One dimensional simulation of an interstellar Flight

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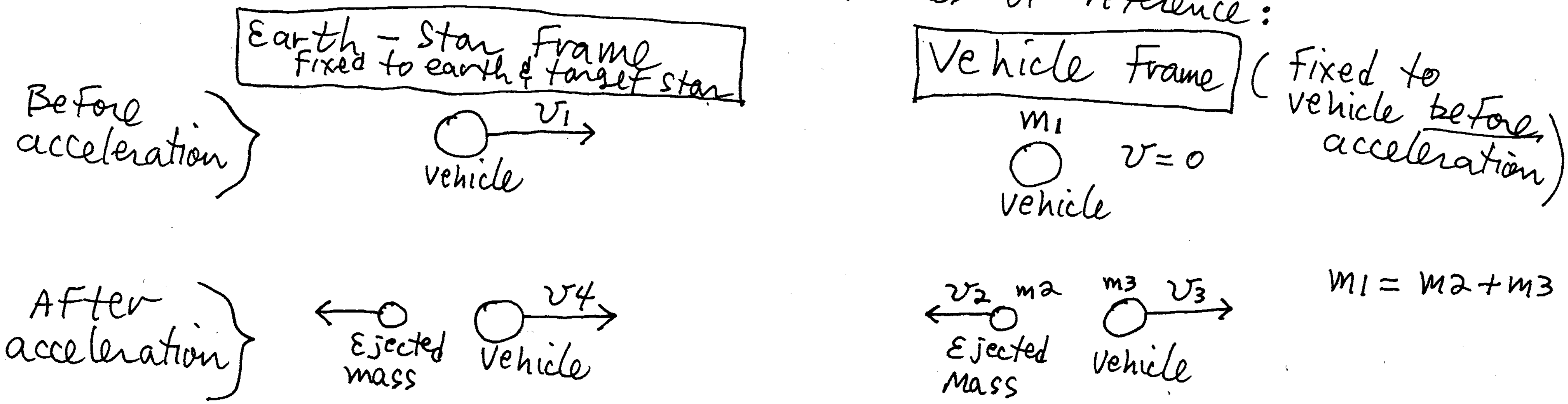
Let's look at the physics in two frames of reference:



We know: v_1, v_2, m_1, m_2, m_3

we want to find v_4, the resultant vehicle velocity in the Earth-Star frame, which will tell us how long it will take to travel from the Earth to the star.

Two steps

1. use conservation of momentum to find v_3 in the vehicle frame:

$$v_3 = \frac{m_2}{m_3} v_2$$

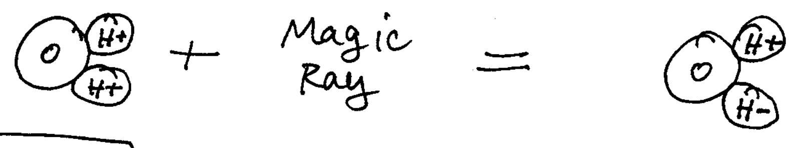
2. use the Einstein velocity addition theorem to add v_1 and v_3 in the Earth-Star frame:

$$v_4 = \frac{v_3 + v_1}{1 + \frac{v_3 v_1}{c^2}}$$

Let's hypothesize a propulsion source

Future tech 1 Low energy matter-to-antimatter pathway

We find a Bertold Ray or Inverse Tachyon Pulse that can flip a proton to an anti-proton and use it on water ~~molecules~~ molecules



Future tech 2 Matter/antimatter based propulsion

The proton and anti-proton combine to release energy ($E=mc^2$), and that energy accelerates the oxygen atom out the tailpipe.

Released energy = Kinetic energy of oxygen atom

$$2 m_p c^2 = \frac{1}{2} 16 m_p v_2^2$$

where m_p = mass of a proton
v_2 = mass ejection velocity

solving, $v_2 = 0.5\,c$

In the real galaxy, we won't be able to convert all of the energy of the matter-antimatter reaction into speed of the ejected oxygen ~~Fa~~ atom, so we introduce a fudge factor called propulsive efficiency (P.E.), which is the fraction of energy that does get transferred into propulsion.

Then $v_2 = 0.5c * P.E.$

Let's set up an Excel File to plot a trip in multiple time steps, where the $v4$ found in one step becomes the v_1 in the next step.

Units:
position = lightyears
v = fraction of c
time = years

Inputs: Initial position, velocity, time in Earth-Star Frame
Initial mass, time in vehicle Frame
Propulsive Efficiency of our propulsion source

Per step Parameters: Mass ejected from vehicle, in tonnes (1 tonne = 1000 kg)
Time step in vehicle frame, in years
Direction of mass ejection, for acceleration or deceleration
(acceleration = +1, deceleration = −1)

Output Parameters: $v4 = ((m2 * 0.5 * P.E. * Direction)/m3 + v1)/$
$(1 + m2 * 0.5 * P.E. * Direction * v1/m3)$

w.r.t. = "with respect to" or "in the frame of"

Position w.r.t. Earth = average speed during step * duration of step + position at beginning of step =

$$\left(\frac{v1+v4}{2}\right) * \text{Duration of step w.r.t. earth} + \text{Position at end of last step}$$

Duration of step w.r.t. Earth =

$$\text{Time step in vehicle frame} * \frac{1}{\sqrt{1 - \frac{(\text{Ave speed during step})^2}{c^2}}} =$$

$$(\text{Time step in vehicle frame}) / (\text{SQRT}(1 - (\frac{v1+v4}{2})\wedge 2))$$

---more output parameters

Vehicle time = summation of vehicle time steps to date

Earth time = summation of Earth time steps to date

Vehicle mass = Initial mass − summation of mass ejections to date

Acceleration felt on vehicle during the time step $= \dfrac{\Delta \text{speed}}{\Delta \text{time}} = \dfrac{v3}{\text{time step}} = \dfrac{m2 * 0.5c * P.E.}{\frac{m1+m3}{2}}$ ← average vehicle mass during time step

Units: Note that

$$\frac{\text{Fraction of } c}{\text{yr}} * \frac{300e6 \ \frac{m/sec}{c}}{31.4e6 \ \frac{sec}{yr}} * \frac{1 \text{ gee}}{9.8 \ \frac{m}{sec^2}} \approx 1 \text{ gee},$$

so a parameter dimensioned in $\frac{\text{Fractions of } c}{\text{yr}}$ is also dimensioned in gees.

Reference: Robert Resnick, Relativity and Early Quantum Theory, John Wiley and Sons, 1972. Equations ~~2-7~~ 2-17 and 2-11

Excel File at www.dadshomeschool.net