Solution to Exercise 6.2.

We have the energy generation rate from the pp-chain (equation 6.6) the same as from the CNO cycle (equation 6.11):

$$2.6 \times 10^{-37} X^2 \rho T^{4.5} = 7.9 \times 10^{-118} X Z \rho T^{16},$$

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hence

$$T^{11.5} = \frac{2.6}{7.9} 10^{118-37} \frac{X}{Z} = \frac{2.6 \times 0.74}{7.9 \times 0.02} \times 10^{81},$$

$$T = 1.37 \times 10^{7}\,\mathrm{K}.$$

$$T^{11.5} = \frac{2.6}{7.9} 10^{118-37} \frac{X}{Z} = \frac{2.6 \times 0.74}{7.9 \times 0.02} \times 10^{81},$$

$$T = 1.37 \times 10^{7}\,\mathrm{K}.$$

For the second star,

$$\varepsilon_{pp} = 2.6\times10^{-37}(0.7)^2\rho\left(1.37\times10^7\right)^{4.5}$$
$$\simeq 1.7\times10^{-5}\rho \quad \text{(in SI units)},$$
$$\varepsilon_{CNO} = 7.9\times10^{-118}\times0.001\times0.7\rho\left(1.37\times10^7\right)^{16}$$
$$\simeq 8.9\times10^{-7}\rho \quad \text{(in SI units)},$$

$$\varepsilon_{pp} = 2.6\times10^{-37}(0.7)^2\rho\left(1.37\times10^7\right)^{4.5}$$
$$\simeq 1.7\times10^{-5}\rho \quad \text{(in SI units)},$$
$$\varepsilon_{CNO} = 7.9\times10^{-118}\times0.001\times0.7\rho\left(1.37\times10^7\right)^{16}$$
$$\simeq 8.9\times10^{-7}\rho \quad \text{(in SI units)},$$

Hence

$$\frac{\varepsilon_{CNO}}{\varepsilon_{pp}+\varepsilon_{CNO}} \simeq \frac{8.9}{170+8.9} \simeq 0.05\,.$$

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Solution to Exercise 6.3.

Using equation (6.10),

$$L \propto X^2 \mu^{\alpha} \frac{M^{2+\alpha}}{R^{3+\alpha}} \propto X^2 \mu^{\alpha}, \quad \mu = \frac{4}{3+5X},$$

$$L \propto X^2 \mu^{\alpha} \frac{M^{2+\alpha}}{R^{3+\alpha}} \propto X^2 \mu^{\alpha}, \quad \mu = \frac{4}{3+5X},$$

hence

$$\frac{L_1}{L_2} = \left(\frac{X_1}{X_2}\right)^2 \left(\frac{3+5X_2}{3+5X_1}\right)^{\alpha}$$

$$= \left(\frac{0.8}{0.4}\right)^2 \left(\frac{3+2}{3+4}\right)^{\alpha} = 4\left(\frac{5}{7}\right)^{\alpha},$$

$$\frac{L_1}{L_2} = \left(\frac{X_1}{X_2}\right)^2 \left(\frac{3+5X_2}{3+5X_1}\right)^{\alpha}$$

$$= \left(\frac{0.8}{0.4}\right)^2 \left(\frac{3+2}{3+4}\right)^{\alpha} = 4\left(\frac{5}{7}\right)^{\alpha},$$

and with $\alpha \simeq 4.5$ $\alpha \simeq 4.5$, we have

$$\frac{L_1}{L_2} \simeq 0.88, \quad \frac{L_2 - L_1}{L_1} = \frac{L_2}{L_1} - 1 \simeq 0.14.$$

$$\frac{L_1}{L_2} \simeq 0.88, \quad \frac{L_2 - L_1}{L_1} = \frac{L_2}{L_1} - 1 \simeq 0.14.$$

Solution to Exercise 6.4.

From equation (6.9), for polytropic stars of the same polytropic index and of the same chemical composition,

$$L \propto T_c^{\alpha} \rho_c^2 R^3.$$

$$L \propto T_c^{\alpha} \rho_c^2 R^3.$$

With $\alpha = 4.5$ $\alpha = 4.5$ and $\rho_c T_c^{4.5} = \text{const}$ $\rho_c T_c^{4.5} = \text{const}$, we have

$$L \propto \rho_c R^3 ,$$

$L \propto \rho_c R^3 ,$

and with $\rho_c \propto M/R^3$ $\rho_c \propto M/R^3$ (see equation 5.16), we have

$$L \propto M .$$

$L \propto M .$