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LECTURE (1)

THE ORIGIN OF ENTROPY

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1. INTRODUCTION

One of the chief challenges in cosmology is to explain the origin of the entropy in the Universe. The entropy is contained primarily in the 3 K background radiation and the entropy per baryon S is therefore just the ratio of the 3 K photon number density ($n_\varphi \sim 300\ \mathrm{cm}^{-3}$) to the baryon number density ($n_b \sim 10^{-5}\ \Omega\ \mathrm{cm}^{-3}$ where Ω is the matter density in units of the critical density; $\varrho_{\mathrm{crit}} = 3H_0^2/8\pi G \sim 10^{-29}\ \mathrm{g\ cm}^{-3}$ for $H_0 = 50\ \mathrm{km\ s}^{-1}\ \mathrm{Mpc}^{-1}$). Thus

$$S \sim \frac{n_\varphi}{n_b} \sim 10^8 \Omega^{-1} . \tag{1}$$

In contemplating why S has the value it does one might just assume that it was fed into the Universe's initial conditions. However, this is not a very enterprising attitude, especially as it now appears possible that its value could be explained "naturally" by processes which occurred in the early Universe.

Attempts to explain the value of S in this way come from several directions. Some people assume that the excess of baryons observed locally is global and try to explain how the photons could be generated in a Universe which starts off cold (i.e. only with baryons and no antibaryons). Other people, also making the global excess assumption, try to explain how an excess of baryons could arise in a Universe which was initially baryon-symmetric. In this case, the fractional baryon excess required is just

$$\frac{\Delta B}{B} \sim \left(\frac{n_b - n_{\bar{b}}}{n_b + n_{\bar{b}}}\right) \sim S^{-1} \tag{2}$$

and the excess baryons would then survive after the other pairs annihilated when the temperature fell below 10^{12} K. A third group of people assume that the apparent excess of baryons is only local and that the Universe is globally baryon-symmetric. In this case the problem is to explain how some of the matter and anti-matter managed to separate before annihilating and the value of S just reflects the efficiency of annihilation [1]. Most cosmologists now regard the symmetric cosmology as rather implausible. One needs separation on at least the scale of galaxies but it is difficult to understand how this could come about [2]. Also it would seem difficult to produce both $S \sim 10^8$ and the observed helium abundance through cosmological nucleosynthesis in a symmetric Universe [3]. Henceforth I will therefore concentrate on the first two approaches.

2. PRODUCTION OF PHOTONS IN AN INITIALLY COLD UNIVERSE

Many schemes have been proposed for producing radiation in a cold universe and some of these make direct recourse to details of particle physics. For example, the entropy could be generated through the decay of various species of exotic particles which might exist at early times (such as Hagedorn's 10^{15}g superbaryons [4]) or as a result of a phase transition (such as the quark soup to nucleon transition which occurs in Lasher's model [5]). Unfortunately, our understanding of such exotic particles and phase transitions is so scanty that it is difficult to make specific quantitative predictions. I will therefore concentrate on four somewhat different sorts of scenario. The first invokes the dissipation of initial anisotropy, the second the dissipation of initial inhomogeneities, the third the production of radiation through pregalactic stars, and the fourth pregalactic black hole accretion. I have selected these for special attention — not because they are necessarily more plausible than the others — but because they illustrate how the values of S may be related to other cosmological parameters.

(a) *The dissipation of anisotropy.* Although the Universe is very isotropic today, it may not always have been and many people have suggested that it may have started off "chaotic" with large anisotropies [6]. In this case the anisotropy energy density would initially dominate the matter density and the Universe would be expanding at different rates in different directions. The rapidly varying curvature means that quantum mechanical particle production would be important at the Planck time [7]

$$t_0 \sim \left(\frac{\hbar G}{c^5}\right)^{\frac{1}{2}} \sim 10^{-43}\ \mathrm{s} \tag{3}$$

and the particles created could then react back on the background and isotropize it [8]. In this way initial anisotropy could be dissipated into photons and particle pairs almost immediately. Even if the initial anisotropy survived the Planck era, it could still be dissipated at a later time due to neutrino viscosity and a host of other highly efficient dissipative processes [9]. This means that most of the original shear energy must have gone into background radiation. However, there seems to be no particular reason why this should produce $S \sim 10^8$. Indeed these sorts of argument place an important limit on how much anisotropy the early Universe could have contained, because if it was too anisotropic dissipation would have produced more background radiation than is observed [10]. This is a consequence of the fact that the anisotropy energy ϱ_σ density falls off much faster with time than the radiation density ϱ_R:

$$\varrho_\sigma \propto z^b, \quad \varrho_R \propto z^4 \tag{4}$$

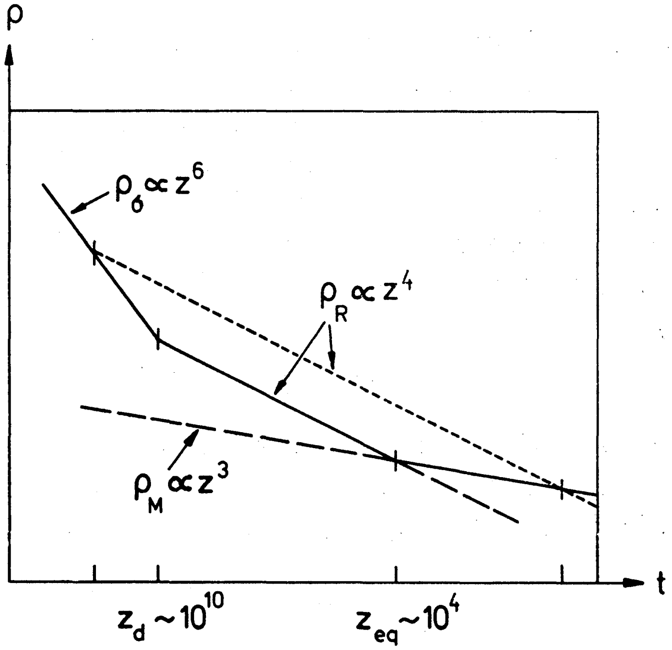

Fig. 1. This illustrates why the difference in redshift dependences of ϱ_σ, ϱ_R and ϱ_M puts an important upper limit on the initial anisotropy of the Universe. At early times the anisotropy will dominate the density but many processes could dissipate it into radiation at some redshift z_d. The generated radiation density will eventually fall below the matter density and the redshift z_{eq} at which it does so determines the resultant photon to baryon ratio ($S \propto z_{eq}^{-1}$). For example, if the anisotropy were dissipated by neutrino viscosity at $z_d \sim 10^{10}$ (solid line), one could choose its initial value so that $z_{eq} \sim 10^4$ and $S \sim 10^8$. However, if the same amount of anisotropy were dissipated earlier (dotted line), the resultant value of S would be much larger than observed. Since one would *expect* dissipative effects to operate at earlier times (for example, at the Planck epoch), this places an important limit on the anisotropy of the early Universe. This argument could be circumvented only if the Universe was initially baryon-symmetric with the baryon excess associated with the present matter density being generated *after* z_d

where z is the redshift. Therefore, as illustrated in figure (1), even a small amount of initial anisotropy may result in a huge photon-to-baryon ratio today. This line of reasoning suggests that the early Universe may have been "quiescent" rather than chaotic [11].

(b) *The dissipation of initial inhomogeneities.* If the early Universe contains inhomogeneities (density fluctuations), as of course are required — at least on large scales — to produce galaxies, then there will be a continuous generation of entropy as these fluctuations fall within the Jeans length. This is because fluctuations turn into acoustical waves when they fall within the Jeans length on account of pressure effects and the energy of these waves will be dissipated by the many sources of viscosity which operate at early times. Whenever the Universe has a hard equation of state ($p = \gamma\varrho$: $0<\gamma\leqslant 1$), the Jeans length is just of order the horizon size $\sim ct$. Therefore if the fluctuations which fall within the horizon at time t then have amplitude $\varepsilon_H(t)$, the acoustical energy generated is

$$\varrho_{ac}(t)\sim\varrho^{-1}(\delta\varrho)^2\sim\varepsilon_H(t)^2\varrho(t)\,. \tag{5}$$

This energy will be dissipated into photons and particle-pairs (which will eventually annihilate to produce more photons) with a $p = \frac{1}{3}\varrho$ equation of state. If the matter itself has $p = \frac{1}{3}\varrho$, one can see that the radiation density will always be of order ε_H^2 times the total density. The photon-to-baryon ratio will therefore always be small since ε_H is necessarily less than 1. However, if the matter has an equation of state stiffer than $p = \frac{1}{3}\varrho$ (due, for example, to a strong repulsion between nucleons at high densities) then the generated radiation density will fall off more slowly than the matter density:

$$\varrho_R \propto z^4, \qquad \varrho_M \propto z^{3(1+\gamma)}\,. \tag{6}$$

It may therefore eventually exceed the matter density (as is required). In this situation one can show that the largest contribution to S comes from the fluctuations which first enter the horizon (at time t_1, say) and that it is of order [12]

$$S\sim\varepsilon_1^{3/2}\varrho_0^{3/4}n_0^{-1}\left(\frac{t_1}{t_0}\right)^{(1-3\gamma)/2(1+\gamma)} \tag{7}$$

where $\varepsilon_1 \equiv \varepsilon_H(t_1)$ and n_0 and ϱ_0 are the baryon number density and energy density at the Planck time t_0. If we assume that the shortest initial wavelength for a fluctuation is $n_0^{-1/3}$ and that the typical baryon mass is the neutron mass $\sim 10^{-24}$ g, then equation (7) becomes

$$S\sim 10^{29(3\gamma-1/3\gamma+1)}\varepsilon_1^{3/2}\,. \tag{8}$$

Zel'dovich [13] first obtained this formula for the $\gamma = 1$ case ($p = \varrho$). In this situation $t_1 \sim 10^{-32}$ s and one generates all the entropy of the Universe ($S \sim 10^8$) if $\varepsilon_1 \sim 10^{-4}$. He claims that the same fluctuations on a larger scale will produce galaxies, so this scenario has the attractive feature of relating the two "fundamental" problems of cosmology. On the other hand, the derivation of equation (8) is questionable since it may obviously be unjustified to assume that the wavelength of the shortest fluctuation is defined by the internucleon distance at a time when nucleons presumably do not exist. Nevertheless, in principle, this sort of process could generate a large value of S.

(c) *The production of radiation through stars.* If the early Universe is cold, the equation of state will be soft after 10^{-4} s (before that it will almost certainly be hard due to strong interactions and degeneracy pressure) and so the Jeans length may be much smaller than the horizon size. This means that fluctuations can fall within the horizon and bind (i.e. $\delta\varrho/\varrho$ can grow to 1) before they fall inside the Jeans length and dissipate. In this situation one expects bound regions of mass M to form prolifically at a time which depends on the value of $\varepsilon_H(M)$.

$$t_B(M) \sim 10^{-5}\left(\frac{M}{M_\odot}\right)\varepsilon_H(M)^{-2/3}\ \mathrm{s}, \tag{9}$$

and these regions should produce stars (either directly or via fragmentation) well before galaxies form. Several people [14, 15] have suggested that the 3K background could be the radiation generated by these stars, the starlight being thermalized by grains which are also produced by the stars. Stars bigger than about $10^2\ M_\odot$ are radiation-dominated and have a mass-independent nuclear-burning timescale [16]

$$t_{MS} \sim \beta\left(\frac{c\sigma_\tau}{4\pi G m_P}\right) \sim \beta \times 10^8\ \mathrm{y} \tag{10}$$

where β is the efficiency with which they produce radiation (for hydrogen to helium burning $\beta \sim 0.007$) and the term in brackets is the so-called "Eddington timescale". The resultant photon-to-baryon ratio is [15]

$$S \sim \left(\frac{\hbar c}{G m_P^2}\right)^{1/4}\left\{\left(\frac{m_P}{m_e}\right)\left(\frac{e^2}{\hbar c}\right)\beta^{5/4}\right\}F^{3/4} \tag{11}$$

where F is the fraction of the Universe which goes into the stars. (If the stars were smaller than $10^2\ M_\odot$, t_{MS} and S would be somewhat larger.) Since the term in curly brackets is of order 1, one naturally ends up with a value for S or order $\alpha_G^{-1/4}$ where

$$\alpha_G \equiv \left(\frac{G m_P^2}{\hbar c}\right) \sim 10^{-38} \tag{12}$$

is the gravitational fine structure constant. This picture therefore has the attractive feature of relating the value of S to the microphysical constants.

(d) *Pregalactic black hole accretion.* One drawback of the last scenario is that it is not clear that the starlight really can be thermalized by grains. An alternative, though closely related, scenario invokes accretion by black holes to generate the 3 K background [12]. The formation of pregalactic black holes in a cold Universe would be almost inevitable: sufficiently large regions could collapse to holes directly and the sort of stars invoked in the last scenario could also leave black hole remnants after burning their nuclear fuel. Because holes produce radiation with a greater efficiency than stars ($\beta \sim 0.1$), they can produce the radiation at an earlier time t_R. In general one needs

$$t_R \sim 10^{11}\,\Omega_B^{-2}\,\beta^{-3/2}\ \mathrm{s} \tag{13}$$

where Ω_B is the present black hole density in units of the critical density. Since t_R can precede the time which conventionally corresponds to decoupling $\sim 10^{13}$ s (before which the Universe is ionized), one can now appeal to free-free processes to thermalize the radiation. However, the value of t_R is constrained very tightly: since $\Omega_B < 1$ and $\beta < 0.1$, one needs $10^{12}\ \mathrm{s} < t_R < 10^{13}$ s. It is striking that the first holes would indeed form in the period required if they were remnants of massive stars. One problem with both the star and the black hole scenarios is that they do not produce the 3 K radiation until after the time at which cosmological production of helium and deuterium occurs ($t \sim 100$ s). Except in rather contrived circumstances, this means that one does not end up with the "standard" helium and deuterium abundances [17]. One has to appeal to nucleosynthesis in stars to produce these elements.

3. GENERATION OF A BARYON EXCESS IN AN INITIALLY SYMMETRIC UNIVERSE

If the Universe starts off "hot", with all the present 3 K photons, the observed baryon density (if global) would correspond to a slight excess of baryons over antibaryons at times sufficiently early that baryon-antibaryon pairs can be produced. One might wonder whether such an excess could arise naturally through non-baryon-conserving processes in an initially baryon-symmetric Universe. This approach has been stimulated recently by the realization that baryon-non-conserving processes are indeed permitted in the grand unified theories of strong, weak and electromagnetic interactions (GUTS) [18—23]. In particular, a baryon excess could be generated through the free decay of the heavy X-bosons which characterize these theories and which would be abundant in the Universe at times sufficiently early that the temperature exceeded their rest mass, $M_X \sim 10^{14}$—10^{16} GeV. However, this effect is important only if the decay rate of the X-bosons,

$$\Gamma_X \sim \alpha_X M_X^2 (k^2 T^2 + M_X^2)^{-\frac{1}{2}} \tag{14}$$

(where α_X is the unification coupling constant), is less than the cosmological expansion rate $\sim G^{\frac{1}{2}}(kT)^2$ when kT first falls below M_X. Otherwise the decay and inverse decay rate is always fast enough to maintain thermal equilibrium and, if equilibrium pertains, one can show from the CPT theorem that no asymmetry develops. The GUT scenario works therefore only if M_X exceeds a critical mass

$$M_C \sim a_X M_0 \tag{15}$$

where $M_0 = (\hbar c/G)^{\frac{1}{2}} \sim 10^{19}$ GeV is the Planck mass. Whether this condition is satisfied is not clear. These are actually two types of X-boson, the gauge boson and the Higgs boson, and since the coupling constant associated with the Higgs particle is around 10^{-5} whereas that associated with the gauge particle is around 10^{-2}, it is probable that one must appeal to the Higgs particle to produce an asymmetry. Even if the Higgs mass condition *is* satisfied, the asymmetry produced (and the consequent value of S) is very uncertain, depending on the size of the CP violation involved and the details of the unification model. Estimates for the final value of S span a range [24] from 10^4—10^{12}, so it is obviously premature to claim that GUTS explains the actual value observed.

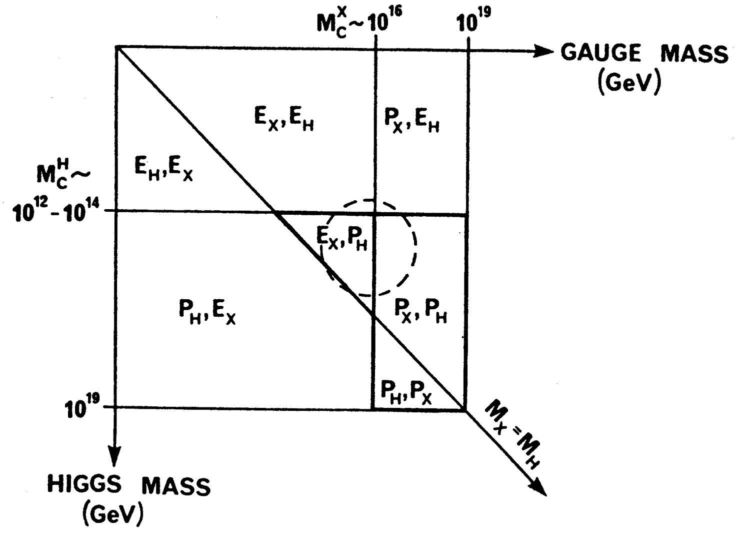

Fig. 2. This shows how the sequence of production (P) and erasure (E) of baryon asymmetries by gauge and Higgs bosons in the GUT scenario depends upon the mass of these particles (M_X and M_H). Only in the triangle are pre-GUT asymmetries erased and fresh ones produced, as the "standard" scenario supposes. In the lower right rectangle, pre-GUT asymmetries survive and may dominate the GUT-produced ones. In the rest of the diagram, one ends up with no final asymmetry. The circle indicates the region where M_X and M_H most probably reside

It should also be born in mind that baryon asymmetries either existing *ab initio* or generated before the GUT epoch are not necessarily erased (although this is sometimes claimed). The condition that GUT processes should destroy pre-existent asymmetries [e.g. by the two-step process $ql \to X \to \bar{q}\bar{q}$ (where q = quark, $\bar{q}$ = antiquark, l = lepton) which reduces the baryon number by 1] is essentially the same condition that it should not produce an asymmetry ($M_X < M_C$). The "standard" scenario [23] suggests that the initial asymmetries are first erased by the gauge particle (because $M_X < M_C^X$) and that fresh asymmetries are then produced by the Higgs particle (because $M_H > M_C^H$). However, as illustrated in figure (2), this only happens in a triangular region of (M_H, M_X) space. If *both* M_X and M_H exceed their respective critical masses (and it is within the bounds of possibility that this is the case), asymmetries existing before the GUT epoch may survive. Since a number of mechanisms could produce such an earlier asymmetry [25] (for example, quantum gravity effects at the Planck time, semiclassical quantum gravity effects — in which gravity is treated classically and particles are described by quantum fields — somewhat later [26] primordial black hole evaporations [27—29]), this is an important possibility. One might indeed *need* to invoke such effects if the GUT-generated S turned out to be much greater than 10^8.

If, on the other hand, the S generated by both GUT and pre-GUT processes turned out to be much less than 10^8, one would have to appeal to subsequent dissipation via one of the mechanisms discussed in section (2) to explain the observed value of S. In this case the intermediate (unboosted) value of S could still play an important role because of its effect on cosmological nucleosynthesis and the evolution of density fluctuations. I will argue in my next lecture that a model in which GUT and pre-GUT processes generate a value for S less than 10^8 actually has some advantages over a model which produces $S \sim 10^8$ from the point of view of galaxy formation. This raises the possibility that the explanation for S may involve *both* baroyn-non-conserving and dissipative processes.

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