Part C Major Option Astrophysics

High-Energy Astrophysics

Michaelmas 2006

Lecture 10

Today’s lecture

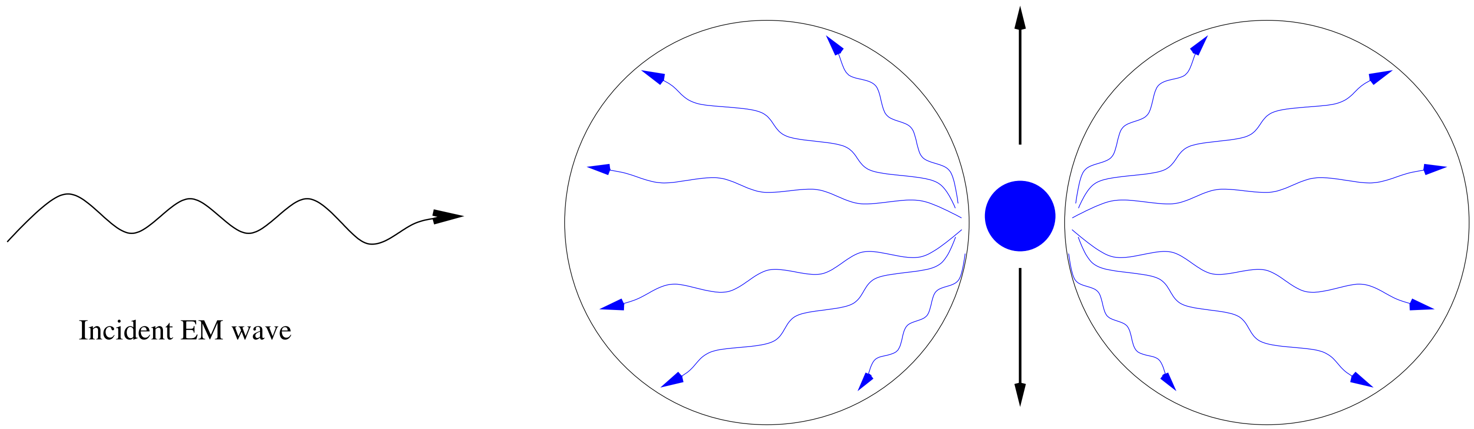
- Inverse-Compton scattering.
- Hot plasma in galaxy clusters and thermal bremsstrahlung.
- Inverse-Compton scattering in radiosources and galaxy clusters.

Thomson, Compton and Inverse-Compton scattering

Inverse-Compton scattering is a mechanism which is seen in both highly-relativistic and “merely” very hot plasma, and today we shall see examples of this in both AGN and in clusters of galaxies.

First however we will remind ourselves of the low-energy limit for electron/photon scattering, namely Thomson scattering. Here the electron has little or no kinetic energy and the incident radiation has very little power; we simply treat the interaction as the oscillation of the electron in response to an incident electromagnetic wave.

Thomson scattering



Electron oscillates sinusoidally: dipole emission uniformly in all azimuthal angles

The scattered light is polarized with its **E** vector parallel to the acceleration of the electron. Recall from classical electromagnetism that we can *define* the Thomson cross-section here, such that the power radiated is

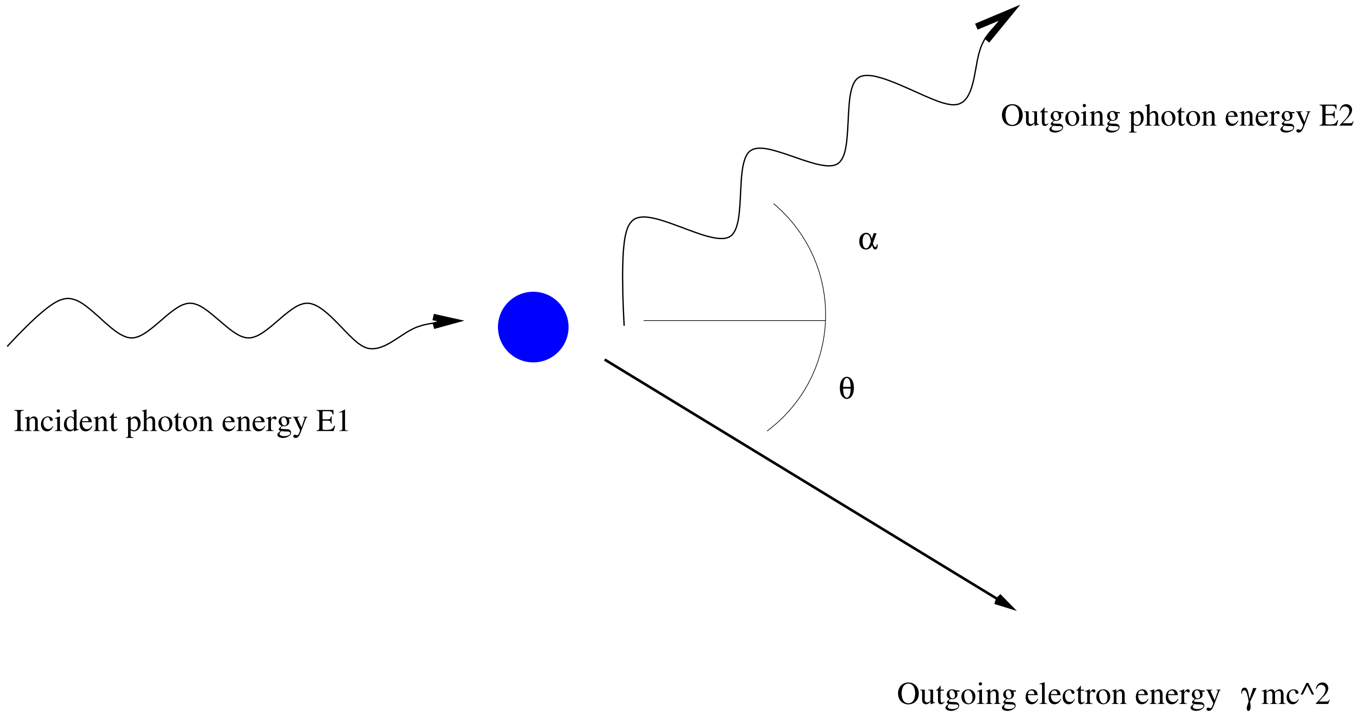
$$P = \sigma_{\mathrm{T}} c u_{\mathrm{rad}}$$

where c is the speed of light and u_{rad} is the energy density of the incoming radiation field.

We see Thomson-scattered light in many astrophysical situations, typically where optical or ultraviolet light is scattered by relatively "cold" electrons (e.g. in the $\sim 10^4$ K phase of the interstellar medium). Scattered, polarized, light from young stellar outflows and in AGN allows us an indirect view of the central regions.

Compton scattering

When the momentum of the incident photons becomes significant, we have to treat the scattering process as a relativistic particle-particle collision. This was originally experimentally demonstrated by Compton’s measurement that the wavelength of X-rays increases when they are scattered against electrons.



Outgoing photon energy E2
α
θ
Incident photon energy E1
Outgoing electron energy γ mc^2

The solution to this is Compton's famous result

$$\frac{E_1}{E_2} = 1 + \frac{E_1}{mc^2}(1 - \cos\alpha)$$

However in high-energy astrophysical situations we find the *inverse* of this process often occurs. With relativistic, or extremely hot thermal electrons, scattering against radio-wavelength photons, the photons are scattered up to high energy at the expense of the electron energy.

To calculate the rate at which electrons lose energy to inverse-Compton scattering we shall use the same trick as we used for synchrotron emission: transform into the instantaneous rest frame of the electron and use the fact that the total power emitted is Lorentz invariant.

Inverse-Compton: calculation

For radio-wavelength photons, the photon energy in the electron rest frame will always be $<< m_e c^2$, even for electrons with Lorentz factors of many thousands. So we know that the power emitted in the electron rest frame is given by

$$P' = \sigma_{\mathrm{T}} c u'_{\mathrm{rad}}$$

and our task is to determine the energy density of the radiation field in the electron rest frame, u'_{rad}.

In the rest frame of the electron, the incident energy density from some *observed* angle θ is boosted by two relativistic Doppler factors; one because the photon energy is increased, and one because the photon arrival rate is increased. So we have

$$u'_{\mathrm{rad}}(\theta) = u_{\mathrm{rad}}[\gamma(1 + \beta\cos\theta)]^2$$

Now we must average over all incident angles

$$\overline{u'_{\mathrm{rad}}} = \frac{4}{3} u_{\mathrm{rad}}(\gamma^2 - \frac{1}{4})$$

$$P = P' = \frac{4}{3} \sigma_{\mathrm{T}} c u_{\mathrm{rad}}(\gamma^2 - \frac{1}{4})$$

Inverse-Compton calculation cont'd.

There is a subtlety here which is easy to miss among the algebra. The power P we have calculated is the rate at which the electron "emits" scattered energy in high-energy photons. But we must remember that the *incident* photons carry a power $\sigma_{\mathsf{T}} c u_{\mathsf{rad}}$. So the net rate at which the electron energy decreases is

$$\frac{dE}{dt}_{\mathrm{IC}} = \frac{4}{3}\sigma_{\mathsf{T}} c u_{\mathsf{rad}}(\gamma^2 - \frac{1}{4}) - \sigma_{\mathsf{T}} c u_{\mathsf{rad}}$$

$$= \frac{4}{3}\sigma_{\mathsf{T}} c u_{\mathsf{rad}}(\gamma^2 - 1)$$

$$= \frac{4}{3}\sigma_{\mathsf{T}} c u_{\mathsf{rad}}\beta^2\gamma^2$$

And we have our final result. Note that it has just the same form as the relation for energy lost by synchrotron electrons:

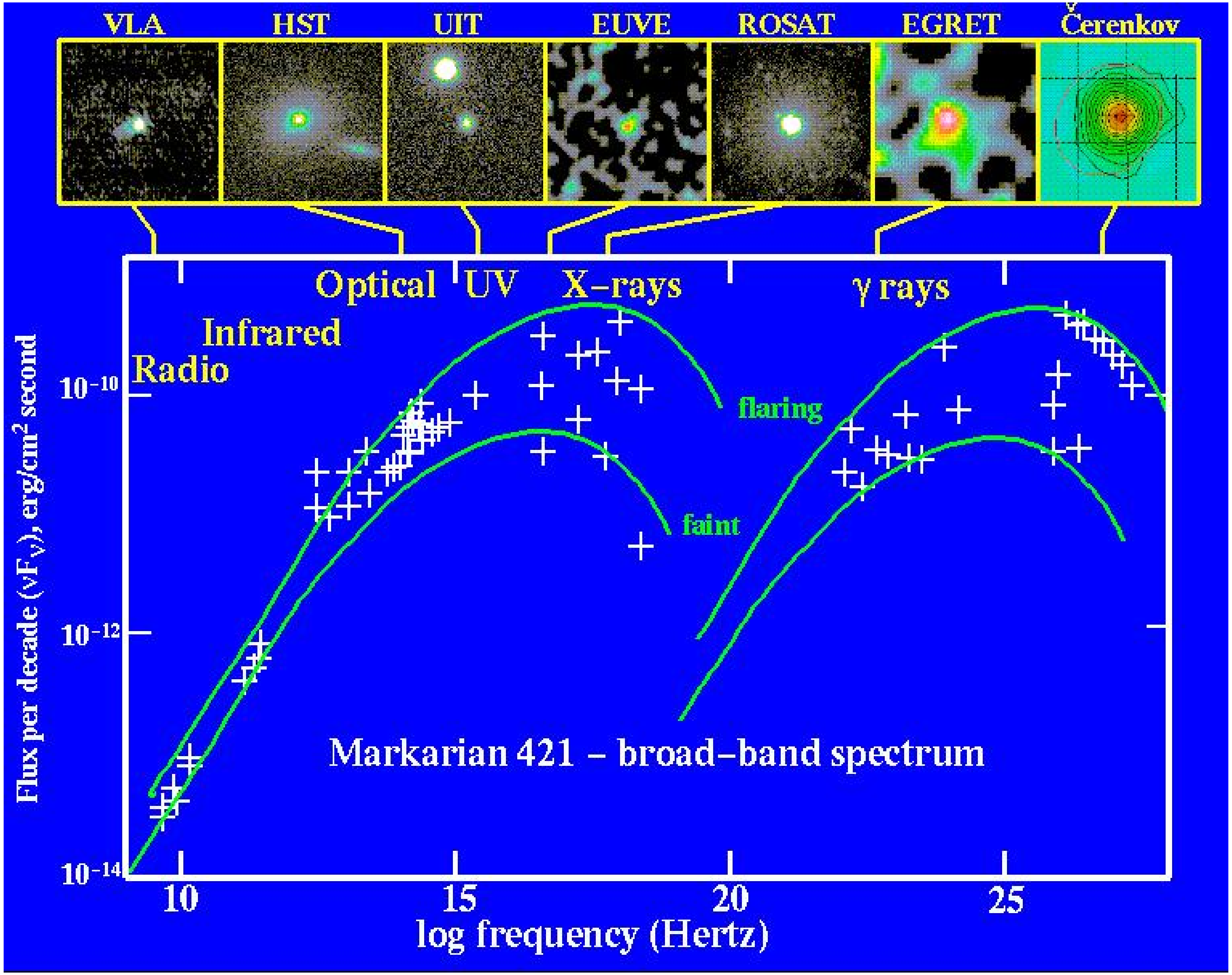
$$\frac{dE}{dt}_{\mathrm{synch}} = \frac{4}{3}\sigma_{\mathrm{T}} c u_{\mathrm{mag}} \beta^2 \gamma^2$$

Synchrotron self-Compton

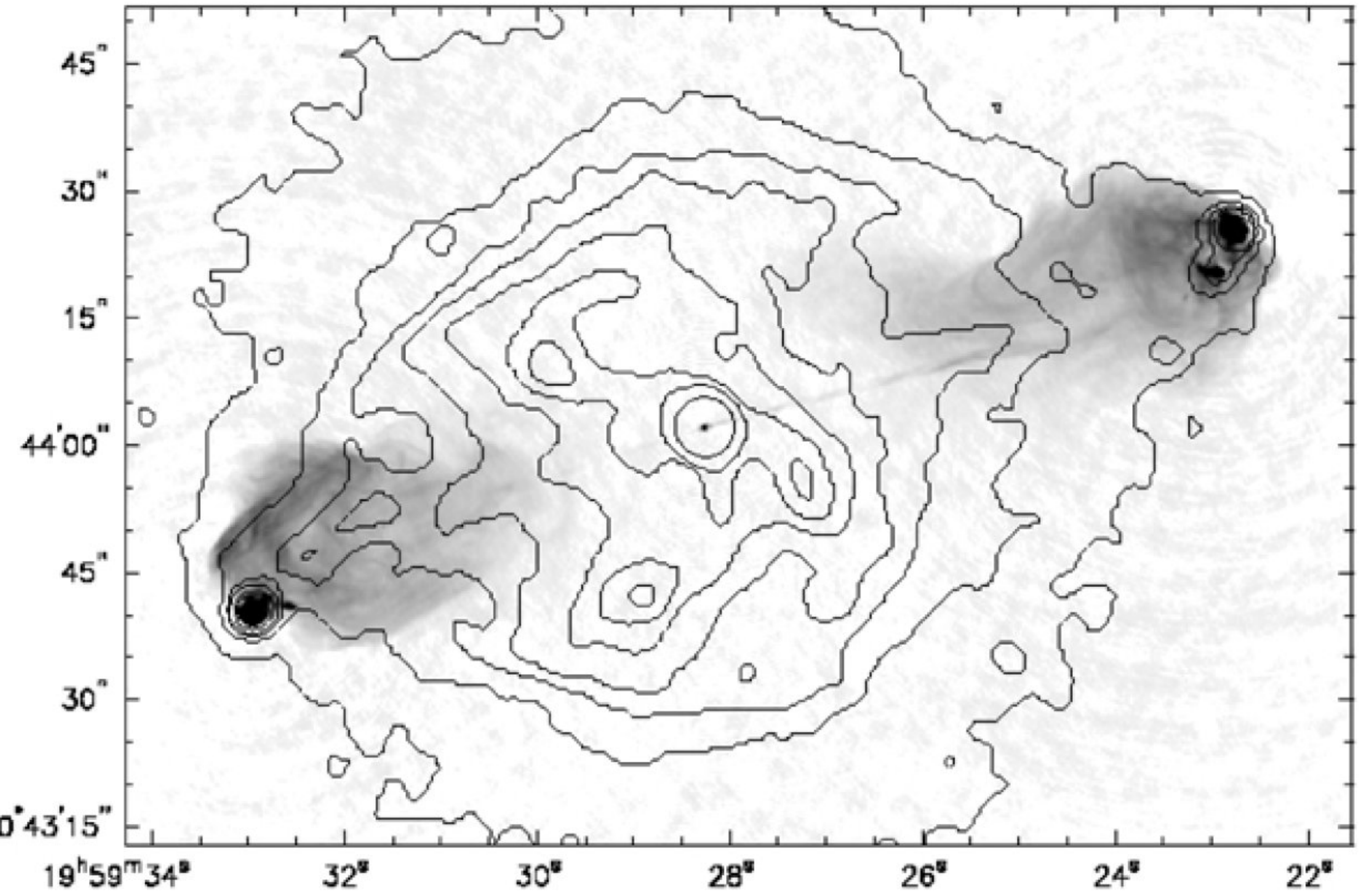
Synchrotron electrons can inverse-Compton scatter low-energy photons up to X-rays and γ-rays. The sources of the photons can be

- The synchrotron radio photons themselves.
- Optical/UV light from the AGN nucleus.
- Cosmic Microwave Background photons.

Self-Compton against the CMB is observationally important because we know u_{rad}, and so we can infer the Lorentz factors of the synchrotron electrons from the observed power in up-scattered X-rays.

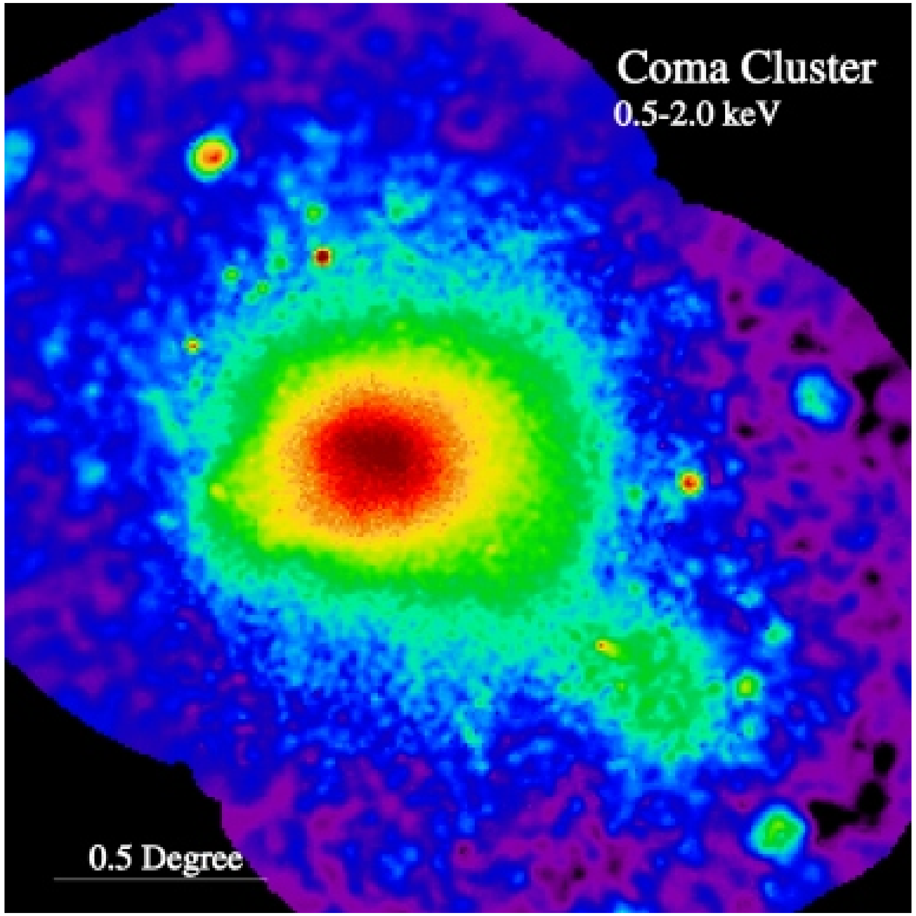

VLA
HST
UIT
EUVE
ROSAT
EGRET
Čerenkov
Radio
Infrared
Optical
UV
X-rays
γ rays
flaring
faint
Markarian 421 – broad-band spectrum
Flux per decade (νF$_\nu$), erg/cm^2 second
10^{-10}
10^{-12}
10^{-14}
10
15
20
25
log frequency (Hertz)

Inverse-Compton X-rays from hotspots in Cygnus A



45"
30"
15"
44'00"
45"
30"
40°43'15"
19h59m34s
32s
30s
28s
26s
24s
22s

X-ray emission from clusters of galaxies: thermal bremsstrahlung

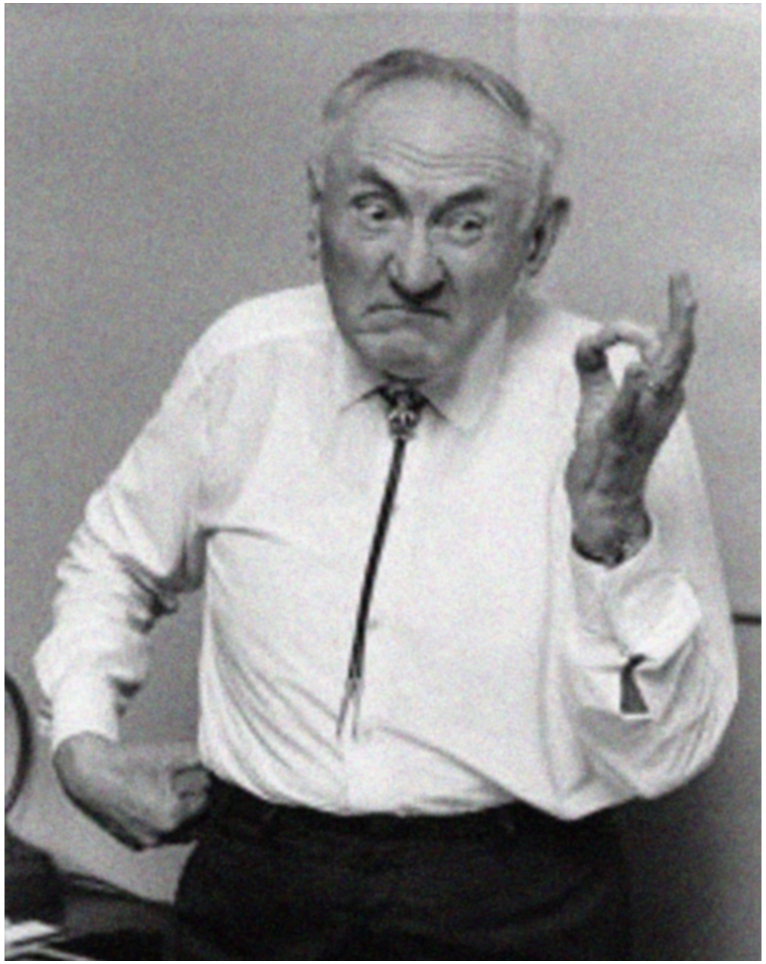

Coma Cluster
0.5-2.0 keV
0.5 Degree

X-ray emission from clusters of galaxies: thermal bremsstrahlung

The notion that the space between galaxies in a cluster must be filled with some material which gravitates but does not emit light (at least, in the optical) was first proposed by Zwicky in the 1930’s. His observations showed that the orbital speeds of galaxies in nearby clusters were too great for the systems to be bound by the stellar mass of the galaxies alone.

We now know that there is typically a factor of ten times as much mass in intra-cluster plasma as there is mass in stars, and that there is a factor of ten times as much mass again in some form of matter which gravitates but interacts via the electro-weak force negligibly, if at all.

Fritz Zwicky



Spherical bastards!
Whatever way you look at them, they're bastards!

X-ray emission from clusters of galaxies: thermal bremsstrahlung

The plasma particles in a galaxy cluster will not have neat circular orbits, but if their random thermal motions are sufficiently energetic, the plasma will not collapse in the cluster's gravitational potential well.

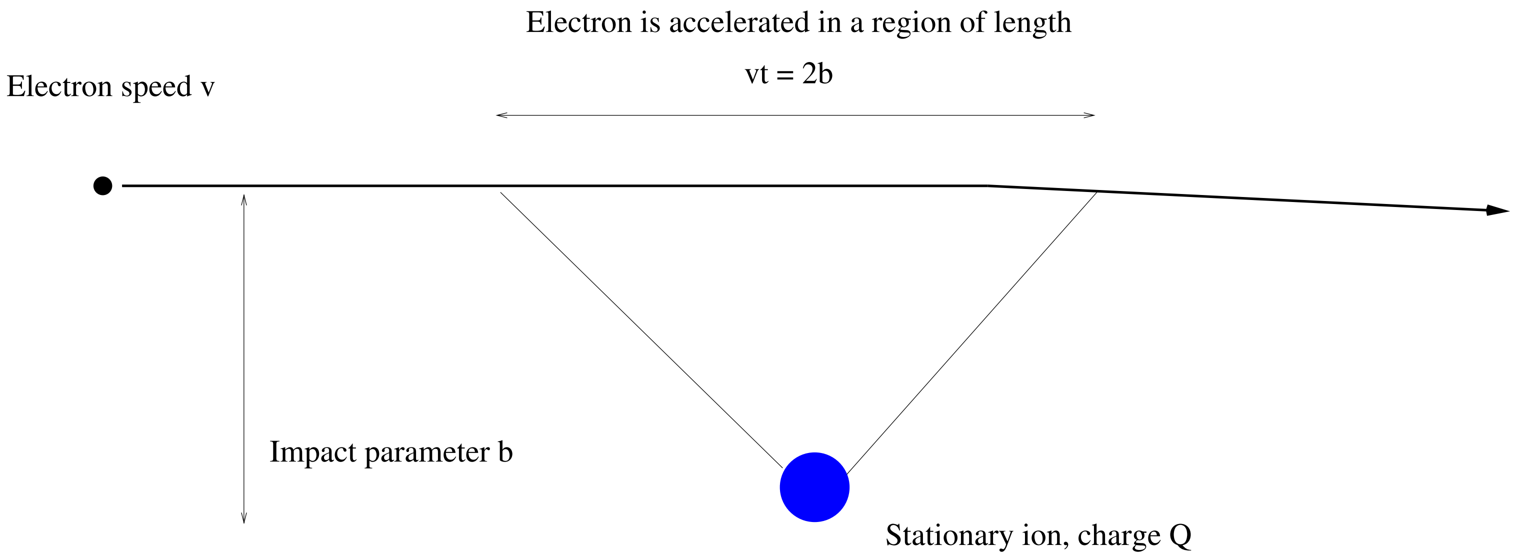
We can estimate this temperature via the virial theorem:

$$k_{\mathsf{B}}T \sim \frac{GMm_H}{R}$$

Here M and R are the total mass and characteristic size of the cluster. For the most massive galaxies clusters known, with total masses on the order of $10^{15} M_{\odot}$, the plasma reaches temperatures of several keV (1 keV $\equiv 1.16 \times 10^7$ K).

Here we are dealing with a thermal distribution of energies in the plasma: even the electrons, at rest mass 511 keV, are only barely relativistic at these temperatures. However because the plasma *density* is so low, we don't get a black-body spectrum. The emitted radiation is in the form of thermal bremsstrahlung.

Bremsstrahlung: calculation



An individual electron being accelerated will emit a very short pulse of radiation.

Bremsstrahlung: calculation cont'd.

Acceleration of the electron is given by:

$$a = \frac{Qe}{4\pi m_e \epsilon_0 b^2}$$

Power from an accelerated electron (Larmor):

$$P = \frac{e^2 a^2}{6\pi \epsilon_0 c^3} = \frac{Q^2 e^4}{96\pi^3 m_e^2 \epsilon_0^3 b^4 c^3}$$

Time taken is $t = 2b/v$ so the total energy radiated by a single acceleration event is

$$E \approx \frac{Q^2 c \sigma_{\mathrm{T}}}{8\pi^2 \epsilon_0 b^3 v}$$

Bremsstrahlung: calculation cont'd.

To get the spectral shape we use the same trick as for synchrotron emission: take the Fourier transform. The FT of a sharp pulse of duration t is broad in frequency space, with a cutoff at $\nu_{\mathrm{max}} \approx 1/t$. So we divide by $1/t$ to estimate the energy per unit frequency

$$E_\nu \mathrm{d}\nu \approx \frac{Q^2 c \sigma_{\mathrm{T}}}{4\pi^2 \epsilon_0 b^2 v^2} \mathrm{d}\nu$$

Recall this is for a single electron. Next we integrate over the electron impact parameters.

Bremsstrahlung: calculation cont'd.

Number of collisions per unit time at impact parameter b per ion:

$$N\mathrm{d}b = n_e v 2\pi b \mathrm{d}b$$

Number of collisions per unit time at b for all ions:

$$N'\mathrm{d}b = n_e n_i v 2\pi b \mathrm{d}b$$

So we get the total emissivity (power per unit volume per unit frequency) via:

$$j_\nu \mathrm{d}\nu \sim \int_{b_{\mathrm{min}}}^{b_{\mathrm{max}}} \frac{Q^2 c n_e n_i \sigma_{\mathrm{T}}}{4\pi\epsilon_0 b v} \mathrm{d}b \mathrm{d}\nu$$

$$j_\nu \mathrm{d}\nu \sim \frac{Q^2 c n_e n_i \sigma_{\mathrm{T}}}{4\pi\epsilon_0 v} \ln\left(\frac{b_{\mathrm{max}}}{b_{\mathrm{min}}}\right) \mathrm{d}\nu$$

Bremsstrahlung: calculation cont'd.

Next estimate the impact parameters. We have an upper limit in frequency of ν_{max} so

$$b_{\mathrm{max}} \sim \frac{v}{2\nu}$$

And the closest impact parameter is governed by the quantum limit

$$mvb_{\mathrm{min}} \sim \hbar$$

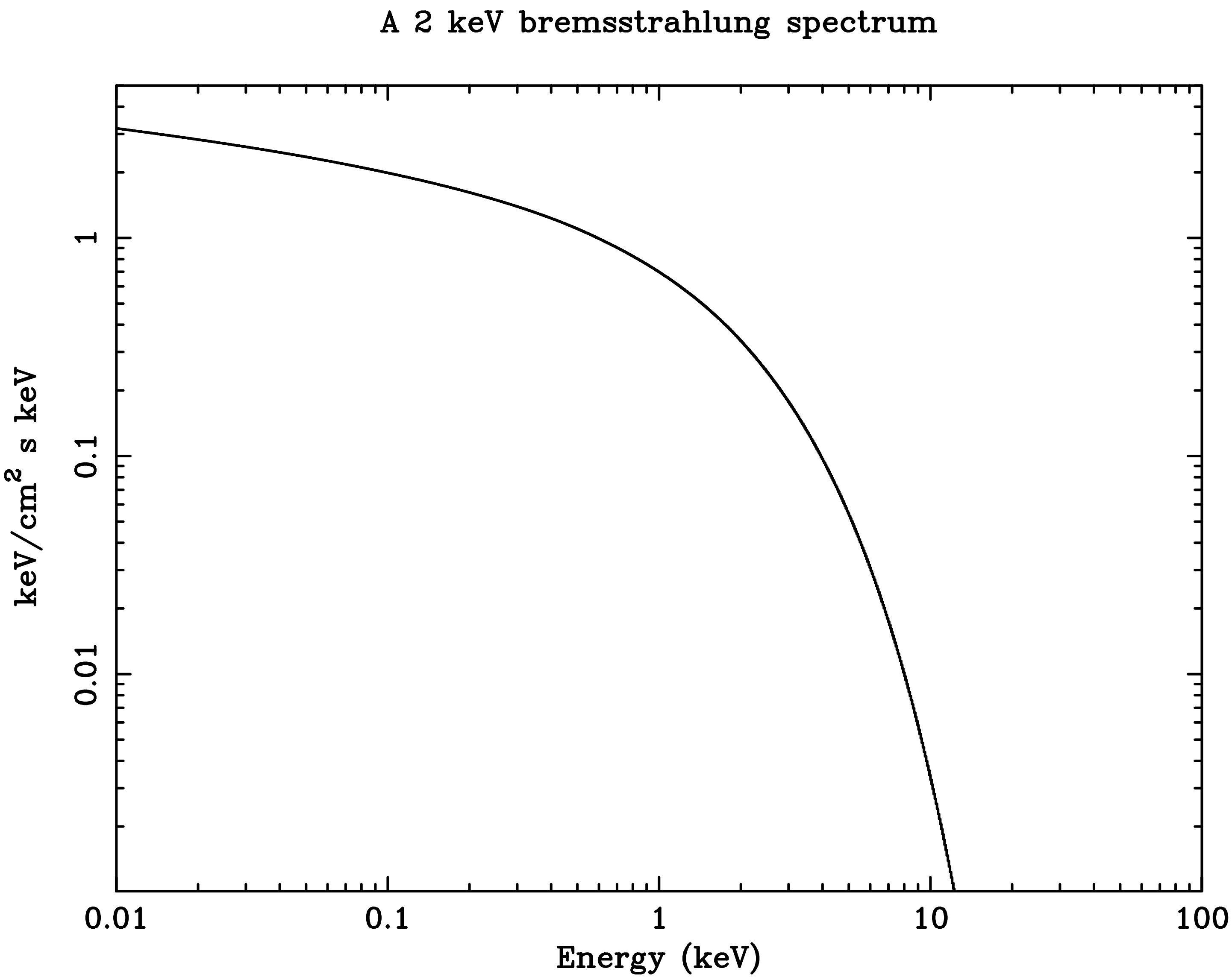
So we get to

$$j_\nu \mathrm{d}\nu \approx \frac{Q^2 c n_e n_i \sigma_\mathrm{T}}{4\pi\epsilon_0 v} \ln\left(\frac{mv^2}{2\hbar\nu}\right) \mathrm{d}\nu$$

Since we are dealing with a thermal distribution of particle energies, we can approximate $\frac{1}{2}mv^2 \sim k_\mathrm{B}T$. Notice immediately that this means photon energies above $k_\mathrm{B}T$ become exponentially unlikely.

So we have shown the significant properties of the thermal bremsstrahlung spectrum:

- Emissivity is proportional to the square of the plasma density.
- Emissivity varies as v^{-1} and hence as $T^{-1/2}$.
- The spectrum has a sharp cutoff above photon energies $\sim k_{\mathrm{B}}T$
- From observation of bremsstrahlung we can measure n_e and T_e.


A 2 keV bremsstrahlung spectrum
keV/cm^2 s keV
1
0.1
0.01
0.01
0.1
1
10
100
Energy (keV)

Inverse-Compton in cluster plasma: Sunyaev-Zel'dovich effect

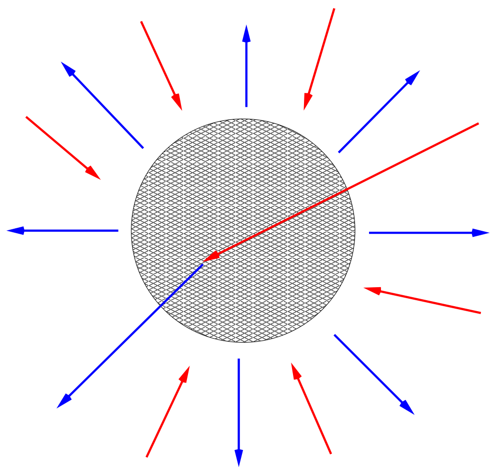
A cluster of galaxies sitting in deep space will be illuminated (almost) isotropically by the CMBR. However, as the CMBR photons pass through the cluster, some fraction of them will be inverse-Compton scattered by the cluster plasma. In effect, the CMBR will be *heated* by the cluster. The rate of change of photon occupancy is given by the Kompaneets equation:

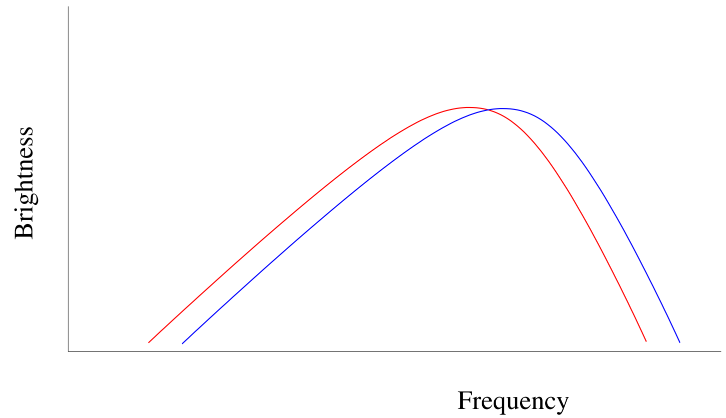
$$\dot{n} = \left(\frac{\sigma_\mathrm{T} n_e \hbar}{m_e c}\right) \frac{1}{\omega^2} \frac{\partial}{\partial \omega} \left\{ \omega^4 \left[n(n+1) + \frac{kT}{\hbar} \frac{\partial n}{\partial \omega} \right] \right\}$$

Happily this simplifies greatly if $T_{\rm gas} \gg T_{\rm CMB}$ and we observe in the Rayleigh-Jeans region. Here, the fractional change in brightness of the CMBR travelling through a column of plasma will be:

$$\frac{\delta I_\nu}{I_\nu} = -2 \int n_e \sigma_{\rm T} \frac{k_{\rm B} T}{m_e c^2} {\rm d}l$$

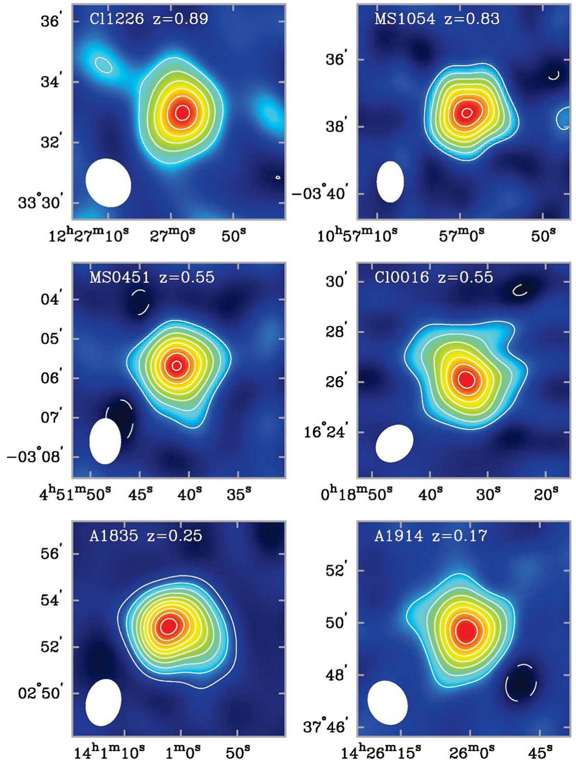
Note that we don't get a higher temperature CMBR spectrum as a result of this process; rather, the black-body spectrum is shifted to the right, as the photon numbers are conserved, but the mean photon energy is slightly increased (and the energy distribution is broadened by the scattering). For typical massive clusters, the energy change is of order 10^{-4}.





So in the Rayleigh-Jeans region we see a small *dip* in brightness towards the cluster. Knowing n_e and T_e from an X-ray observation, we can calculate the line-of-sight depth of the cluster.

There is another very profound consequence: whatever the distance to the cluster, the *fractional* change in the CMBR spectrum is the same. A cluster at $z = 0.01$ gives the same Sunyaev-Zel'dovich effect as one at $z = 1$ or more!



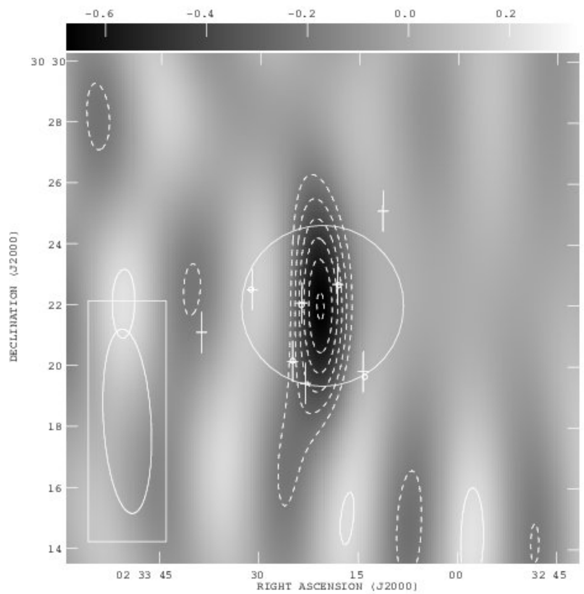


Figure 4. CLEANed 0.65–1.0 kλ Ryle Telescope map of the TOC J0233.3+3021 field after subtraction of the point sources listed in Table 2.1. The greyscale runs from -0.675

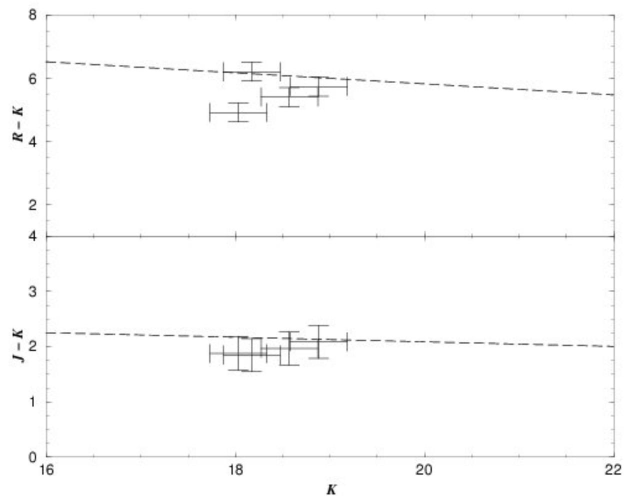


Figure 6. $R - K$ and $J - K$ colour-magnitude plots for the host galaxies of the radiosources; cf Fig. 4 of Stanford et al. (1997). The dashed lines show the red sequence of the galaxies in the $z = 1.27$ cluster identified by Stanford et al. (1997). The similar red colours of the TOC J0233.3+3021 galaxies suggests that they are old elliptical galaxies, with the R and J filters straddling the observed-frame 4000-Å break.