Topic 30 — Curved Mirrors

We should be fairly familiar with reflection and refraction at plane interfaces, so now we move on to ask how curved interfaces affect wavefronts – that is, we find explore the phenomena of focussing, image formation, and magnification. The interfaces we look at will be, as in the lens surface we treated in our discussion of Newton's rings, parts of spherical surfaces. Of course, if we really want to design an optical instrument we will probably use a computer program, which will not only allow us to explore the performance of the instrument but will even print out an order for the appropriate components.

T30.1 Approximations and Sign Conventions

Approximations

Throughout the treatment of optical instruments we make the *paraxial approximation*, that the rays make a small enough angle α to the axis that

$$\sin(\alpha) \approx \tan(\alpha) \approx \alpha,$$

although most of the figures we draw will exaggerate the angles in order to make the angles visible. These small-angle rays are known as *paraxial rays*.

Sign Conventions

In order to get unambiguous formulae, we need to adopt a sign convention.

We choose a so-called Cartesian convention, in which

- the origin of the Cartesian system is located at the vertex of the curved boundary or mirror, and at the centre of a thin lens, with the x axis directed along the optical axis from left to right.
- object, image, and centre of curvature distances are defined to be the x coordinates of the y, z planes which contain them. Thus distances to points or planes to the right of the vertex or lens centre are positive, those to the left are negative.
- light sources and objects are placed to the left of the first surface in the system, so that the light rays travel from left to right, but the object has a negative x coordinate and the object distance will thus be negative.
- angles are taken to be positive or negative dependent on whether their tangents are positive or negative.

The way we use these sign conventions is two-fold

- when *deriving* equations, just use geometry and put in the signs at the end to get general-purpose equations;
- when *using* the general-purpose equations, substitute distances *with the appropriate signs.*

[1]

T30.2 Image formed by curved mirror on and off axis *AF876- 882*

To start with, we look at the formation of images in curved mirrors. Qualitatively, we can see what happens by looking at a concave mirror as shown in

[1]Note that we shall use the Cartesian sign convention, that is Group I case 1 of T. Smith's report on the Teaching of Geometrical Optics (1934). This appears to be the one in most common use. One user of an alternative (Group II case 1) scheme, in which distances are counted positive if they are actually traversed by a light beam, negative if not (i.e. if they lead to a virtual image) is S.C. Strong Concepts of Classical Optics, Freeman (1958). Note that Hecht uses a virtual image negative convention, as do Alonso and Finn.

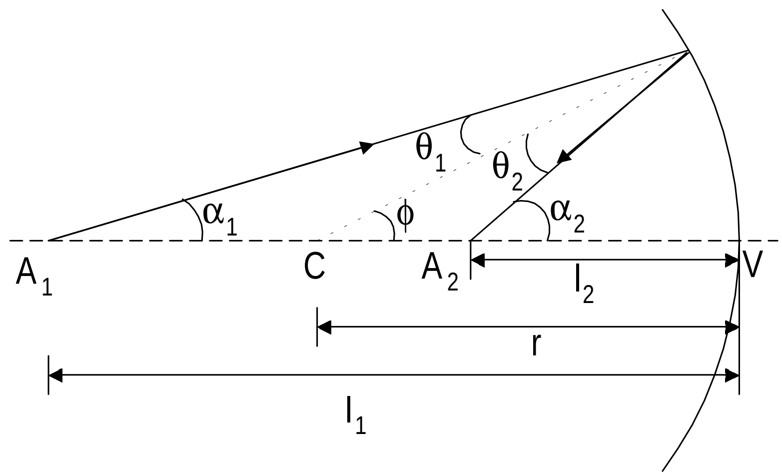

Figure T30.1: Reflection in a concave spherical mirror.

figure T30.1. A ray of light from A_1 is reflected off the mirror at a distance y from the optical axis, to cross the axis again at A_2. If the centre of curvature is at C, it is obvious that the two points A_1 and A_2 must be on opposite sides of C.

Now let us make this more quantitative. In the figure, the laws of reflection tell us that $\theta_1 = \theta_2$. Now using the fact that the exterior angle of a triangle is equal to the sum of the two opposite internal angles we have

$$\theta_1 = \phi - \alpha_1 = \frac{y}{r} - \frac{y}{l_1}$$
$$\theta_2 = \alpha_2 - \phi = \frac{y}{l_2} - \frac{y}{r}$$

but as $\theta_1 = \theta_2$

$$\frac{y}{r} - \frac{y}{l_1} = \frac{y}{l_2} - \frac{y}{r}$$

or

$$\frac{1}{l_1} + \frac{1}{l_2} = \frac{2}{r}$$

and as all the distances in this case are negative, this is the final result.

focus, focal length *AF876-7*

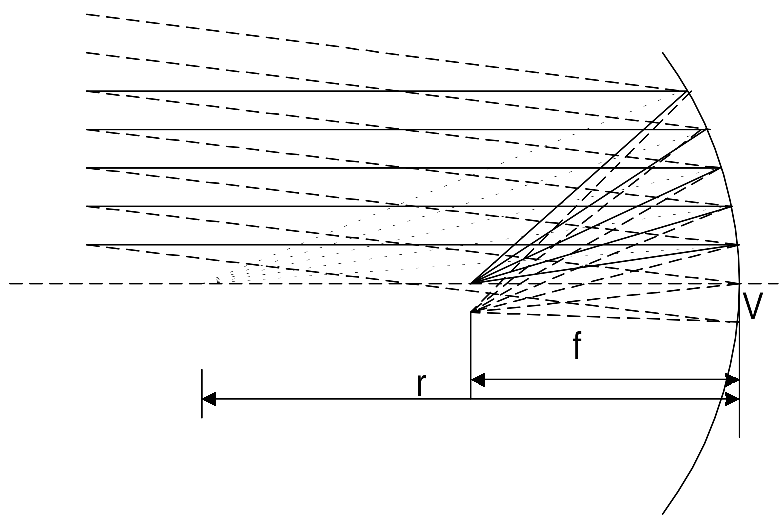

Figure T30.2: The focal length of a concave spherical mirror.

If light is incident from infinity, as shown in figure T30.2, it will be focused at a distance of $2/r$, so the *focal length* is $2/r$. In this case the radius r is negative (distance to centre of curvature negative) so the focal length is also negative.

convex mirror

In the case of a convex mirror, shown in figure T30.3, we have a similar set of equations

$$\begin{aligned} \theta_1 &= \phi + \alpha_1 = \frac{y}{r} + \frac{y}{l_1} \\ \theta_2 &= \alpha_2 - \phi = \frac{y}{l_2} - \frac{y}{r} \end{aligned}$$

but as $\theta_1 = \theta_2$

$$\frac{y}{r} + \frac{y}{l_1} = \frac{y}{l_2} - \frac{y}{r}$$

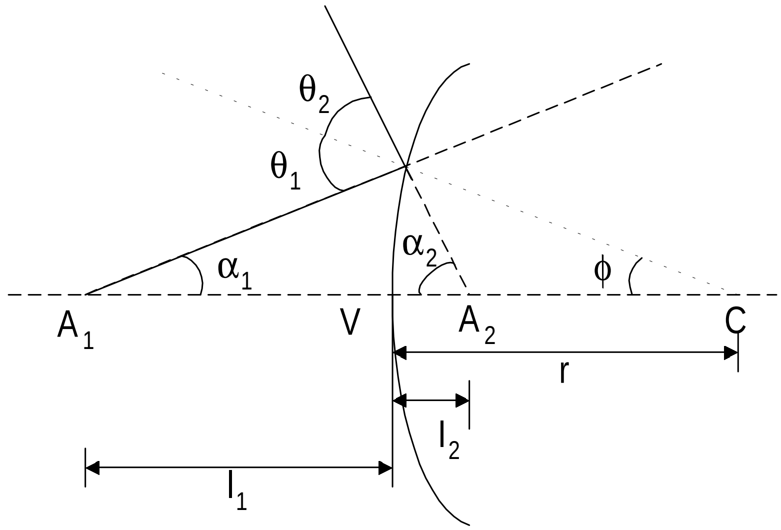

Figure T30.3: Reflection in a convex spherical mirror.

or

$$\frac{1}{l_2} - \frac{1}{l_1} = \frac{2}{r}$$

but in this case l_1 is negative, but r and l_2 are positive, so putting the correct signs in we recover

$$\frac{1}{l_1} + \frac{1}{l_2} = \frac{2}{r}.$$

Again, the focal length is $2/r$, but this time r is positive so the focal length is positive.

size of image *AF878*

What we generally want to do with an optical system is to form an image, and one of the things we want to know about an image is its size, so that we can compute the magnification of the system. An image may be real (the rays pass through the image) or virtual (the rays do not pass through the image, but the rays produced do).

In general we assume that the image is 'perfect', as defined by Maxwell:

- all rays from a point on the object which enter the optical system pass (really or virtually) through a single image point;
- if the object lies in a plane perpendicular to the axis of the system, the image will lie in a parallel plane;
- the image is geometrically similar to the object.

magnification *AF881*

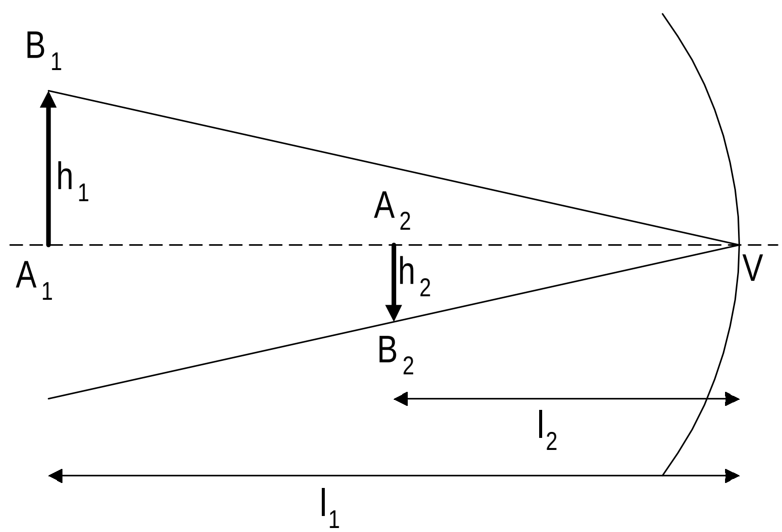

Figure T30.4: Magnification of the image reflected in a concave spherical mirror.

The way to find the magnification is to locate the object and image, and then use the ray which is reflected from the centre of the mirror and passes through the top of the image. Referring to figure T30.4, the triangles A_1VB_1 and A_2VB_2 are similar (all angles equal) so

$$\frac{h_1}{l_1} = \frac{h_2}{l_2}$$

and the magnification will be

$$M = \frac{-h_2}{h_1} = -\frac{l_2}{l_1}$$

which is negative because we have measured all heights in the Cartesian system, and the fact that it is negative shows that the image is inverted with respect to the object.

For example, consider a shaving or make-up mirror which is concave with a radius of curvature of 400 mm. If it is desired to form an image 250 mm from the eye, how far from the mirror should one place one's face?

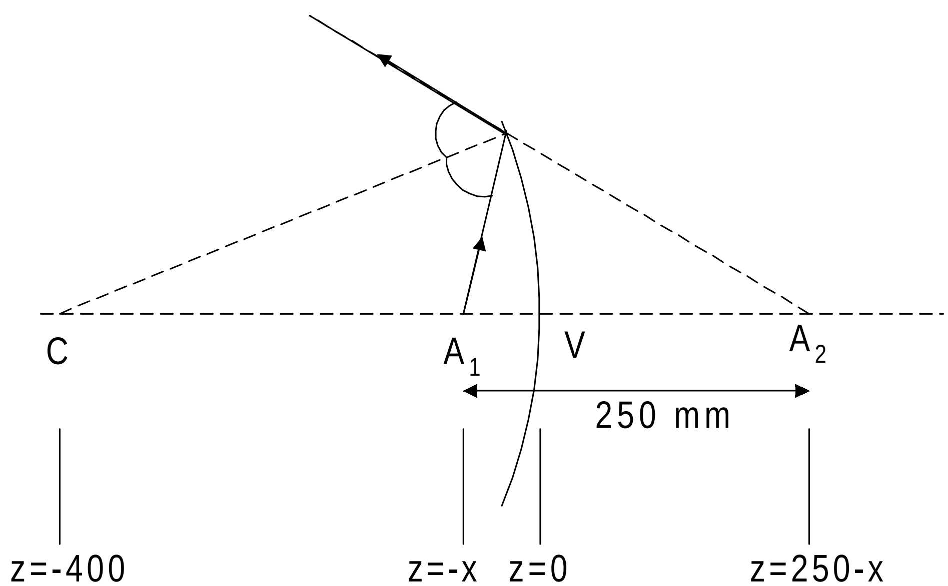

Figure T30.5: A concave spherical mirror used as a make-up or shaving mirror.

If the face is x from the mirror as shown in figure T30.5, then the image will be $250 - x$ on the other side, so that (with the sign convention)

$$-\frac{2}{400} = -\frac{1}{x} + \frac{1}{250 - x}$$

which gives us a quadratic in x to solve, with solutions

$$x = 561$$

which gives us an inverted, reduced image ($l_1 = -561$, $l_2 = -311$, $M = -\frac{-311}{-561} = -\frac{311}{561}$) or

$$x = 89$$

($l_1 = -89$, $l_2 = 161$), which is the solution we seek.

The magnification of the face is then

$$-\frac{250 - 89}{-89} = 1.8,$$

that is, the image is upright and magnified.

The distance of 250 mm is the typical nearest distance at which the eye may comfortably focus – the so-called *near point*. For children the distance is shorter, typically 70 mm or so, and in old age it gets greater – hence the tendency of old people without glasses to hold a book at arm's length to read it, and the comment to the doctor 'there is nothing wrong with my eyes, it is just that my arms are not long enough.'

spherical aberration *AF878*

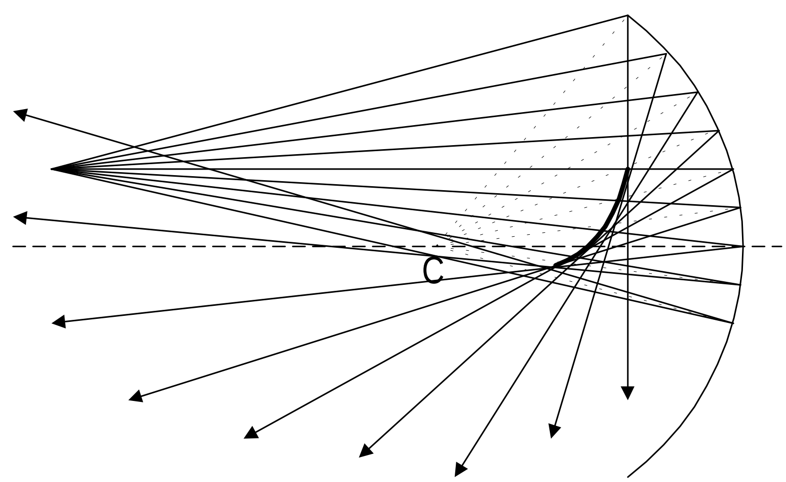

Figure T30.6: Spherical aberration of a concave spherical mirror.

Note that all this only works properly within the paraxial approximation. If we include rays at larger angles, as in figure T30.6, the focus is no longer a point. There is a line of high intensity, the envelope of the rays which we would expect to go through the focus, forming what is called a *caustic*. You may have noticed such a cusp-shaped pattern on the surface of a cup of tea when the cup is standing in bright sunlight and one side of the cup is acting as a cylindrical mirror. This deviation from a point focus is known as *spherical aberration*. If we have a parabolic rather than a spherical mirror, then we *can* bring light from infinity to a point focus – or alternatively form a parallel beam from a point source. This is the geometry used in car headlights, for example.