state of stellar material

Having determined an expression for the minimum mean temperature of a star, we can now determine the physical state of the material of which stars are composed.

We have the inequality:

$T_{av} > GM_s m / 6kr_s$.

We know the values of M_s and r_s for the Sun. We can estimate the mean particle mass, m, by assuming that the Sun is composed of nothing other than hydrogen atoms, the most abundant element in stars, which have a mass of 1.67×10^{-27} kg. These values can be inserted into the inequality to give:

$T_{av\odot} > 4 \times 10^{6}$ K.

This is much higher than the measured surface temperature of the Sun ($T_{e\odot}$ = 5780 K). We can obtain a more accurate constraint by noting that the above minimum mean temperature far exceeds the temperature at which hydrogen is fully ionized. There will then be two particles for each hydrogen atom - a proton and an electron - and the mean particle mass will hence become ½(1.67×10^{-27}) kg, resulting in a revised inequality:

$T_{av\odot} > 2 \times 10^{6}$ K.

We can also estimate the mean density of the Sun using the relation:

$\rho_{av} = 3M_\odot / 4\pi r_\odot^3 = 1.4 \times 10^3$ kg m^{-3}.

So, the above figures tell us that the mean density of the Sun is similar to that of ordinary liquids like water. We know that liquids like water turn to gases at temperatures far lower than 2×10^{6} K. Furthermore, at these extreme temperatures, the average kinetic energy of the particles is much higher than the energy required to remove all but the most tightly bound electrons from the atoms, hence the gas will be highly ionised. Such a gas is known as a *plasma*. Since the gas is ionised, it can withstand much greater compression without deviating from an ideal gas (which, in effect, demands that the distances between particles are much greater than their sizes, so that the inter-particle forces are negligible except during a collision) because a typical nuclear dimension is 10^{-15} m compared with a typical atomic dimension of 10^{-10} m.

But what about our assumption that the radiation pressure is negligible? Radiation pressure is the pressure exerted by photons on the particles in a gas and is given by:

$P_{rad} = aT^4 / 3$,

where a is the radiation density constant. We can estimate the importance of radiation pressure at a typical point in the Sun by comparing it with the gas pressure, as follows:

$P_{rad} / P_{gas} = (aT^4/3) / (kT\rho / m) = maT^3 / 3k\rho$.

Taking $T \sim T_{av} = 2 \times 10^6$ K, $\rho \sim \rho_{av} = 1.4 \times 10^3$ kg m^{-3} and $m = ½(1.67 \times 10^{-27})$ kg, we obtain:

$P_{rad} / P_{gas} \sim 10^{-4}$.

It does appear as if radiation pressure is indeed negligible at an average point in the Sun.

In summary, with no knowledge of how energy is generated in stars, we have been able to derive a value for the Sun's internal temperature and deduce that:

Stars are composed of near ideal gases (plasmas) with negligible radiation pressure.

This is not true of all stars, however. Radiation pressure is of importance in the hottest stars, and some stars are much denser than the Sun and hence corrections to the ideal gas law are very important.
