## Outline Solutions to Coursework 5 :        Week 5

**QUESTION 1: (20 marks)** This is a standard engine analysis question.

**a)** The P - V diagram looks like this:
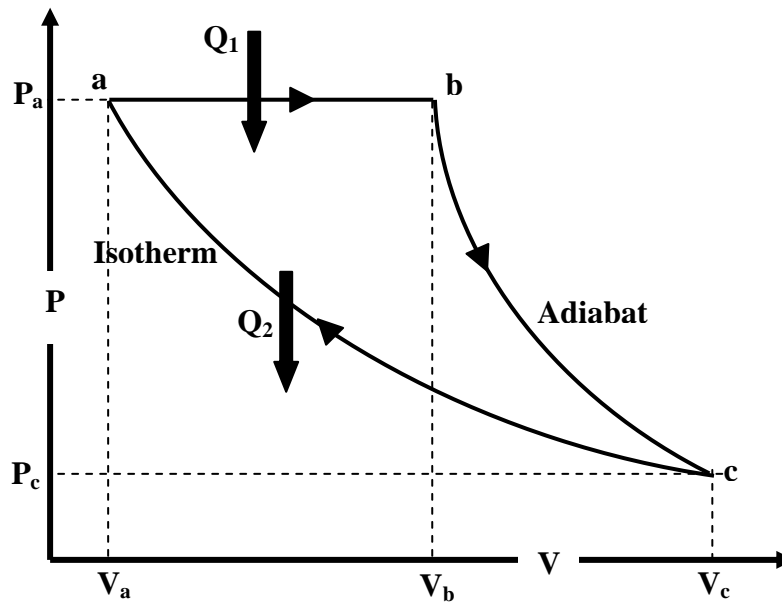


**[3mks]**

**b)** First Law applied to a → b process

$$\Delta U_{ab} = \Delta Q_{ab} + \Delta W_{ab}$$

$$\frac{3}{2}\left(P_b V_b - P_a V_a\right) = Q_1 - \int_a^b P dV = Q_1 - P_a \int_a^b dV = Q_1 - P_a\left(V_b - V_a\right)$$

Since P is constant on a → b, P = P$_a$ ,

Therefore

$$Q_1 = \frac{5}{2} P_a\left(V_b - V_a\right)$$       **[3mks]**

First Law applied to the process c → a

$$\Delta U_{ca} = \Delta Q_{ca} + \Delta W_{ca}$$

Because c → a is an isothermal process $U_{ca} = 0$ and the First Law becomes

$$0 = -Q_2 - \int_c^a PdV$$

noting that the heat flow $Q_2$ is OUT of the system and so enters the First Law with a minus sign. We may use the equation of state on the isotherm to write

$$0 = -Q_2 - \int_c^a \frac{nRT}{V}dV = -Q_2 - nRT_a \int_{V_c}^{V_a} \frac{dV}{V} = -Q_2 - nRT_a \ln\left(\frac{V_a}{V_c}\right)$$

Hence, since $nRT_a = P_aV_a$ (the equation of state)

$$Q_2 = -P_aV_a \ln\left(\frac{V_a}{V_c}\right) = P_aV_a \ln\left(\frac{V_c}{V_a}\right) \qquad \text{[3mks]}$$

c) For the efficiency we have

$$\eta_E = 1 - \frac{Q_2}{Q_1} = 1 - \frac{P_aV_a \ln\frac{V_c}{V_a}}{\frac{5}{2}P_a(V_b - V_a)} = 1 - \frac{2}{5}\frac{\ln\left(V_c/V_a\right)}{\left(V_b/V_a - 1\right)} \qquad \text{[1mk]}$$

Along the isotherm $c \to a$ the equation of state gives us
$$P_aV_a = nRT = P_cV_c,$$
Hence
$$\frac{V_c}{V_a} = \frac{P_a}{P_c}$$

Along the adiabat $b \to c$ we have

$$P_bV_b^{5/3} = P_cV_c^{5/3} \qquad \Rightarrow \qquad \left(\frac{V_b}{V_c}\right)^{5/3} = \frac{P_c}{P_b} \qquad \text{[2mks]}$$

Hence

$$\frac{V_b}{V_c} = \left(\frac{P_c}{P_b}\right)^{3/5} = \left(\frac{P_c}{P_a}\right)^{3/5}$$

To handle the ratio $\dfrac{V_b}{V_a}$ we now write

$$\frac{V_b}{V_a} = \frac{V_b}{V_c}\frac{V_c}{V_a} = \left(\frac{P_c}{P_a}\right)^{3/5}\frac{P_a}{P_c} = \left(\frac{P_a}{P_c}\right)^{2/5} = \left(\frac{P_a}{P_c}\right)^{1-1/\gamma} \qquad \textbf{[1mk]}$$

Putting it all together gives for the efficiency

$$\eta_E = 1 - \frac{2}{5}\frac{ln\left(P_a/P_c\right)}{\left(P_a/P_c\right)^{1-1/\gamma} - 1} \qquad \textbf{[1mk]}$$

**d)** To increase $\eta_E$ we must increase the high pressure $P_a$.

$$\eta_E^{Max} = 1 - \frac{2}{5}\frac{ln\,32}{(32)^{2/5} - 1} = 1 - \frac{2}{5}\frac{ln\,32}{3} = 0.54 \qquad \textbf{[2mks]}$$

Max rate of working is given by

$$\eta_E^{Max}\left(Rate\ of\ heat\ \ sup\,ply\right) = 0.54 \times 5\ \ kW = 2.7\ \ kW. \qquad \textbf{[2mk]}$$

One could say two things at least to the environmentalist. First, that the Second Law of Thermodynamics tells us that there must always be some waste heat. **[1mk]** Secondly, the magnitude of the waste heat is $Q_2 = (1 - \eta_E)Q_1$. Thus to reduce the waste heat, we must invest in research in materials to enable the engine to run at the highest possible pressure $P_a$, thus improving the efficiency. **[1mk]**

**QUESTION 2: (20 Marks)**

**a)**

$$\Delta S = \int_i^f \frac{dQ_R}{T} = \int_{T_i}^{T_f} mc_P \frac{dT}{T} = mc_P \int_{T_i}^{T_f} \frac{dT}{T} = mc_P\, ln\left(\frac{T_f}{T_i}\right)$$

$$\Delta S = 5kg \times 4.2 \times 10^3 J \ kg^{-1} \ K^{-1} \ ln\left(\frac{278.15}{298.15}\right) = -1455 \ J \ K^{-1} \quad \textbf{[2mks]}$$

**b)**

(i) Here we have water cooling again, just as in **a)** so

$$\Delta S = mc_P \ ln\left(\frac{T_f}{T_i}\right) = 10 \times 10^{-3} kg \times 4.2 \times 10^3 J \ kg^{-1} \ K^{-1} \ ln\left(\frac{273.15}{373.15}\right) = -13.07 J \ K^{-1}$$

**[2mks]**

(ii) Freezing occurs with a loss of latent heat by the water while the temperature remains fixed, so

$$\Delta S = \int_i^f \frac{dQ_R}{T} = \frac{1}{T_{Freezing}} \int_{T_i}^{T_f} dQ_R = \frac{1}{T_{Freezing}} \ m\ell_{L \to S}' = \frac{10g \times \left(-333 Jg^{-1}\right)}{273.15} = -12.19 J \ K^{-1}$$

**[3mks]**

**c)**

(i) Entropy as a function of state is defined by

$$dS = \frac{dQ_R}{T} \quad \textbf{[1mk]}$$

*or for finite change*

$$\Delta S = S_f - S_i = \int_i^f \frac{dQ_R}{T}$$

The entropy form of the Second Law states that for a thermally isolated system, the entropy can only increase or stay the same ($\Delta S \geq 0$) but cannot decrease. For irreversible processes, the entropy increases ($\Delta S > 0$) while for reversible processes it stays unchanged ($\Delta S = 0$). **[1mk]**

(ii) The water will cool to 0 $^{\circ}$C then freeze to ice at 0 $^{\circ}$C and the ice will then cool to the ambient temperature of -5 $^{\circ}$C when it is in equilibrium with the surroundings.

**[2mks]**

(iii) The total entropy change of water is

$$\Delta S_{Tot}^{Water} = \Delta S_{Cooling}^{Water} + \Delta S_{Freezing}^{Water} + \Delta S_{Freezing}^{Ice} \quad \textbf{[1mk]}$$

Calculating each component in turn

$$\Delta S_{Cooling}^{Water} = mc_P^{Water} \ln\left(\frac{T_f^{Water}}{T_i^{Water}}\right) = 4 \ kg \times 4.2 \times 10^3 J \ kg^{-1} \ K^{-1} \ln\frac{273.15}{283.15} = -604 \ J \ K^{-1}$$

**[1mk]**

$$\Delta S_{Freezing}^{Water} = \frac{ml_{L \to S}}{T_{Freezing}} = \frac{4 \ kg \times \left(-3.33 \times 10^5 J \ kg^{-1}\right)}{273.15} = -4876 \ J \ K^{-1}$$

**[2mks]**

$$\Delta S_{Cooling}^{Ice} = mc_P^{Ice} \ \ln\left(\frac{T_f^{Ice}}{T_i^{Ice}}\right) = 4 \ kg \times 2.1 \times 10^3 J \ kg^{-1} \ K^{-1} \ln\frac{268.15}{273.15} = -155 \ J \ K^{-1}$$

**[1mk]**

$$\Delta S_{Tot}^{Water} = \Delta S_{Cooling}^{Water} + \Delta S_{Freezing}^{Water} + \Delta S_{Freezing}^{Ice} = \left(-604 - 4876 - 155\right) J \ K^{-1} = -5634 \ J \ K^{-1}$$

**[1mk]**

**NB *Marks to be lost if signs are forgotten (inclusing + sign)***

(iv)

The surroundings whilst remaining at 268.15 K will **receive heat from** the water as it cools and as it freezes and also as the ice freezes.

We begin by calculating the heat lost by the water in the three process

$$\Delta Q_{Tot}^{Water} = m\left(c_P^{Water} \Delta T_{Cooling}^{Water} + l_{L \to S} + c_P^{Ice} \Delta T_{Cooling}^{Ice}\right) = -1.542 \times 10^6 J$$

**[1mk]**

The surroundings have received tjhis heat at constant temperature and therefore

$$\Delta S_{Tot}^{Surr} = \frac{\Delta Q}{T_{Surr}} = \frac{1.542 \times 10^6 \ J}{268.15 \ K} = +5751 \ J \ K^{-1}$$

**[1mk]**

(v) The water plus its surroundings is a thermally isolated system for which the second law requires that

$$\Delta S = \Delta S_{Tot}^{Water} + \Delta S_{Tot}^{Surr} > 0$$

since we have an irreversible process. From the calculation above we have

$$\Delta S = -5634 \ J \ K^{-1} + 5751 \ J \ K^{-1} = +117 \ J \ K^{-1}$$

which is positive confirming that the second law is satisfied. **[1mk]**