Dimensional analysis 1

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Purpose: This paper will introduce the idea that dimensions like length, time, and mass can be multiplied and divided to form new dimensions. In fact, anything that can be measured can be written as products and ratios of simple quantities like length, time, and mass. This is one of the secrets that scientists and engineers use every day but the general public does not know or understand. Be warned – if you read further, there might be no turning back.

In the tables below, we will introduce and describe some of the units of measurement of the physical world. Learn them well, so they are like old friends. They are called "derived" units because they can be expressed as products and ratios of the "fundamental" units of length (L), time (T), and mass (M). For each derived unit, the table shows how it is made of fundamental units, its SI (metric) units, and an example of its English system units.

Derived unit	Description	Fundamental units	SI (metric) units	English units example
Area	You know what area is.	L^2	m^2	ft^2
Volume	You know what volume is.	L^3	m^3	ft^3
Velocity	This is a measure of how fast something moves. How many miles or kilometers a car moves in an hour. Speed. If you divide distance by time you get velocity. $$Velocity = \frac{Distance}{Time} = \frac{Miles}{Hour} = \frac{Kilometers}{Hour}$$ Note that we often use the word "per" to talk about dividing one dimension by another, so "miles per hour" means "miles divided by hours"	$\frac{L}{T}$	$\frac{m}{s}$	$\frac{ft}{s}$
Acceleration	This is a measure of how fast something changes its velocity. For example, a car that goes from zero to 60 miles per hour in 5 seconds increases its speed by 12 miles per hour every second on average, so its acceleration is 12mi/hr/s. If you stand on the Earth and drop an object, it will accelerate at 9.8m/s/s = $9.8 m/s^2$, meaning its downward velocity will increase by 9.8m/s every second.	$\frac{L}{T^2}$	$\frac{m}{s^2}$	$\frac{ft}{s^2}$

<table>
<tr><th>Derived unit</th><th>Description</th><th>Fundamental units</th><th>SI (metric) units</th><th>English units example</th></tr>
<tr><td>Momentum</td><td>This is a measure of how much “goomph” a moving body has. It’s equal to mass times velocity, so you can increase the momentum of an object by increasing its mass or its velocity. Newton’s First Law states that a body in motion will tend to remain in motion in a straight line unless acted upon by an outside force. The larger the momentum of a body, the more force it will take to change its motion.

Newton’s Third Law states that, for every action, there is an equal and opposite reaction, and that’s a good description of how a rocket works in space. A rocket throws mass out the tailpipe at high velocity and moves forward as a result. It turns out that the forward momentum that the rocket acquires is exactly equal to the reverse momentum of the mass thrown out the tailpipe. Another term for the third law is “conservation of momentum.” A rocket in space can be called a “momentum machine”

In the movie Harry Potter and the Prisoner of Azkaban, when Harry climbs high during the Quidditch match, gets tangled up with the dementors, falls off his broom and plummets toward Earth, Dumbledore rises, points to him and bellows “Arresto Momentum” to save his life.</td><td>$\frac{ML}{T}$</td><td>$\frac{kg-m}{s}$
a dash (–) is added sometimes for clarity between units that are multiplied together, like kg-m instead of kgm</td><td>$\frac{ft-lbm}{s}$</td></tr>
<tr><td>Force</td><td>You know what a force is – it’s a push or a pull. If you apply a force to a free object, it will accelerate. Newton’s Second Law of Motion defines the acceleration that is produced, and is often expressed by the equation:

Force = mass * acceleration, or

F=ma

In proper physics, weight is the measure of the force of gravity on an object. A bathroom scale is a force measuring device, using a calibrated spring. If you took your scale to the Moon, it would show your weight as about one sixth your weight</td><td>$\frac{ML}{T^2}$
1 Newton = 1nt = 1kg-m/s^2</td><td>$\frac{kg-m}{s^2}$</td><td>$\frac{ft-lbm}{s^2}$

1 pound-force = 1lbf = 32.2 ft-lbm/s^2</td></tr>
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on Earth. What changed? Your mass stayed the same; it was the acceleration of gravity on the Moon that changed:

F(Moon) = m*a(Moon), where a(Moon) is one sixth of a(Earth)

In everyday use, weight is often expressed in units of mass. You say a twelve-year-old student weighs 35kg. That means a force gauge (bathroom scale) that is calibrated to show the correct mass numbers on the surface of the Earth points to 35 when loaded by the student. Just don't expect that calibration to be accurate on the Moon or Mars.

On the other hand, the kind of nurse's office scale where you move a metal mass until the beam points to the middle of its range *does* measure mass directly. This kind of scale is called a balance, because it balances your mass through a lever arm against the metal mass that you slide. A balance will be accurate on the Earth or on the Moon.

OK, there is one more complication to talk about. In the English system, the basic unit of force and the basic unit of mass are both called the pound. We distinguish them by writing lbf = "pounds-force," and lbm = "pounds-mass." By definition, a pound force is equal to the weight of a pound mass on earth. To say it another way, a pound force is the force of gravity that a pound mass will feel on the surface of the earth. In English units, the acceleration of gravity on the surface of the earth is 32.2 ft/s^2, so by the equation F=ma,

$$1\text{lbf} = 1\text{lbm} * 32.2\text{ft/s}^2 = 32.2\ \text{lbm-ft/s}^2$$

Note that the definition of a pound force is Earth-centered, but that the definition of a Newton = 1kg-m/s^2 is not. Both systems are equally accurate, but the metric system is easier to use because there are no multiplying factors in its fundamental units. You can multiply a mass in kilograms by an acceleration in m/s^2 and get a result in Newtons. In the English system, if you multiply a mass in lbm by an acceleration in ft/s^2 you have to further divide by 32.2 to get an answer in lbf. Because of this feature, the metric system is called "coherent," and the English

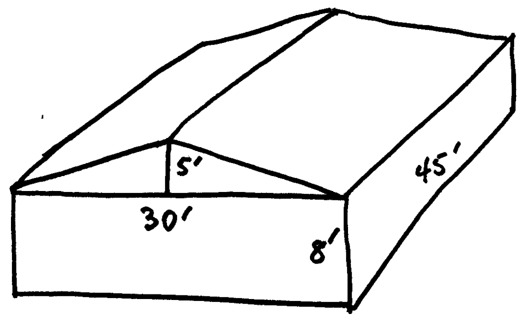
	system is not. A unit of mass called the slug, equal to 32.2 lbm, was introduced to help make the English system coherent, but it's not used much in the United States. How many types of forces are there? Four, as far as we know. The weakest one is gravity, and next is the electric force. These two are the forces we experience in everyday life. It is the electric force that connects atoms into molecules and holds matter together, creates friction, causes chemical reactions, moves motors and the wind, and powers our bodies. There are two other forces that work at subatomic distances, called the weak force and the strong force. They are responsible for nuclear reactions, and for holding the nuclei of atoms together.			
Derived unit	**Description**	**Fundamental units**	**SI (metric) units**	**English units example**
Pressure	Pressure is kind of interesting. The important characteristic of a fluid is that it cannot withstand any steady force. If you push on air or water, it moves out of the way. As a result, the force that a container's walls put on a fluid is the same throughout the container, because the fluid does not allow a difference in force to exist. An exception to this is the force of gravity on a fluid. The force at the bottom of the ocean or bottom of the atmosphere is greater than the force at the top because of the extra weight of water or air overhead. The force on a hard object can be measured as a simple force in newtons. On a fluid, container wall force is measured in newtons per square meter of wall surface or pounds-force per square inch, and this measure is called "pressure." Under steady conditions, fluid pressure is constant everywhere in the container. A bicycle tire at 35psi is 35psi everywhere inside the tube. So think of fluids in terms of pressure. Cool.	$\frac{M}{LT^2}$	$\frac{nt}{m^2}$ 1 kiloPascal (kPa) = 1000nt/m^2	$\frac{lbf}{in^2}$ (psi)
Density	Density is a pretty simple concept – mass of material per unit of volume. Take a box. The more protons and neutrons you can pack into the box, the higher the	$\frac{M}{L^3}$	$\frac{kg}{m^3}$	$\frac{lbm}{ft^3}$

	density of the stuff in the box. Uranium and lead atoms have a lot of protons and neutrons in their nuclei (we don't count the electrons in the atoms because they don't weight very much). When their atoms are packed tightly in a solid metal block, density is very high. On the other side, oxygen and nitrogen have just a few protons and neutrons in their nuclei. They are light, fast moving atoms that bounce back and forth off each other in the gaseous state. Oxygen and nitrogen are the gases that make up air, which has a low density.			
Derived unit	**Description**	**Fundamental units**	**SI (metric) units**	**English units example**
Energy	Now, energy is an interesting unit. The fundamental units that make up energy (M, L, L, T, T) can be grouped in different ways to describe different forms of energy. If they are grouped as $(ML/T^2)*L$, the results are force * distance, and the form of energy is called "work." You perform work on an object when you exert a force on it through a distance. If you apply a force of 1 newton to a rock and lift it 1 meter to a bench top, you have done work of 1nt-m on that rock. If they are grouped as $M*(L/T^2)*L$ the results are mass * acceleration * distance. If the acceleration used is the acceleration of gravity, and the distance is the height to which a mass is raised, then E=mgh is called the gravitational potential energy that is stored in the mass. In the example above of the rock lifted to the bench top, the work that was done on the rock was stored in the rock as a gravitational potential energy of 1 joule. If they are grouped as $M*(L/T)^2$ the result is mass * velocity^2. $E=(1/2)(mv^2)$ is called the kinetic energy of a moving mass. The speed of light is given the label "c" by physicists, and we then see that Einstein's formula $E=mc^2$ is a measure of how much nuclear energy is locked up in a mass. Potential and kinetic energy are brother and sister in many systems. Think of a swinging pendulum. When it zooms up and reaches the top of its swing, it stops	$\frac{ML^2}{T^2}$	1 joule (j) = 1 nt-m	ft-lbf

	for a moment before starting back down. Its kinetic energy goes to zero when it is stopped, but it has a lot of potential energy because it is at its highest point above the ground. When it comes back down, it reaches maximum velocity at the bottom of its swing and it has a lot of kinetic energy. It cannot get any closer to the ground than the bottom, so it has zero potential energy at this point. So the pendulum swings back and forth, converting potential energy into kinetic energy and back on every swing. A lot of science and engineering work is spent in converting one form of energy into another. There are many forms of energy that you will learn later – electrical, chemical, heat – and they will all have the fundamental dimensions of ML^2/T^2.			
Derived unit	**Description**	**Fundamental units**	**SI (metric) units**	**English units example**
Power	Power is simply the rate at which energy is consumed or generated. A toaster consumes about 1000 joules of energy every second, so it requires a kilowatt of electric power to operate. A furnace that produces 100,000 Btu (British thermal units, a unit of energy) of heat energy for every hour it burns is a 100,000 Btu/hr furnace.	$\frac{ML^2}{T^3}$	1 Watt = 1 j/s	1 horse-power (hp) = 550 ft-lbf/s

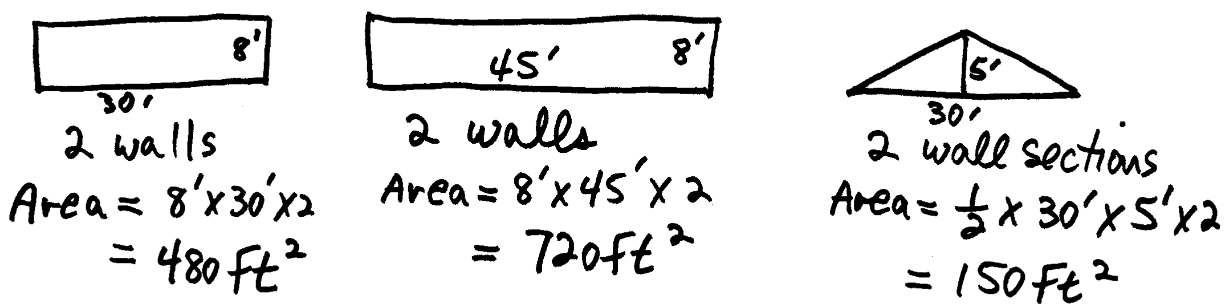
OK, let’s work some problems to help drive these physical units into our skulls.

Problem 1 – You have been sent to the store to get paint for the outside walls of a wooden house. The label on the can says one gallon of paint will cover 440 square feet. The house has the dimensions shown:



There are doors and windows that do not need to be painted, and their total area is 200ft^2 (note that ft^2 is the cool way to write “square feet”). Everything else except the roof is to be painted. How many cans of paint should you get?

Solution 1 – There are six shapes to be painted – four rectangles and two triangles:



Then the total area to be painted is 480ft^2 + 720ft^2 + 150ft^2 –200ft^2 = 1150ft^2. You would need 3 gallons of paint to cover 1150ft^2, but here is a further question – do you really believe that a gallon of paint will cover 440ft^2 or is that some ideal coverage performed by a professional painter with a thin coat on a perfect surface? Better to get an extra gallon. That way there will be some left for touch-up work, too.

Problem 2 – The south-facing roof of the house measures16 feet by 47 feet. You have a bunch of solar cells that can collect 10 Watts of electrical power for every square foot of cell area in the noonday sun. The house has a central air conditioner that requires 10kW of peak power to start its compressor and fan motors. If you cover the roof with solar cells, will you be able to collect enough power to start the motors of the air conditioner?

Solution 2 – It’s a pretty easy calculation. Area of the roof is 16ft x 47ft = 752ft^2. Then the total power that can be collected is 10W/ft^2 x 752ft^2 = 7.52kW, so you could not start your air conditioner. That’s why solar collectors often have some form of energy storage, like lead-acid storage batteries, to supply peak power to loads (and also to maintain power to the loads when the sun goes behind a cloud or at night).

On the other hand, 7.5kW is a lot of power for just a house. If your house were connected to the power company's lines, you could run your meter backwards and sell your excess power back to them. As an exercise, you might want to find out how much electrical power is used in your area and estimate how much power would be generated if every house had solar cells on its roof.

Problem 3 – I read somewhere that one third of the weight of human feces (poop) is bacteria. Let's assume that the density of a bacterium is the same as the density of water = 1.0g/cm^3 (grams per cubic centimeter).

Now, before we go on with this problem, let's talk about a few things.

This problem is going to get into some big numbers. They've probably already taught you that the way to represent big numbers is with scientific notation, so 1,670,000,000,000 is $1.67\text{x}10^{12}$. Well, when people started programming computers, it was too complicated to write exponents on primitive terminals and printers, so they developed a shorthand 1.67e12. Lately, this shorthand has become the cool way to write the powers of 10. It's so new that a lot of old people haven't caught on to it yet, so you can be a pioneer in coolness by using it. The rest of this paper and papers to come will use the shorthand notation for powers of 10.

Also, while we are talking about exponents, you should know that $(A^x)(A^y) = A^{x+y}$ and $(A^x)/(A^y) = A^{x-y}$ and $A^1 = A$ and $A^0 = 1$. For example:

$(10^1)(10^2) = 10 * 100 = 1000 = 10^3$
$(10^5)(10^3) = 100{,}000 * 1000 = 100{,}000{,}000 = 10^8$
$(10^0)/(10^3) = 1/1000 = 0.001 = 10^{-3}$
$(10^4)(10^{-2}) = 10{,}000 * .01 = 10{,}000/100 = 100 = 10^2$
$(4.4^2)(4.4^3) = 1649 = 4.4^5$

But wait, there's more! Beyond scientific notation, there's a variation called engineering notation that only uses exponents that are multiples of three. So

3e7 becomes 30e6
2.655e-2 becomes 26.55e-3
7.768e5 becomes 776.8e3, you get the picture.

What's the point of making exponents into multiples of 3? Well, it turns out that there are Greek names for each of the multiple-of-three exponents, and those are what people who *really* know what they are talking about use as casually as you would use your dog's name. Let's list these Greek names and give some examples:

Scientific notation example	Engineering notation example	Supercool Greek name	Symbol	Greek notation example	Comments
1e12 bytes	1e12 bytes	tera	T	1 Tbyte	A large hard disk drive in 2008 would store 1 terabyte of data.
2.45e9 Hertz	2.45e9 Hertz	giga	G	2.45GHz	The frequency of the electrical cooking power in a microwave oven is 2.45 gigaHertz.
3e8 Hertz	300e6 Hertz	mega	M	300MHz	The wavelength of a 300 megaHertz radio wave is 1 meter (and also the speed of light is 300,000 kilometers per second).
2.5e4 Watts	25e3 Watts	kilo	k	25kW	It takes about 25 kiloWatts of power to accelerate an electric car out from a stop sign.
5 grams	5000e-3 grams	milli	m	5000mg	A U.S. nickel (a 5 cent coin) weighs about 5000 milligrams
1e-5 meters	10e-6 meters	micro	μ (Greek letter mu)	10μm (sometimes just a lower case u is used, 10um)	Note: a micrometer is commonly called a micron. Human body cells are about 10 microns in diameter.
6e-7 meters	600e-9 meters	nano	n	600nm	The wavelength of visible green light is about 600 nanometers.
3e-10 seconds	300e-12 seconds	pico	p	300ps	The time it takes a beam of light to cross the palm of your hand is about 300ps. Note: A unit of distance called the Angstrom (written as an A with a circle on top) is 100pm. Atoms have radii in the range of 0.6 to 5 Angstroms.
-1.6e-19 Coulomb	-0.00016e-15 Coulomb	femto	f	-0.00016fC	The electrical charge of an electron is -0.00016 femtoCoulombs

Now, there are also two Roman names that sneaked into usage, and they are not multiple-of-three exponents:

Scientific notation example	**Engineering notation example**	**Kind of cool Roman name**	**Symbol**	**Roman notation example**	**Comments**
14g/dL	14g/dL	deci, means one-tenth (1/10)	d	14g/dL, this unit is used a lot in medicine	The concentration of hemoglobin in an adult's blood stream might be about 14 grams per deciliter of blood.
2.540cm	2.540cm	centi, means one one-hundredth (1/100)	c	2.540cm	An inch is defined to be *exactly* 2.540 centimeters, because the metric system is the primary system of measurement worldwide, and the English system units are then defined from the metric system units.

So we will use the Greek notation whenever we can.

An "order of magnitude" is the term given to a factor of 10. A mosquito is 2mm long and an adult human is 2m long, so a human is said to be three orders of magnitude longer than a mosquito. The Greek names in the table above encompass 27 orders of magnitude, from 1e-15 to 1e12. That is approximately the ratio of the mass of a U.S. nickel (5 cent coin) to the mass of the Earth. That's a big range, and the Greek names are good for most problems, but some large number problems will go outside these ranges and we will resort to engineering notation to show the numbers.

OK, now we can get back to Problem 3.

<u>If</u>

1. An average bacterium is a sphere with a diameter of 2 microns (2e-6m),
2. The formula for the volume of a sphere is $(4/3)\pi r^3$, where r is the radius,

3. The nucleus occupies on the order of 1% of the volume of the typical bacterium,
4. Under optimal conditions, the typical bacterium will reproduce by splitting in two when it's 20 minutes old,
5. Many bacteria that live in humans are optimized to live at the human body temperature of about 37°C (98.6°F),
6. Bacteria are not mobile and can be classified as a plant,

<u>Then</u> How many bacteria are present in a 500 gram, uh, specimen of feces?

Solution 3 – OK, 500 gram specimen and one third of it is bacteria, so there are 167 grams of bacteria.

Now, each bacterium has a diameter of 2 microns and a radius of 1 micron = 1e-6m = 1e-4cm, so it's volume is $(4/3)\pi(1e\text{-}4cm)^3 \approx 4e\text{-}12cm^3$. We don't have to be too exact here. Some of our input data is approximate and the answer only needs to be approximate. The symbol ≈ means "is approximately equal to."

Since the bacterium has a density of $1g/cm^3$, then the mass of a $4e\text{-}12cm^3$ bacterium is 4e-12g.

167 grams of bacteria at 4e-12g for each one means that there are 42e12 bacteria in our specimen. That's 42 trillion bacteria down the pipes in one sitting. That's more germs than you have brain cells.

All right, what about facts 3, 4, 5, and 6? We didn't use them in this problem, so why were they written down? Ha ha, welcome to the real world, where useful information and not pertinent information are all mixed up and you are the one who has to sort them out. Some of the problems from now on will have extraneous information, so keep on your toes.

Real world problems have two other complications that we will not be adding to practice problems until later – sometimes the information you are given is incorrect, and sometimes it is not clear what the problem is that you should be solving. For now, you can rest easy that we will not lie to you, unless it is by our own ignorance.

As long as we're here, let's have some fun with powers of 2. Fact 4 says each bacterium divides every 20 minutes. How long would it take a single bacterium to divide up to the 42e12 bacteria in our specimen, under optimal conditions? Let's make a table:

Time (hours)	# of bacteria
.33	2
.67	4
1	8
1.33	16
1.67	32
2	64
2.33	128
2.67	256
3	512
3.33	1024
3.67	2048
4	4096
4.33	8192
4.67	16384
5	32768
5.33	65536
5.67	131e3
6	262e3
6.33	524e3
6.67	1.05e6
7	2.10e6
7.33	4.19e6
7.67	8.39e6
8	16.8e6
8.33	33.6e6
8.67	67.1e6
9	134e6
9.33	268e6
9.67	537e6
10	1.07e9
10.33	2.15e9
10.67	4.29e9
11	8.59e9
11.33	17.2e9
11.67	34.4e9
12	68.7e9
12.33	137e9
12.67	275e9
13	550e9
13.33	1.10e12
13.67	2.20e12
14	4.40e12
14.33	8.80e12
14.67	17.6e12

15	35.2e12
15.33	70.4e12

Bingo. 42e12 was our target. Only 15 hours to grow a big lump of bacteria.

Diarrhea is one of the biggest killers of babies and young children in the world. It happens when bacteria in the guts eat the food there and multiply out of control. The body reacts to the waste products emitted by the bacteria by flushing them out of the intestines as a runny poop, but that takes water and nutrition away from the body. If the infection goes on for too long, the body can die.

Does this mean that we would be better off if there were no bacteria at all in our guts? Oh, mercy no! We have to have bacteria there to help us digest food with complex molecular structures. They break the molecules down and get some energy from doing so, and our blood stream takes up and delivers the broken down molecules to various work sites in the body. It’s a partnership, or, more accurately, a symbiosis. At any given time, our bodies contain more bacterial cells than human cells. We live together peacefully as long as they don’t run amok and create problems like diarrhea.

Here’s a game you can play that uses powers of 2. Tell someone to pick a number between 1 and 1000, and you can guess it in 10 tries if they tell you whether you are too low or too high. The trick is to keep splitting the remaining range of numbers in half until you zero in on the chosen number. Here’s an example – suppose your mark chooses the number 687. The game would go like this:

Guess #	**You guess**	**He or she says**	**Your next guess will be halfway between**	
			Low &	**High**
1	500	“too low”	500	1000
2	750	“too high”	500	750
3	625	“too low”	625	750
4	688	“too high”	625	688
5	656	“too low”	656	688
6	672	“too low”	672	688
7	680	“too low”	680	688
8	684	“too low”	684	688
9	686	“too low”	686	688
10	687	“correct!”		

These numbers are small enough that you should be able to work them in your head. (Are you crazy, Jaquish? No! You should learn to add and subtract 3 digit numbers in your head and multiply and divide at least 2 digit numbers. If you don’t do mental arithmetic, how will you know if the numbers you get out of a computer or a calculator are anywhere near correct? To have command of numbers, you have to develop a gut feel for numbers.)

For a real exercise, tell someone you can guess any number between 1 and 1,000,000 in 20 tries.

4. The piston inside a bicycle pump has a diameter of 1 inch. If you push downward with a force of 40lbf, to what pressure could you pump up a bike tire?

Answer: The area of the piston is πr^2, where r is ½ inch, so the area is $0.79in^2$. To this area is applied 40lbf, so the pressure is $40lbf/0.79in^2 = 51psi$. That's enough pressure for a fat bike tire, but a skinny tire needs more pressure to hold up the rider. Notice that the smaller the area of the piston, the higher the pressure you can get from a 40lbf push on the handle. That's why pumps for higher pressure tires often have smaller diameter pistons.

5. A pitch in a baseball game flies toward home plate at 135 feet per second. The batter steps into the pitch and hits the ball with the meat of the bat, slamming a long drive into center field. The velocity of the ball coming off the bat is 180 feet per second. If the total time of the encounter of the ball with the bat is 5 milliseconds (I just guessed at this number), what is the acceleration of the baseball?

Answer: The momentum of the bat changes the velocity of the baseball from 135ft/s toward home plate to 180ft/s away from home plate, so the change in velocity is 315ft/s. This change in velocity takes place in 5ms, so the acceleration is

$$\frac{315\,ft/s}{5ms} = \frac{315\,ft/s}{.005s} = 63000\,ft/s^2$$

From the first table in this paper, note that the acceleration due to gravity at the surface of the Earth is $32ft/s^2$, so the acceleration of the baseball is about 2000 times the acceleration of gravity, which is also called 2000 gees. The maximum acceleration that is felt by a passenger on the Space Shuttle is about 3 gees, and a fighter pilot can get up to about 8 gees on a tight maneuver, so 2000 gees is pretty high. High enough to knock the cover off a ratty old ball. But not as high as the acceleration of a golf ball or a bullet.

6. Someone did a measurement and found that every square meter at the top of the earth's atmosphere receives 1.4kW of continuous solar power. The Earth is 149e6km (=149e9m) from the sun. The formula for the surface area of a sphere is $A=4\pi r^2$. The radius of the Earth is 6.4e6m. How much power does the sun put out, and what percentage of that power hits the Earth?

Answer: A sphere that has a radius as big as the Earth's orbit has a surface area of $4\pi(149e9m)^2 = 279e21m^2$. (Note that $(149e9m)^2 = (149e9)^2(m)^2 = 22.2e21m^2$.) We know that each square meter of that sphere receives 1.4 kilowatts of power. Therefore, the total power put out 24/7 (twenty four hours a day, seven days a week) by the sun is

$$\left(279e21m^2\right)\left(1.4e3\frac{W}{m^2}\right) = 391e24W$$

That's a lot of power. The sun is a large hydrogen bomb that is constantly converting mass into energy down in its core. How much mass? Well, we know that $E=mc^2$, so $m=E/c^2$, where
m = mass converted per second, in kilograms

E = energy output per second by the sun = 391e24W * 1second = 391e24 joules = $391e24\frac{kg-m^2}{s^2}$
c = speed of light = 300e6m/s
$c^2 = (300e6m/s)^2 = (300e6)^2(m)^2/(s)^2 = 90e15m^2/s^2$,

so now we can calculate the mass converted:

$$m = \frac{E}{c^2} = \frac{391e24\frac{kg-m^2}{s^2}}{90e15\frac{m^2}{s^2}} = 4.3kg$$

So 4.3kg of matter is converted to energy every second in the center of the sun. And we figured that all out from a measurement of the solar power reaching the earth. This shows the power of modern science to reach farther than we can see – out to the stars, down into the atom. Most human beings are content to live in the world of their ancestors, but there is a restless minority that thirsts for what is over the horizon and around the bend. Which type do you think you are?

OK, now to finish up this problem. We found above that the surface area of a sphere that has the radius of the earth's orbit is $279e21m^2$. On this large surface bathed in solar energy sits the little earth in its journey around the sun. The area taken up by the earth on the surface of this sphere is the area of a circle that has the radius of the earth, which we said above is 6.4e6m. So

Area of earth seen by sun (called the planform area) = $\pi r^2 = \pi*(6.4e6m)^2 = 129e12m^2$

Then the ratio of the solar energy intercepted by the earth to the total energy emitted by the sun is

$$\frac{129e12m^2}{279e21m^2} = 0.46e-9 = .000000046\%$$

Well, that doesn't tell much of a story. One of the challenges in posing and answering problems is to get the results in a form that people can comprehend and make use of. Let's look at the ratio of the total energy emitted by the sun to the energy intercepted by the earth:

$$\frac{279e21m^2}{129e12m^2} = 2.2e9$$

Now this tells a story. It says that for every ray of sunshine that smacks the earth, there are 2 billion more that fly off into the universe and serve no useful purpose for human beings. Freeman Dyson hypothesized that an advanced civilization would build a wall of collectors and habitats all around its sun to harness that big hydrogen bomb. The wall came to be know as a Dyson Sphere, and it pops up in a science fiction story from time to time.

All right, let's finish this paper up with some simpler problems in the physical units introduced in the first table above.

7. The density of liquid water at 0°C is $0.99987g/cm^3$ and the density of ice at 0°C is 0.917. Does water expand or contract when it freezes?

Answer: Water is one of the few materials that expands when it freezes. Since the density of ice is about 92% the density of water, about 92% of a floating ice cube or iceberg will be under water. It's a good thing that ice floats; otherwise, many life forms would die in the winter at the higher latitudes as the bodies of water filled with ice from the bottom up.

8. How much force does it take to hold up a 50kg mass in the earth's gravitational field? The acceleration of gravity at the surface of the earth is 9.8m/s^2.

Answer: The force is mass x acceleration = 50kg x 9.8m/s^2 = 490nt.

9. Atmospheric pressure is caused by the weight of the layer of oxygen and nitrogen molecules surrounding the earth. At sea level, the pressure of the earth's atmosphere is 14.7psi (lbf/in^2), which means that for every square inch column rising up 100 miles or so through the atmosphere, there is 14.7lbm of nitrogen and oxygen molecules. This much mass does a good job of blocking high-speed cosmic and solar particles and keeping the surface of the earth relatively free of nuclear radiation. Suppose we wanted to make a space station with aluminum walls to give us the same amount of shielding that the earth's atmosphere does. A square inch column of aluminum has a mass of 0.095lbm per inch of length. How many inches thick would we need to make the walls?

Answer: We want 14.7lbm aluminum for each square inch of wall. The mass of a square inch bar of aluminum is 0.095lbm per inch of length, so we would need 154 inches of aluminum to get earth-like shielding in space. That's a lot of metal! The first people to settle on the Moon or Mars would do well to find a cave to live in to get a lot of rock over their heads.

10. Let's say your top fuel dragster goes a quarter mile in 4.420 seconds, and crosses the finish line at a speed of 484ft/s. What is the average acceleration of the vehicle during the run?

Answer: It's $\dfrac{484\frac{ft}{s}}{4.420s} = 110\frac{ft}{s^2}$

Since the acceleration of gravity (one gee) is 32.2ft/s^2, then this average acceleration is 3.4gees.

Well, that's about it for this paper. Make up and solve problems until you understand everything we've talked about. There's more to discuss on this subject, and we'll take it up again in the future.

Note for parents or other persons who have accepted the responsibility of a teacher: As kids mature, their ability to understand complex ideas matures. It might take your kids a while to wrap their minds around the ideas in this topic, and you will have to be their guide. The younger a kid is, the more he or she learns through the ears rather than the eyes. Don't be shy about inventing different ways to pour these concepts into your kids' ears. And make up as many sample problems as you need – it might take tens of problems spread over a number of weeks to transfer this material from abstract concept to trusty implement in your student's toolbox. Remember that learning in the brain is biological, not electrical, and it takes time for the brain to grow the structures that can crank out a new skill. Print these sheets, let kids write all over them, and date them and keep them in a notebook, so they can one day look back over the journey they've taken.