example problems

1. **The synodic period of Venus was measured to be 583.92 days. If the sidereal period of the Earth is 365.26 days, calculate the sidereal period of Venus.**

   Venus is an inferior planet, so we must use the relation

   $1/S = 1/P - 1/E$.

   Rearranging and substituting gives

   $1/P = 1/583.92 + 1/365.26 = 0.00171 + 0.00274 = 0.00445$

   and hence

   $P = 224.7$ days.

2. **If the greatest elongations of Mercury and Venus are 28° and 48° respectively, calculate the distances of these planets from the Sun in AU.**

   We have seen from Figure 28 that Copernicus derived a relationship between the greatest elongation of an inferior planet, SEP_{max}, and its distance from the Sun in AU, SP / SE:

   $SP / SE = \sin (SEP_{max})$.

   Hence, the distance of Mercury from the Sun is simply sin (28°) = 0.47 AU and the distance of Venus from the Sun is sin (48°) = 0.74 AU.

3. **The synodic period of Mars was measured to be 779.9 days. 105.9 days after opposition, the elongation of Mars was measured to be 90°. Calculate the distance of Mars from the Sun in AU.**

   We have seen from Figure 28 that Kepler derived the following relationship between the angle between the Earth and a superior planet when viewed from the Sun t days after opposition, θ, and the synodic period of the planet, S:

$\theta = 360° t/S$.

Hence for Mars we obtain

$\theta = 360° t/S = 360° \times 105.9 / 779.9 = 48.9°$.

The angle between the Sun and the Earth as viewed from the superior planet t days after opposition, ϕ, is related to the elongation, ε, and θ through the relation:

$\phi = 180° - \varepsilon - \theta$.

Hence for Mars we obtain

$\phi = 180° - 90° - 48.9° = 41.1°$

The distance of Mars to the Sun in units of the Earth-Sun distance is then equal to $\sin \varepsilon / \sin \phi = \sin 90° / \sin 41.1° = 1.52$ AU.

4. **In problem 1, we calculated the sidereal period of Venus from its synodic period, and obtained a value of 224.7 days. Given this, calculate the mean distance of Venus from the Sun in AU.**

We must use Kepler's third law to solve this problem:

$a^3 = T^2$,

where a is the mean distance of Venus from the Sun in AU and T is the sidereal period of Venus in years. Hence

$a = T^{2/3} = (224.7/365.26)^{2/3} = 0.72$ AU.