PHY101 Electricity and Magnetism I
Course Summary

TOPIC 1 – ELECTROSTATICS

- **Coulomb's Law**
  The magnitude of the force between two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them.
  $$\mathbf{F} = \frac{qQ}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}}$$

- **Principle of Linear Superposition**
  $$\mathbf{F}_1 = \mathbf{F}_{12} + \mathbf{F}_{13} + \cdots + \mathbf{F}_{1N}$$

TOPIC 2 – ELECTRIC FIELDS

- **Electric field** is force per unit (test) charge.
- Field due to a point charge
  $$\mathbf{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}}$$

- Force exerted on a charge q by a field
  $$\mathbf{F} = q\mathbf{E}$$

- **Conductors**
  Under static conditions, there is no net macroscopic field within the material of a conductor.

  Under static conditions, the electric field at all points on the surface of a conductor is normal to that surface.

  Under static conditions, all the (unbalanced) electric charge resides on the surface of the conductor.

- The field due to a continuous charge distribution is
  $$\mathbf{E} = \int \frac{\mathrm{d}q}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}}$$

- **Electric Dipoles**
  The electric dipole moment of a pair of equal and opposite charges separated by a distance **d** is
  $$\mathbf{p} = Q\mathbf{d}$$

  In an electric field, the torque on an electric dipole is
  $$\boldsymbol{\tau} = \mathbf{p}\times\mathbf{E}$$
  and the potential energy of the dipole is
  $$U = -pE\cos\theta = -\mathbf{p}\cdot\mathbf{E}$$

TOPIC 3 – GAUSS'S LAW

- **Electric Flux**
  The electric flux (or number of field lines) intercepted by an area is given by
  $$\Phi = \int \mathbf{E}\cdot\mathrm{d}\mathbf{A} = \int E\cos\theta\,\mathrm{d}A$$

- **Gauss's Law**
  The electric flux through a closed surface is equal to the total charge contained within that surface divided by ε_0.

$$\oint \mathbf{E} \bullet \mathrm{d}\mathbf{A} = \frac{Q}{\varepsilon_0}$$

<u>TOPIC 4 – Potential</u>

- **Electric Potential**
  The change in electrostatic potential is the change in potential energy per unit charge

$$\Delta V = \frac{\Delta U}{q}$$

The potential at a point is the external work required to bring a positive unit charge from a position of zero potential to the given point, with no change in kinetic energy.

- In general

$$\Delta V = V_\mathrm{B} - V_\mathrm{A} = -\int_\mathrm{A}^\mathrm{B} \mathbf{E} \cdot \mathrm{d}\mathbf{s}$$

- The electrostatic potential due to a point charge is

$$V = \frac{Q}{4\pi\varepsilon_0 r}$$

- The electrostatic potential due to a charged conducting sphere of radius R is

  for $r > R$ $$V = \frac{Q}{4\pi\varepsilon_0 r}$$

  for $r < R$ $$V = \frac{Q}{4\pi\varepsilon_0 R}$$

- By linear superposition, the total potential at a point due to a system of charges is the sum of the potentials due to the individual charges.
- All points within and on the surface of a conductor in electrostatic equilibrium are at the same potential.
- **Calculating Electric Field from Potential**
  In one dimension, the field can be derived from the potential distribution using

$$E = -\frac{\mathrm{d}V}{\mathrm{d}r}$$

In vector form, $$\mathbf{E} = -\nabla V \qquad \text{where } \nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

- **Potential Energy of a System of Charges**
  The potential energy of a charge q at an electrostatic potential V is

$$U = qV$$

The potential energy of a pair of charges, Q_1 and Q_2, separated by a distance r is

$$U = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r}.$$

Note that this is the energy of the <u>system</u>, not of each charge individually.

- A continuous charge distribution will have potential energy due to each element of charge experiencing the field due to the rest of the charge. For example, a conducting sphere of radius R carrying a charge Q_{tot} will have potential energy

$$U = \int_0^{Q_{\text{tot}}} \frac{q}{4\pi\varepsilon_0 R} \mathrm{d}q = \frac{1}{2} \frac{Q_{\text{tot}}^2}{4\pi\varepsilon_0 R} \qquad \textit{(Note the factor ½!)}$$

- **Energy of Moving Charges**
  For a freely moving charge, the gain in kinetic energy is equal to the loss in potential energy

$$\Delta K = -q\Delta V$$

TOPIC 5 – Capacitors

- **Capacitance**
  Capacitors store electric charge, with the capacitance C equal to the charge stored per unit potential difference, so

$$Q = CV$$

- The capacitance of a parallel plate capacitor is

$$C = \frac{\varepsilon_0 A}{d}$$

- Capacitors in parallel have a combined effective capacitance

$$C = C_1 + C_2 + \cdots + C_N$$

- Capacitors in series have a combined effective capacitance given by

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_N}$$

- The energy stored in a capacitor (equalling the work done by a battery or other source in charging it) is

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV = \frac{1}{2}CV^2$$

- The energy density in an electric field is

$$u = \tfrac{1}{2}\varepsilon_0 E^2$$

- **Dielectrics**
  A dielectric material reduces the electric field by a factor called the *dielectric constant, k*.

  The capacitance is therefore increased by k, e.g. for the parallel plate capacitor

$$C = \frac{k\varepsilon_0 A}{d}$$

TOPIC 6 – Current and Resistance

- **Electric Current**
  Current is the rate of flow of charge

$$I = \frac{\mathrm{d}Q}{\mathrm{d}t}$$

- (Conventional) current flows from high to low potential. (In a metal, negative electrons move in the opposite direction.)

- The current density in a wire with charge carriers of charge q and density n, with drift velocity v_d, is

$$J = \frac{I}{A} = nqv_\mathrm{d}$$

- **Resistance and Resistivity**
  The resistance of a conductor depends on the material and its geometry, and is given by

$$R = \frac{V}{I}$$

- The conductivity σ and resistivity ρ, which depend only on the material, are defined by

$$\mathbf{J} = \sigma\mathbf{E} = \frac{1}{\rho}\mathbf{E}$$

- The relationship between resistance and resistivity is

$$R = \frac{\rho\ell}{A}$$

- The temperature coefficient of resistivity α describes how resistivity changes with temperature

$$\rho(T) \simeq \rho_0\left(1+\alpha(T-T_0)\right)$$

- **Electrical Power**
  The power dissipated by a current flowing in a resistive medium is

$$P = IV = I^2R = \frac{V^2}{R}$$

- **Ohm's Law**
  Ohm's law states that, under certain circumstances, the current flowing through a conductor is proportional to the potential across it. That is, the electrical resistance is a constant.

- **The Drude Model of Conduction**
  Charge carriers scatter of the lattice with a mean time between collisions τ. When an electric field is applied, the mean drift velocity is hence

$$\mathbf{v}_\mathrm{d} = -\frac{e\mathbf{E}}{m}\tau$$

  and the resistivity is

$$\rho = \frac{m}{ne^2\tau}$$

__**TOPIC 7 – Electric Circuits**__

- **Electromotive Force and Batteries**
  The EMF $\mathcal{E}$ of a battery is the work done per unit charge moved from negative to positive terminals. It is thus measured in volts.

- Real batteries have an internal resistance, so their terminal voltage is reduced when current flows,

$$V_\mathrm{T} = \mathcal{E} - I\,r$$

- **Resistors in Series and Parallel**
  Resistors in series have a combined effective resistance

$$R = R_1 + R_2 + \cdots + R_N$$

Resistors in parallel have a combined effective resistance given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$$

- **Kirchhoff's Rules**
  - The junction rule states that: *The algebraic sum of the currents entering and leaving a junction is zero.* $\Sigma I = 0$
  - The loop rule states that: *The algebraic sum of the changes in potential around any closed loop is zero.* $\Sigma V = 0$

- **RC Circuits**

  When a capacitor discharges through a resistor, its charge varies as

$$Q = Q_0 e^{-t/RC}$$

  When a capacitor is charged through a resistor, its charge increases as

$$Q = Q_0 \left(1 - e^{-t/RC}\right)$$

Copies of lecture handouts and PowerPoint displays are available from the Web, at http://www.cbooth.staff.shef.ac.uk/phy101E&M/