Topic 32 — Applications of Refraction II

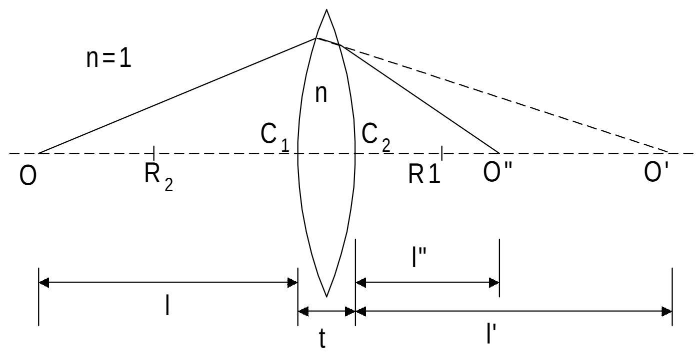

Figure L32.1: The refraction of light by a thin lens.

Now that we know how spherical surfaces behave, we are in a position to stick them together to make lenses[1]. A lens is just a piece of transparent material bounded by two spherical surfaces (or one spherical and one plane surface). Light passing through a lens is therefore refracted twice, as shown in figure L32.1.

L32.1 Lenses *AF882-889*

We shall assume that the medium on each side of the lens is the same (air).

We shall also assume initially that the lens is *thin* compared with any other lengths involved.

[1]It is a curious fact (discussed very nicely by D. Park, *The fire within the eye*, Princeton Press 1997) that scholars had been walking around with spectacles on their noses for hundreds of years before they bothered to ask how they worked. It may well be true that their operation was lumped with many other phenomena under the heading of magic. In 1589 Giovanni Battista della Porta (1535-1616) began to ask questions: "...spectacles, which are most necessary for the use of man's life; whereof no man yet hath assigned the effects, nor yet the reasons of them." Despite much thought and experimentation. Porta never got to the bottom of the matter. The first complete theory of lenses was given by Johannes Kepler in *Dioptrice* in 1610 (Kepler performed the rather gruesome experiment of taking an eyeball, scraping away its back wall until only a thin layer remained, and observing that an inverted image was cast on the retina). Some fine details were missing (for example, he did not have Snel's law, and had to use the adequate approximation that the angle of refraction is a constant multiple of the angle of incidence). In *Dioptrice* Kepler went as far as a theory of the telescope.

Consider a biconvex lens. Light from an object at O is refracted by the first surface and would, if the glass continued indefinitely to the right of the first surface, cross the optical axis again at O', at z coordinate l'. Then, with refractive index 1 for air and n for glass,

$$\frac{1}{l} - \frac{n}{l'} = \frac{1-n}{r_1}.$$

If the thickness of the lens can be neglected, we can take O' as the object for refraction at the second surface, and write

$$\frac{n}{l'} - \frac{1}{l''} = \frac{n-1}{r_2}.$$

Eliminating l' by adding the equations we get

$$\frac{1}{l''} - \frac{1}{l} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right).$$

This is the basic equation for the behaviour of a thin lens[2].

Another common notation uses u for the object position, v for the image position: in this notation

[2]Note that here we let the sign convention take care of positions, so that l, l' etcetera all include signs. Alternatively, we could work through the specific case of a biconvex lens putting in all the signs, then put the sign convention back in at the end. In this case, with the diagram as shown, l and r_2 would be negative, l', l'' and r_1 would be positive. Then we would have

$$\frac{1}{-|l|} - \frac{n}{|l'|} = \frac{1-n}{|r_1|}.$$

If the thickness of the lens can be neglected, we can take O' as the object for refraction at the second surface, and write

$$\frac{n}{|l'|} - \frac{1}{|l''|} = \frac{n-1}{-|r_2|}.$$

Eliminating l' we get

$$\frac{1}{|l''|} + \frac{1}{|l|} = (n-1)\left(\frac{1}{|r_1|} + \frac{1}{|r_2|}|\right).$$

Now we can put the sign convention back in, to obtain

$$\frac{1}{l''} - \frac{1}{l} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right).$$

$$\frac{1}{v} - \frac{1}{u} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right).$$

principal foci and focal length *AF885*

It is clear that if the object is at minus (or plus) infinity, we have

$$\frac{1}{v} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = \frac{1}{f_2},$$

which gives us the position of the second focal point. Note that this automatically leads to a

sign convention for focal lengths: the focal length of a *converging* (convex) lens is *positive*, that of a *diverging* (concave) lens is *negative*.

We can rewrite the lens equation, then, as

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}.$$

$P = 1/f$ is also known as the *power* of the lens: if f is measured in metres, P is in dioptres.

Drawing ray diagrams

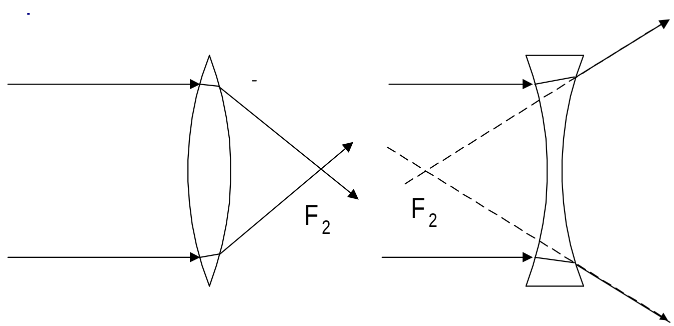

Figure L32.2: The principal rays for a thin lens: rays incident through the first principal focus and exiting parallel to the optical axis.

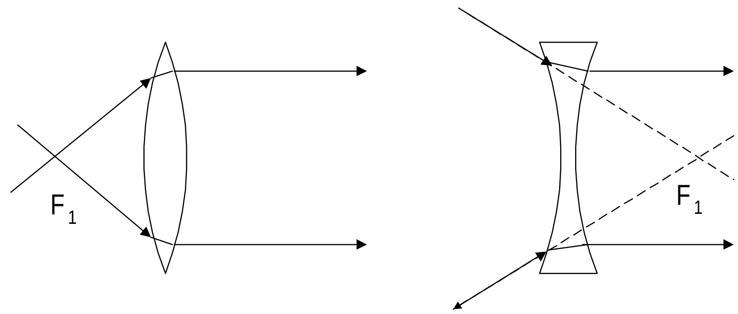

Figure L32.3: The principal rays for a thin lens: rays incident parallel to the axis and exiting through the second principal point.

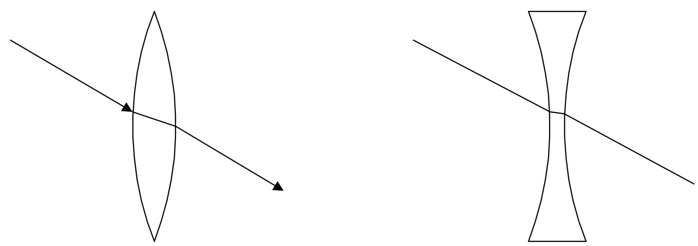

Figure L32.4: The principal rays for a thin lens: rays passing undeviated through the centre of the lens.

In drawing ray diagrams, there are three *principal rays*, as shown in figure L32.2 to L32.4:

- one enters parallel to the optical axis and is refracted so as to pass directly or by projection through the second focal point;
- one passes directly or by projection through the first focal point, and emerges parallel to the optical axis;
- one passes undeviated through the centre of the lens (where the lens surfaces are locally parallel).

Note that it is only for a converging lens that light from infinity is focussed to a small image on the other side of the lens – a fact overlooked by William

Golding in *Lord of the Flies*, in which Piggy (who, from evidence elsewhere in the book, is seriously short-sighted) has his spectacles snatched away and used as burning glasses to light a fire. Unfortunately, the lenses used to correct short sight are diverging lenses.

chromatic aberration ***AF900***

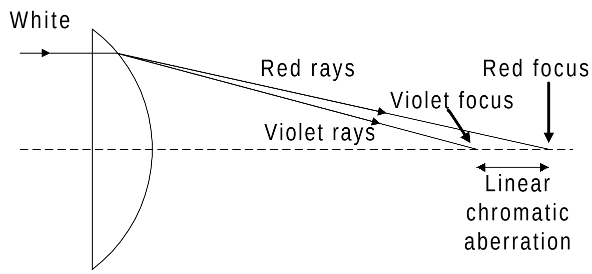

Figure L32.5: Chromatic aberration by a thin lens.

No lens system will be free of aberrations: chromatic aberration, shown in figure L32.5 is the result of dispersion. It may be cured to some extent by an achromatic doublet, in which some of the focussing effect of a high-index lens is undone by a second lower-index, higher-dispersion lens (figure L32.6).

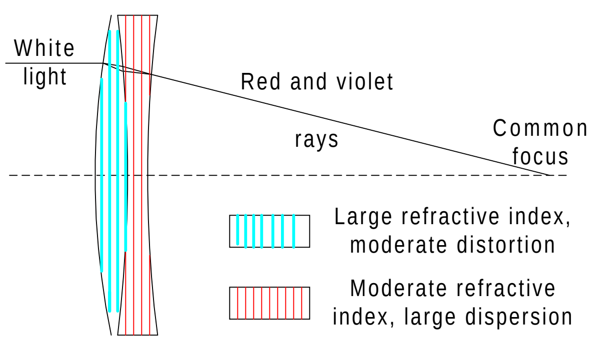

Figure L32.6: Correction of chromatic aberration by an achromatic doublet.

spherical aberration *AF887*

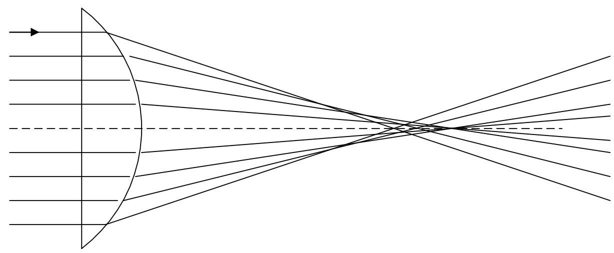

Figure L32.7: Spherical aberration in a large-aperture lens.

With spherical lenses (and also with spherical mirrors) a large aperture leads to rays which fail to focus exactly – spherical aberration, as shown in figure L32.7.

magnification

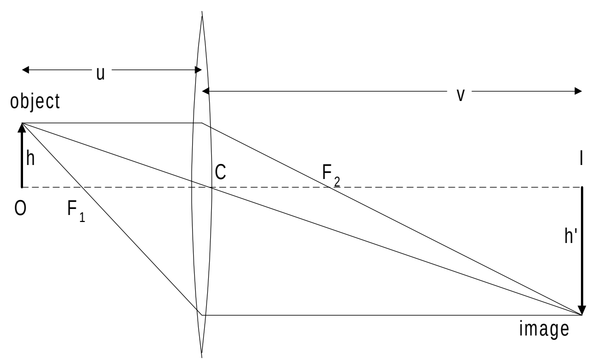

Figure L32.8: The formation of an image by a converging lens.

The principal rays are the key to determining the magnification of a lens system. As an example, consider a converging lens (positive f) with $f =$ 100 mm, and place an object 150 mm from it, i.e. at $u = -150$ mm, as if figure L32.8. Then the image will be at v where

$$\frac{1}{v} - \frac{1}{-150} = \frac{1}{100}$$

so

$$v = 300 \text{ mm}$$

and then, immediately, from the central ray, we may use the similar triangles enclosed by the axis, the ray and the object, the axis, the ray and the image to find the transverse (or linear) magnification which is

$$M = \frac{h'}{h} = \frac{v}{u} = \frac{300}{-150} = -2.$$

The negative sign shows that the image is imverted, and the fact that the magnitude of M is greater than one shows that it is magnified.

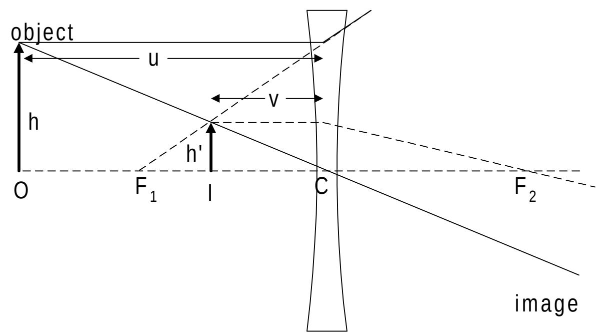

Figure L32.9: The formation of an image by a diverging lens.

Again, as in figure L32.9,consider a diverging lens (negative f) with $f = -100$ mm, and place an object 150 mm from it, i.e. at $u = -150$ mm. Then the image will be at v where

$$\frac{1}{v} - \frac{1}{-150} = \frac{1}{-100}$$

so

$$v = -60 \text{ mm}$$

and then, again from the central ray, the transverse magnification is

$$M = \frac{h'}{h} = \frac{v}{u} = \frac{-60}{-150} = +0.4,$$

upright, but diminished in size.

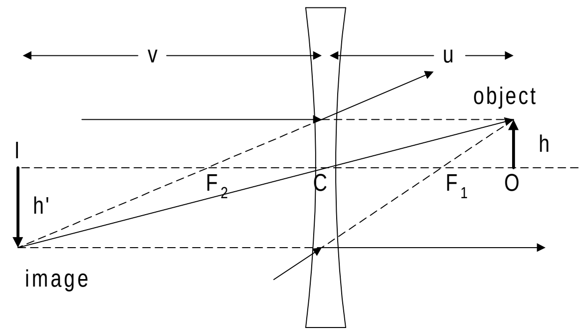

Figure L32.10: The formation of an image of a virtual object in a concave lens.

As a final example, consider the situation shown in figure L32.10. Take a diverging lens (negative f) with $f = -50$ mm, and place an object 80 mm from it but to its right, i.e. at $u = 80$ mm to act as a *virtual object* – in reality, of course, this would only happen in the middle of an assembly of lenses, in which a converging lens further to the left produced the converging incident rays. Then the image will be at v where

$$\frac{1}{v} - \frac{1}{80} = \frac{1}{-50}$$

so

$$v = -133 \text{ mm}$$

and then, immediately, from the central ray, the transverse magnification is

$$M = -1.66.$$

the magnifying glass *AF892*

We can use a simple lens as a magnifying glass. The key to understanding this system is to note that the eye itself has a shortest range at which it can focus. As shown in figure L32.11, the usual mode of operation is to place the lens as close as possible to the eye, with the image at the standard near point (250 mm from the eye). In that case if the object is $l$mm from the lens,

$$\frac{1}{l} - \frac{1}{250} = \frac{1}{f}$$

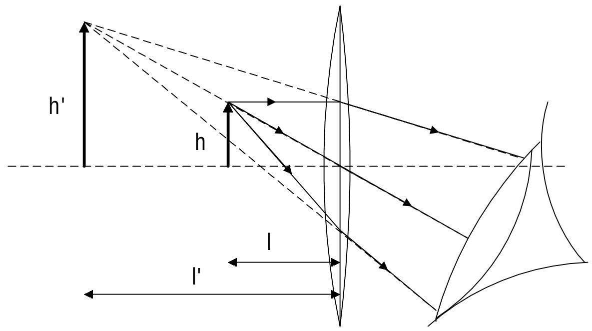

Figure L32.11: The simple lens used as a magnifying glass.

or

$$M = \frac{h'}{h} = \frac{250}{l} = 1 + \frac{250}{f}$$

for a magnifying glass with the image at the near point.

Another way of setting up the system is with the image at infinity, in which case the relevant magnification is an *angular* magnification, the ratio of the angle subtended by the image at infinity to the angle the object would subtend at the standard near point.

In that case, an image at infinity corresponds to an object at the first focal point, with angular height $\frac{h}{f}$, the object at the standard near point has angular height $\frac{h}{250}$, so

$$M = \frac{250 \text{ mm}}{\text{f}}$$

for a magnifying glass with the image at infinity,

which is rather less than the magnification with the image at the near point.

L32.2 Thick lenses

For a thick lens system, we again have the same basic quantities as before – focal points and vertices of lenses – but where do we measure them from?

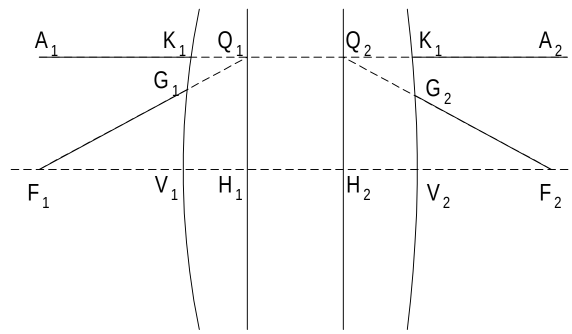

Figure L32.12: The key features of a thick lens system.

We define the so-called *Cardinal Points* as follows (see figure L32.12):

- two *focal points*, defined in terms of entry or exit rays parallel to the axis, F_1 and F_2;
- *principal planes*, defined by the locus of the points of intersection of the incident ray through the focus and the exit ray parallel to the axis (Q_1 and Q_2);
- the *principal points*, H_1 and H_2,being the intersections of the principal planes with the axis;
- the *nodal points* where the ray through the optical centre of the lens (the ray which emerges parallel to its incident direction) intersects the axis.

One *can* derive formulae which resemble

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f},$$

but the algebra is horrendous[3], and one has to measure the distances u and v from the principal planes, which leaves unsolved the question of where those planes are relative to identifiable points such as the vertices of the lens surfaces.

[3] If you can't resist it, try J. Morgan's *Introduction to Geometrical and Physical Optics* (McGraw-Hill 1954) page 57, or R.S. Longhurst's *Geometrical and Physical Optics* (Longman 19570 page 44.

Of course, for a *thin lens*, there is only one principal plane and one nodal point, in the plane of the lens (the two planes and points of the thick lens merge for the thin lens).