E. Course work 3

CW3

Q1. *[25 Marks]*

a)Formulate the equivalence principle and explain what is the difference in interpretation of this principle in Newtonian theory and in General relativity.

b) Explain the similarity between an "actual" gravitational field and a non-inertial reference system. Give the definition of a locally Galilean coordinate system.

c) Explain why an "actual" gravitational field cannot be eliminated by any transformation of coordinates over all space-time.

d) Formulate the covariance principle and explain the relationship between this principle and the principle of equivalence.

Q2. *[25 Marks]*

a) Give the definition of a contravariant vector in terms of the transformation of curvilinear coordinates.

b) Give the definition of a covariant vector in terms of the transformation of curvilinear coordinates.

c) What is the mixed tensor of the second rank in terms of the transformation of curvilinear coordinates (you can assume that a mixed tensor of the second rank is transformed as a product of covariant and contrvariant vectors).

d) Explain why the principle of covariance implies that all physical equations should contain only tensors.

Q3. *[25 Marks]*

a) Prove that the metric tensor is symmetric. Give a rigorous proof that the interval is a scalar.

a) Give the definition of the reciprocal tensors of the second rank. What is the contravariant metric tensor g^{ik}.

b) Show that in an arbitrary non-inertial frame

$$g^{ik} = S^i_{(0)0}S^k_{(0)0} - S^i_{(0)1}S^k_{(0)1} - S^i_{(0)2}S^k_{(0)2} - S^i_{(0)3}S^k_{(0)3},$$

where $S^i_{(0)k}$ is the transformation matrix from locally inertial frame of reference (galilean frame) to this non-inertial frame.

c) Demonstrate how using the reciprocal contravariant metric tensor g^{ik} and the covariant metric tensor g_{ik} you can form contravariant tensor from covariant tensors and vice versa.

d) Show with the help of straightforward differentiation that if A^i is a vector then dA^i is not a vector.

Q4. *[25 Marks]*

a) Motivate the necessity to introduce parallel translation of a vector. Explain the meaning of the Christoffel symbols. Explain why the Christoffel symbols do not form a tensor.

b) Show that

$$\Gamma^i_{km} = \frac{1}{2} g^{in} \left(g_{kn,m} + g_{mn,k} - g_{km,n} \right). \tag{E.1}$$

c) Explain why for the derivation of physical equations in the presence of a gravitational field one can simply replace partial derivatives by covariant derivatives, and as an example show that the motion of a particle in a gravitational field is given by the geodesic equation

$$\frac{d^2x^i}{ds^2} + \Gamma^i_{kl}\frac{dx^k}{ds}\frac{dx^l}{ds} = 0.$$