equation of state of an ideal gas

We have seen that stars are composed of highly ionized gases known as plasmas. Hence, although the density is so high that the typical inter-particle spacing is of the order of an atomic radius, the effective particle size is more like a nuclear radius, i.e. 10^5 times smaller. This means that, despite the high pressures and densities in the interior, the stellar material behaves like an ideal gas with a gas pressure given by:

$P_{gas} = nkT$,

where k is Boltzmann's constant and $n = N / V$, i.e. the number of particles, N, per cubic metre. We need to rewrite this equation in the form

$P = P(\rho, T, \text{composition})$,

which means that we require an expression for n in terms of density and chemical composition. Noting that *density = mass / volume*, we can immediately write:

$\rho = nm$,

where m is the mean particle mass. This can be rewritten in terms of the mass of the hydrogen atom, m_H, as follows:

$\rho = nm_H \mu$,

where μ is known as the *mean molecular weight* of the stellar material and is the mean mass of the particles in the gas in terms of the mass of the hydrogen atom, $m_H = 1.67 \times 10^{-27}$ kg. Hence the equation for the gas pressure becomes:

$P_{gas} = kT\rho / m = kT\rho / m_H\mu$.

If we now define the *gas constant*, R, as

$R = k / m_H$,

we obtain:

$P_{gas} = R\rho T / \mu$.

This is known as the *equation of state* of an ideal gas and has been written in the form that we shall use for much of the remainder of this course.

We calculated earlier that we can ignore the effects of radiation pressure in stars like the Sun and hence the gas pressure is equal to the total pressure. This is not true of all stars, however, in which case we can generalize the above equation as follows:

$$P = P_{gas} + P_{rad}$$

and hence

$$P = R\rho T / \mu + (aT^4/3),$$

where a is the radiation density constant and we have introduced no new variables.
