energy generation

Having determined the physical state of stellar material, we are now in a position to consider the source of stellar energy; where, of course, we mean the conversion of energy from some other form in which it is not immediately available to the star to radiate.

How much energy does a star like the Sun need to generate in order to shine as it does? We know that the luminosity of the Sun is $L_\odot = 4 \times 10^{26}$ W $= 4 \times 10^{26}$ Js^{-1}. We have already noted that fossil and geological records indicate that the properties of the Sun have not changed significantly for at least 10^9 years (3×10^{16} s), and hence in that time the Sun must have lost approximately 1.2×10^{43} J. Using Einstein's formula for the equivalence of mass and energy,

$E = mc^2$,

we find that in this time the Sun must have converted about 10^{26} kg of mass into energy and hence its mass will have decreased by about $10^{-4}\ M_\odot$. What can have been the source of this energy? There are four possible sources which we might consider and we shall address each of them in turn:

- Cooling or contraction
- Chemical reactions
- Nuclear reactions

cooling or contraction

Cooling and contraction are very closely related, so we shall consider them both together.

Cooling is perhaps the simplest idea of all. Let us suppose that the energy radiated by the Sun at the present time is simply due to the fact that the Sun was much hotter when it was formed than it is now and it has since been cooling down. We can test the plausibility of this idea by determining how long the present thermal energy content of the Sun could supply its present rate of energy loss.

Another possibility is that the Sun is slowly contracting with a consequent release of gravitational potential energy and this energy is then converted into radiation.

The gravitational energy and the thermal energy of a star can be related using a very simple equation which we shall now derive. In an ideal gas, the thermal energy of a particle is obtained by multiplying $kT/2$ by the number of degrees of freedom. As the particles move in three dimensions, we obtain $3kT/2$. The thermal energy per unit volume is then given by $3nkT/2$, where n is the number of particles per unit volume.

Now, we have seen that the virial theorem can be written:

$$3\int_0^{V_s} P \, \mathrm{d}V + \Omega = 0,$$

Assuming that the stellar material is an ideal gas with negligible radiation pressure, we can substitute the expression $P = nkT$ in the above equation, to give:

$$3\int_0^{V_s} nkT \, \mathrm{d}V + \Omega = 0,$$

Defining the total thermal energy of a star, U, as the integral over volume of the thermal energy per unit volume, $3nkT/2$, we can write:

$$2U + \Omega = 0.$$

Thus the gravitational potential energy of a star is equal to twice its thermal energy. This means that the time for which the present thermal energy of the Sun can supply its radiation and the time for which the past release of gravitational potential energy could have supplied its present rate of radiation differ only by a factor of two, and to get an approximate value for this time we need consider only one.

Let us consider the gravitational potential energy. We have seen that the gravitational potential energy of a star, Ω, is given by the inequality:

$$-\Omega > GM_s^2 / 2r_s.$$

We can then approximate a value for the gravitational potential energy of a star by writing:

$- \Omega \sim GM_s^2 / r_s$.

The total release of gravitational potential energy would have been sufficient to provide radiant energy at a rate given by the luminosity of the star, L_s, for a time:

$GM_s^2 / L_s r_s$.

This equation is known as the *thermal timescale*, t_{th}, i.e.:

$t_{th} = GM_s^2 / L_s r_s$.

If we insert values for the mass, radius and luminosity of the Sun in the above equation we obtain a time of 3×10^7 years. This means that if the Sun were powered by either contraction or cooling (we can consider either as the factor of 2 is insignificant), it would have changed substantially in, say, the last ten million years, a factor of 100 times too short to account for the constraints on the age of the Sun imposed by fossil and geological records.

chemical reactions

Perhaps chemical reactions such as the combustion of coal, gas and oil could supply enough energy to power the Sun? We know that such reactions release approximately 5×10^{-10} of the rest mass energy. But we calculated above that we need to find a process which can release at least 10^{-4} of the rest mass energy of the Sun, so this immediately rules out chemical reactions as the source of the Sun's energy (which could only power the Sun for a few thousand years).

nuclear reactions

The only known way in which quantities of energy sufficiently large to power stars can be produced is through nuclear reactions. There are two types of nuclear reaction - fission and fusion. Fission reactions, such as occur in nuclear reactors and the atomic bomb, can release 5×10^{-4} of the rest mass energy through the splitting (or fission) of heavy nuclei. Fusion reactions, such as occur in the hydrogen bomb, can release up to 0.01 (i.e. 1%) of the rest mass energy by combining (or fusing) light nuclei. This can be proved very simply by considering that the atomic weight of hydrogen is 1.008172 (defining the weight of O^{16} as 16) and that of the isotope

He^4 is 4.003875. The difference between the atomic weight of He^4 and four times the atomic weight of hydrogen is 0.0288. This mass difference is transformed into energy when four protons are fused into one He^4 nucleus. The mass fraction which is transformed into energy in this process is hence given by 0.0288 / (4 x 1.008172) = 0.007, or 0.7%.

It can be seen that both fusion and fission could in principle power the Sun. As light elements are much more abundant in the Sun than heavy ones, one would expect nuclear fusion rather than fission to be the main source of energy. Given the limits on the pressures and temperatures of the stars that we have just obtained, however, is it possible for nuclear fusion to occur in stars? We shall return to answer this question in detail later in the course.
