnification *AF892*

If the microscope is set up with the image at infinity, it is the angular magnification which is relevant. For an image at infinity, the intermediate image is at the focal point of the eyepiece. Its angular height, then, is

$$\frac{\text{height of intermediate image}}{f_E} = \frac{h'}{f_E}$$

and this intermediate image height is related to the object height by

$$\frac{\text{height of intermediate image}}{\text{height of object}} = -\frac{L}{f_O}.$$

The angular height of the object, unmagnified, is that seen at the standard near point, so

$$M_A = -\frac{L}{f_O}\frac{250 \text{ mm}}{f_E}.$$

Note that many manufacturers standardize on L, the distance from the second focus of the objective to the first focus of the eyepiece, at 160 mm. This enables eyepieces and objectives to be marked separately with their own magnifications, with the magnification of the whole microscope being the product of the two.

resolving power *AF892*

The resolving power is determined, as in all optical instruments, by the aperture. In this case the limit of resolution is given by

$$\delta = \frac{0.61\lambda}{n_1 \sin(\alpha_1)}.$$

Here n_1 is the refractive index of the material *outside* the objective, α_1 is the angle subtended at the objective. The object is often very close to the lens, so here we do have to worry about angles large enough for the paraxial approximation to be questionable. Resolution may therefore be improved by an oil immersion objective.

L33.3 The telescope - *AF893*

the astronomical refractor

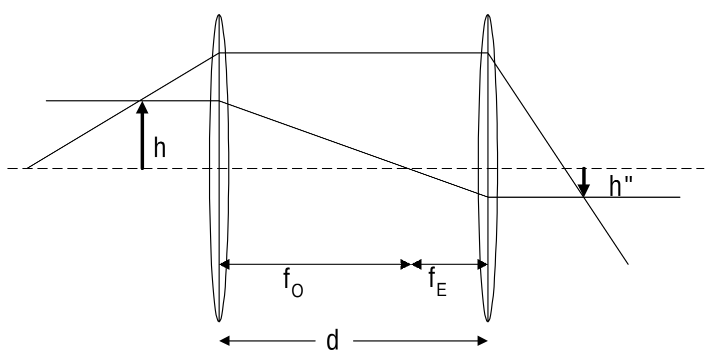

Figure L33.5: The astronomical telescope: linear magnification.

The astronomical refractor (figure L33.5) produces an inverted image. Both the object and the image are at infinity, so the second principal focus of the objective is in the same place as the first principal focus of the eyepiece - that is, the separation of the two lenses is equal to the sum of their focal lengths.

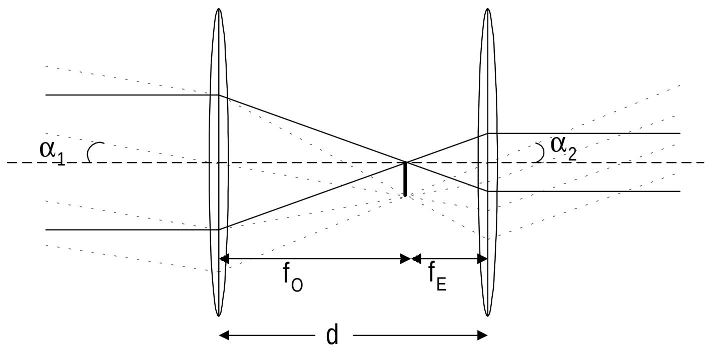

Figure L33.6: The astronomical telescope: angular magnification.

magnification *AF894*

The transverse magnification $M_T = f_E/f_O$ is not a useful quantity in this case. The relevant magnification (figure L33.6) is the angular magnification which can be deduced readily (see diagram) using triangles each having as its height the height of the intermediate image:

$$M_A = \frac{\alpha_2}{\alpha_1} = \frac{h'/f_E}{h'/f_O} = \frac{f_O}{f_E}.$$

resolving power *AF895*

The resolving power of the telescope depends entirely on the linear diameter of the objective. The angular separation of two points on an object that can just be resolved is

$$\delta\theta = \frac{1.22\lambda}{D}$$

where D is the objective diameter.

the terrestrial telescope

In a terrestrial telescope, one wants an upright image. One approach is the Galilean one (figure L33.7). Here again the separation of the lenses is set to the sum of the focal lengths (but the focal length of the eyepiece is negative).

More often, a more complex arrangement of lenses is used to make the image erect.

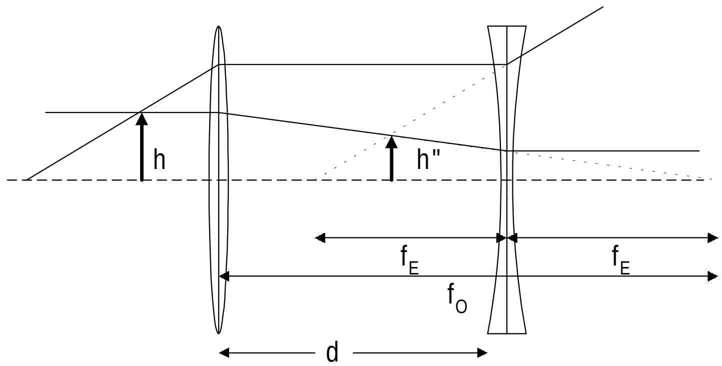

Figure L33.7: The Galilean telescope: linear magnification.

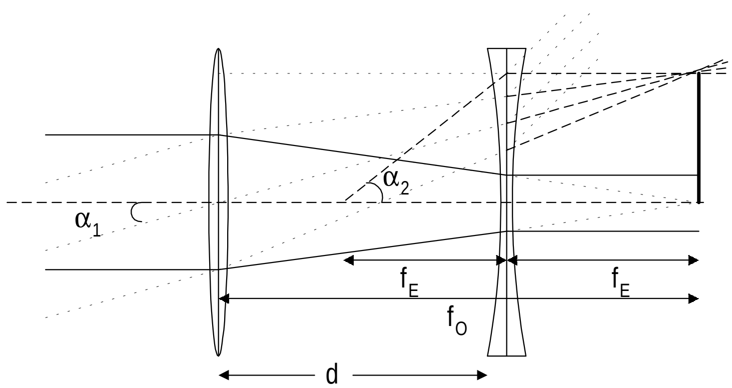

Figure L33.8: The Galilean telescope: angular magnification.

the telephoto lens

This is in some ways similar to the Galilean telescope, but with the light brought to a real focus (in the plane of the film). The lens separation, then, is larger than in the Galilean telescope with similar lenses.

prismatic binoculars

In prismatic binoculars, total internal reflection in prisms is used to extend the path length between objective and eyepiece, effectively 'folding' the optical path.

the astronomical reflector

The first reflecting telescope was designed by Isaac Newton, to avoid the problems of chromatic aberration in glass lenses.

Hubble Space Telescope *AF902-904*

With the Hubble telescope, history has almost come full circle – though the design is actually Cassegrainian rather that Newtonian.