Extrasolar Planets and Astrophysical Discs
Problem Set 4

February 2010

Problem 1

An accretion disc has an effective temperature profile

$$T = T_c \left(\frac{R_c}{R}\right)^{1/2}$$

in the region $R_{in} \leq R < R_c$, which changes at radius R_c to become

$$T = T_c \left(\frac{R_c}{R}\right)^{3/4}$$

in the region $R_c \leq R \leq R_{out}$. The flux emitted at a particular frequency by a black body of effective temperature T is given by

$$F_\nu = \frac{2\pi h\nu^3}{c^2}\frac{1}{\exp\left(h\nu/kT\right) - 1}.$$

Derive an expression for the emitted luminosity, L_ν, of such a disc, and show that it takes the form of a sum of two power laws in ν. By considering how each of these power laws in ν varies as ν changes, sketch the behaviour of L_ν as a function of ν. In particular, illustrate how L_ν varies in the vicinity of ν_c, where $h\nu_c \simeq kT_c$.

A disc in a C.V. system has temperature varying as

$$T \;=\; 2\times 10^4 \left(\frac{5\times 10^7\ \mathrm{m}}{R}\right)^{3/4}\ \ \mathrm{K}$$

in those regions that lie outside a radius $R = 5\times 10^7$ m. Inside this radius, the temperature profile changes to

$$T \;=\; 2\times 10^4 \left(\frac{5\times 10^7\ \mathrm{m}}{R}\right)^{1/2}\ \ \mathrm{K}$$

between the radii $1\times 10^7 \leq R < 5\times 10^7$ m.
Calculate the deficit in the emitted energy that results from this change in temperature profile from the standard $T \propto R^{-3/4}$ found at larger radii.
If this energy is emitted from a hot, optically thin corona in the form of soft X-rays, instead of from the optically thick disc, estimate the flux of emitted 1 keV photons.

[The Stefan constant is $\sigma = 5.670\,\times\,10^{-8}\ \mathrm{W\,m^{-2}\,K^{-4}}$. The electron-volt is $1\ \mathrm{eV} = 1.602\,\times\,10^{-19}$ J.]

Problem 2

Derive an estimate of the torque acting on a geometrically thin accretion disc due to a dipolar magnetic field that originates in a rotating central star. Assume that the disc has an inner edge located away from the star at radius R_{min}.

By equating this magnetic torque to the internal viscous torque acting at the inner radius R_{min}, derive an expression for the inner radius of an accretion disc that is truncated by the stellar magnetic field.

Consider a $1M_\odot$ T Tauri star of radius $R_* = 1R_\odot$, with a magnetic field strength at its surface of $B_z(R_*) = 10^{-1}$ Tesla. If the star is accreting at a rate $\dot{m} = 10^{-8}$ $M_\odot$ yr^{-1}, calculate the radius of the inner edge of the accretion disc.

One class of young stars, known as FU Orionis stars, are known to undergo outbursts in which the apparent accretion rate increases substantially above the canonical value of $\dot{m} = 10^{-8}$ $M_\odot$ yr^{-1}. If the accretion rate during outburst increases by a factor of 10^4 above this value, then calculate the radius of the inner edge of the accretion disc using the above stellar parameters. What do you think happens to the magnetic field in this case ?