## A. Course work 1

For all calculations you are recommended to use **Astro-Physical Calculator.**

### Q1 *[25 Marks]*

A gravitationally bound system of mass $M$ and radius $R$ has a characteristic velocity

$$V \sim \sqrt{\frac{GM}{R}}. \tag{A.1}$$

[This neglects a numerical factor which depends upon the precise distribution of the matter and on whether the speed is associated with rotational or random motion.] Use this formula to estimate the following:

(a) The velocity dispersion of stars in a globular cluster, assumed to have a mass of $10^6 M_\odot$ and a radius of 10 pc. *[9 Marks]*

(b) The rotation velocity of stars in a spiral galaxy, assumed to have a mass of $10^{11} M_\odot$ and a radius of 10 kpc. *[8 Marks]*

(c) The velocity dispersion of galaxies in a rich cluster, assumed to have a mass of $10^{14} M_\odot$ and a radius of 3 Mpc. *[8 Marks]*

### Q2 *[30 Marks]*

Although structures larger than clusters cannot be regarded as gravitationally bound, groups and clusters of galaxies will have peculiar motions due to the gravitational attraction of nearby superclusters.

(a) Explain why, over the age of the Universe $t_0$, a supercluster of mass $M$ and distance $D$ will induce a velocity in the Local Group of roughly

$$V \sim \frac{GMt_0}{D^2}, \tag{A.2}$$

where $D$ is the distance between the supercluster and the Local Group. *[20 Marks]*

(b) If $t_0 = 10^{10}$ y, calculate the velocity induced by the Great Attractor, assumed to have a mass of $10^{17} M_\odot$ and a distance of 100 Mpc. *[10 Marks]*

### Q3 *[45 Marks]*

One usually accounts for the uncertainty in the Hubble parameter by writing it as

$$H_0 = 100h\text{kms}^{-1}\text{Mpc}^{-1}, \tag{A.3}$$

where the dimensionless number h lies between 1/2 and 1.

(a) Show that the quantity $H_0^{-1}$ has the dimensions of time and calculate this in terms of h, expressing the result in years. [As shown later, this is roughly the age of the Universe in the Big Bang picture.] *[10 Marks]*

(b) Show that the quantity $3H_0^2/(8\pi G)$ has the dimensions of density and calculate this in terms of h, expressing the results in $\text{kgm}^{-3}$. [As shown later, this corresponds to the critical density required for the Universe to recollapse.] *[10 Marks]*

(c) Galaxies and their constituent baryons (mainly protons) have roughly a tenth of the critical density. If a typical galaxy has a mass of $10^{11} M_\odot$, infer the average distance between galaxies. *[15 Marks]*

(d) If all the baryons in the Universe were spread out uniformly (instead of being clumped into galaxies), what would be the average separation between them? *[10 Marks]*