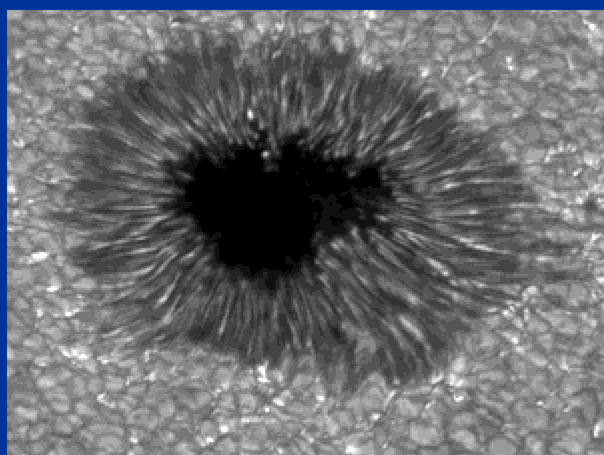
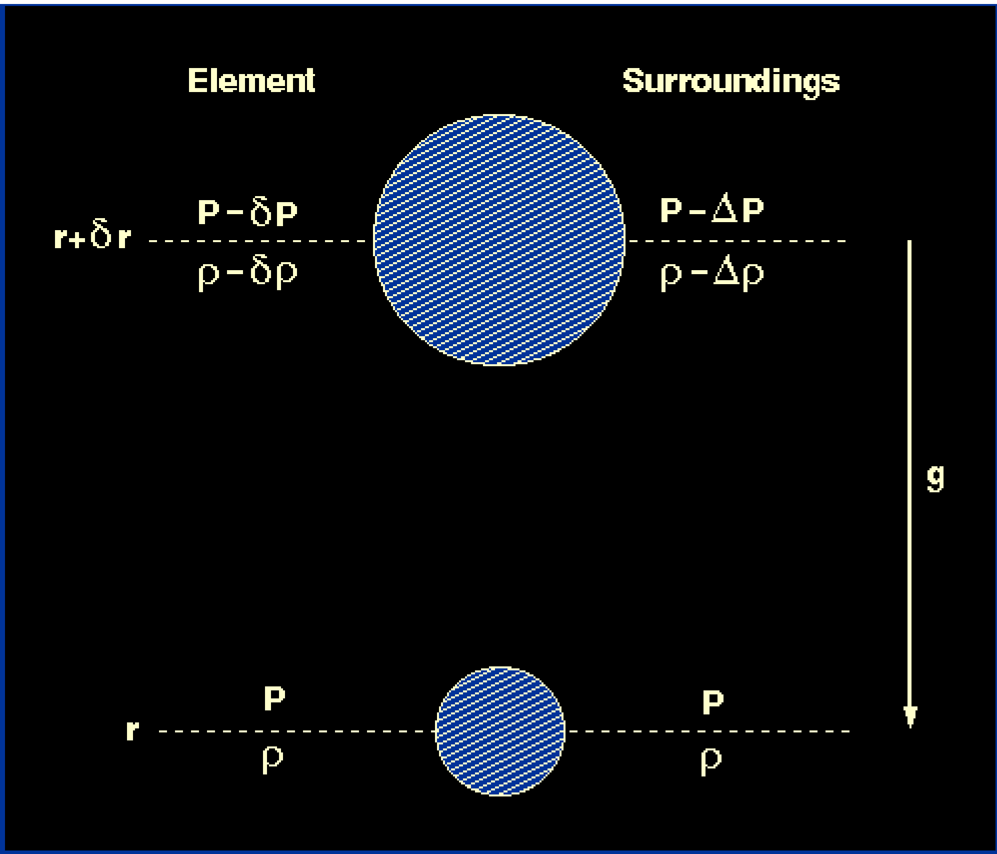
convection

Energy transport by conduction and radiation occurs whenever a temperature gradient is maintained in any body. Convection, on the other hand, which is the mass motion of elements of gas, only occurs when the temperature gradient exceeds some critical value. We will now derive an expression for this critical value.

Figure 10: Top: Solar granulation around a sunspot. Each convective cell is approximately the size of Britain. Bottom: A convective element of stellar material.





Consider a convective element of stellar material a distance r from the centre of the star, as shown in figure 10. The element is in equilibrium with its surroundings at r, i.e. its pressure P and density ρ are the same as its surroundings. Let us suppose that the element rises a distance δr towards the surface of the star, expanding (and hence reducing in pressure and density) as it does so. The pressure and density of the element at $r+\delta r$ may or may not be the same as its surroundings, so let us define δP, $\delta\rho$ as the change in pressure and density of the element and ΔP, $\Delta\rho$ as the change in pressure and density of the surroundings. If the blob is denser than its surroundings at $r+\delta r$ it will tend to sink back to its original position and the gas is said to be stable against convection. However, if the blob is less dense than its surroundings at $r+\delta r$ then it will keep on rising and the gas is said to be convectively unstable. The condition for this instability is therefore:

$$\rho - \delta\rho \quad < \quad \rho - \Delta\rho.$$

Whether or not this condition is satisfied depends on two things; the rate at which the element expands (and hence decreases in density) due to the decreasing pressure exerted on it and the rate at which the density of the surroundings decreases with height.

Having obtained a condition for the occurrence of convection, we now need to rewrite it in terms of a temperature gradient for compatibility with the equations of stellar structure. We can do this by making two assumptions about the motion of the element:

1. the element rises *adiabatically*, i.e. it moves fast enough to ensure that there is no exchange of heat with its surroundings;
2. as the element rises, it does so at a speed which is much less than the speed of sound. This means that, during the motion, sound waves have plenty of time to smooth out the pressure differences between the element and its surroundings and hence $\delta P = \Delta P$ at all times.

The first assumption means that the element must obey the adiabatic relation between pressure and volume:

$$PV^{\gamma} = \text{constant},$$

where $\gamma = c_p / c_v$ is the specific heat (i.e. the energy in J required to raise the temperature of 1 kg of material by 1 K) at constant pressure divided by the specific heat at constant volume. Note that the specific heat of most substances measured at constant pressure is greater than the specific heat at constant volume, because at constant pressure some of the energy input goes into increasing the volume of the gas and hence more energy is required to raise the temperature by 1 K. Given that V is inversely proportional to the density ρ, we can equivalently write for the element:

$$P / \rho^{\gamma} = \text{constant}$$

This implies that:

$$(P - \delta P) / (\rho - \delta\rho)^{\gamma} = P / \rho^{\gamma}.$$

If $\delta\rho$ is small, we can expand $(\rho - \delta\rho)^{\gamma}$ using the binomial theorem as follows:

$$(\rho - \delta\rho)^{\gamma} \sim \rho^{\gamma} - \gamma\rho^{\gamma-1}\,\delta\rho.$$

Combining these last two equations gives an expression for the change in density experienced by the rising element:

$$\delta\rho = (\rho / \gamma P)\,\delta P.$$

We now need to evaluate the change in density in the surroundings, $\Delta\rho$. If we consider an infinitesimal rise, δr, we can write:

$\Delta\rho = (d\rho / dr)\,\delta r$.

Substituting these expressions for $\delta\rho$ and $\Delta\rho$ into the condition for convective instability derived above, we obtain:

$(\rho / \gamma P)\,\delta P \;\; > \;\; (d\rho / dr)\,\delta r$.

In words, the left-hand side of this inequality is the change in density of the element at $r+\delta r$ assuming an adiabatic relation between the pressure and density of the element. The right-hand side of this inequality is the change in density of the surroundings at $r+\delta r$. For convection to occur, therefore, the adiabatic gradient must be greater than the stellar gradient. This is shown graphically in figure 6.3 of the text book by Prialnik.

The above inequality can be rewritten by recalling our second assumption that the element will remain at the same pressure as its surroundings, so that in the limit $\delta r \to 0$, $\delta P / \delta r = dP / dr$:

$(\rho / \gamma P)\,(dP / dr) \;\; > \;\; d\rho / dr$.

We can now convert the density gradient to a temperature gradient by first dividing both sides by dP / dr. This gives:

$(P / \rho)\,(d\rho / dP) \;\; < \;\; 1 / \gamma$.

For an ideal gas in which radiation pressure is negligible, we have seen that:

$P = kT\rho / m$,

where m is the mean mass of the particles in the stellar material. Taking logarithms, we obtain:

$\log P = \log \rho + \log T + \text{constant}$.

This can be differentiated to give:

$dP / P = (d\rho / \rho) + (dT / T)$.

Combining this expression with the inequality, we obtain

$(P / T)\,(dT / dP) \;\; > \;\; (\gamma-1) / \gamma$, which is the **condition for the occurrence of convection**.

If the condition for occurrence of convection is satisfied, then large-scale rising and falling motions occur in the gas. Rising elements are less dense than their surroundings and are therefore hotter than their surroundings (this follows, for an ideal gas, from the formula $P = kT\rho \, / \, m$), while falling elements are denser and cooler than their surroundings. These rising and falling motions therefore transport energy (upwards).

The criterion for convection derived above can be satisfied in two ways: either the ratio of specific heats, γ, is close to unity or the temperature gradient is very steep. If a large amount of energy is released in a small volume at the centre of a star, it may require a large temperature gradient to carry the energy away. This means that convection may occur at the centres of stars where nuclear energy is being released. Such regions are known as convective cores. Alternatively, in the cool outer layers of a star, where the gas is only partially ionized, much of the energy passing through goes into ionization, i.e. changing the state of the material from a gas into a plasma. Hence the specific heat of the gas at constant volume is nearly the same as the specific heat at constant pressure and $\gamma \sim 1$. In such a case a star can have an outer convective layer. We will return to the subject of convective cores and outer convective layers when we discuss the structure of main-sequence stars.

Convection is an extremely complicated subject and it is true to say that the lack of a good theory of convection is one of the worst defects in our present understanding of stellar structure and evolution. In particular, although we know the conditions under which convection is likely to occur, we do not know how much energy is carried by convection. Fortunately, as we shall see later, there are occasions when we can manage without this knowledge.
