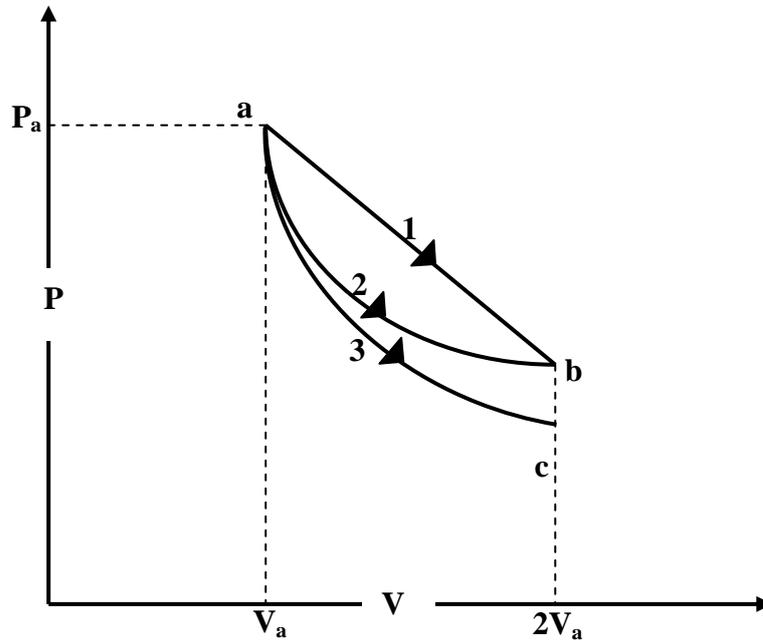


THERMAL AND KINETIC PHYSICS 2010, (PHY 214)

Outline Solutions to Coursework 3 :

Week 3

QUESTION 1: (10 marks)



This is the same series of paths that were considered in week 2 exercises when the work and internal energy changes were calculated for each of the three paths.

Path 1.

From week 2 we found that

$$\Delta W_1 = - \int_{V_a}^{2V_a} \left(-\frac{P_a}{2V_a} V + \frac{3}{2} P_a \right) dV = -0.75 P_a V_a$$

and

$$\Delta U_1 = \frac{3}{2} (P_b V_b - P_a V_a) = \frac{3}{2} \left[\frac{P_a}{2} 2V_a - P_a V_a \right] = 0$$

We find from this the heat flow by using the first law as follows

$$\Delta Q_1 = \Delta U_1 - \Delta W_1 = 0 + \frac{3}{4} P_a V_a = 0.75 P_a V_a \quad \text{[4mks]}$$

Path 2

From week 2 we found that

$$\Delta W_2 = -P_a V_a \log_e 2 = -0.693 P_a V_a$$

and

$$\Delta U = \frac{3}{2}(P_b V_b - P_a V_a) = \frac{3}{2} \left[\frac{P_a}{2} 2V_a - P_a V_a \right] = 0$$

We find from this the heat flow by using the first law as follows;

$$\Delta Q_2 = \Delta U_2 - \Delta W_2 = 0 + 0.693 P_a V_a = +0.693 P_a V_a \quad [3\text{mks}]$$

Path 3

From week 2 we found that

$$\Delta W_3 = -0.555 P_a V_a$$

and

$$\Delta U_3 = -0.555 P_a V_a$$

We find from this the heat flow by using the first law as follows;

$$\Delta Q_3 = \Delta U_3 - \Delta W_3 = -0.555 P_a V_a + 0.555 P_a V_a = 0 \quad [2\text{mks}]$$

as we would expect for an adiabatic process.

QUESTION 2: (14 Marks)

(a) From $PV^\gamma = c$ we can express the pressure as $P = cV^{-\gamma}$. Then the work calculation is

$$\Delta W_1 = - \int_{V_1}^{V_2} P dV = -c \int_{V_1}^{V_2} V^{-\gamma} dV = -c \left[\frac{-V^{-\gamma+1}}{\gamma-1} \right]_{V_1}^{V_2} \quad [3\text{mks}]$$

$$\Delta W_1 = \frac{c}{\gamma-1} \left[V_2^{-\gamma+1} - V_1^{-\gamma+1} \right] = \frac{1}{\gamma-1} \left[cV_2^{-\gamma+1} - cV_1^{-\gamma+1} \right]$$

and using $P = cV^{-\gamma}$ gives

$$\Delta W_1 = \frac{1}{\gamma-1} \left[P_2 V_2 - P_1 V_1 \right] \quad [4\text{mks}]$$

(b) For monatomic argon $\gamma_{Ar} = \frac{5}{3}$ while for diatomic nitrogen $\gamma_{N_2} = \frac{7}{5}$. In an adiabatic process $P_a V_a^\gamma = P_b V_b^\gamma$ and if $V_b = 3V_a$, $P_b = (3^{-\gamma})P_a$. Remembering that in any process

$$\text{Work BY gas} = -\text{Work ON gas}$$

we can use the result in (a) to calculate for argon

$$\text{Work BY Argon} = -\left(\frac{1}{\frac{5}{3}-1}\right)(P_b V_b - P_a V_a) = \frac{3}{2}\left[1-3^{-2/3}\right]P_a V_a = 0.779P_a V_a$$

[3mks]

and for nitrogen we obtain

$$\text{Work BY Nitrogen} = -\left(\frac{1}{\frac{7}{5}-1}\right)(P_b V_b - P_a V_a) = \frac{5}{2}\left[1-3^{-2/5}\right]P_a V_a = 0.889P_a V_a$$

[3mks]

Thus the diatomic gas does more work on the surroundings.

[1mk]

QUESTION 3:(16 marks)

(a)

$$PV^\gamma = P V V^{\gamma-1} = nRT V^{\gamma-1} = \text{const} \quad \Rightarrow$$

thus the adiabatic law can also be written as

$$TV^{\gamma-1} = \text{const} \quad [2\text{mks}]$$

$$\text{For } \gamma = \frac{7}{5}, \quad TV^{2/5} = \text{const} \text{ or } T = \frac{c}{V^{2/5}}.$$

$$\text{For a small volume change, } \Delta T \approx \left(\frac{dT}{dV}\right)_{\text{adiabatic}} \times \Delta V$$

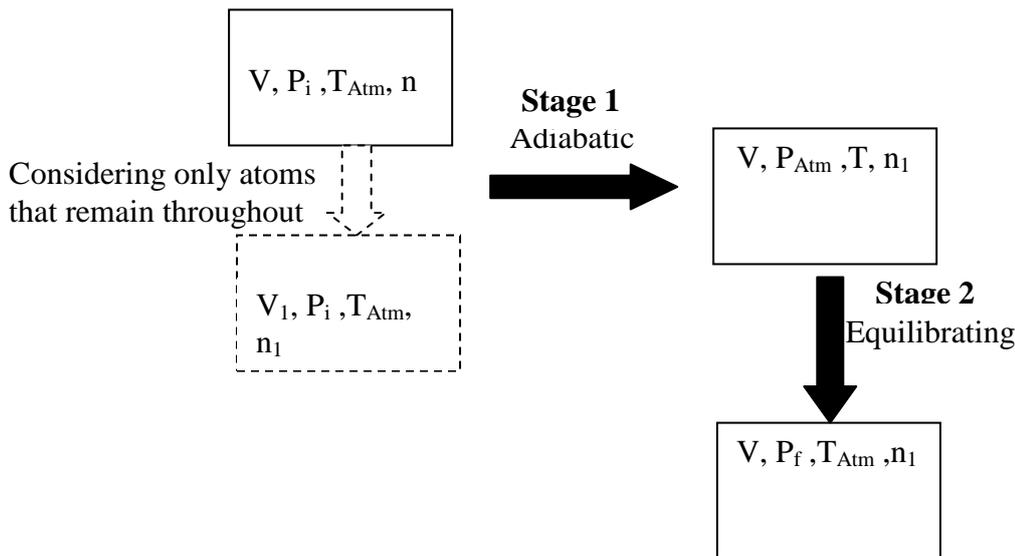
$$\left(\frac{dT}{dV}\right)_{adiabatic} = -\frac{2}{5} \frac{c}{V^{7/5}} = -\frac{2}{5} \frac{T}{V} \quad [2\text{mks}]$$

thus

$$\Delta T \approx \left(\frac{dT}{dV}\right)_{adiabatic} \times \Delta V \approx -\frac{2}{5} (300 \text{ K})(-0.1) = +12 \text{ K} \quad [4\text{mks}]$$

(b)

(i) There are three systems to use throughout the two stage process;



- 1) the initial state, tyre is at T_{atm} and P_i
 - 2) the state immediately after the valve is opened and shut, T, P_{atm}
- and
- 3) the final equilibrium state, T_{atm}, P_f

Let V be the fixed total volume of the tyre and let V_1 be the initial volume of gas which remains inside the tyre throughout. We can apply the equation of state to each stage for the n_1 moles of gas which remain inside .

NB. We are using the fact that V and n are extensive variables and in general we can divide any volume V of a gas with n molecules into (V_1, n_1) and (V_2, n_2) and write $PV_1 = n_1RT$ and $PV_2 = n_2RT$ (P and T are intensive variables being the same in each of the divided volumes).

Thus in **stage 1**, $P_i V_1 = n_1 R T_{Atm}$ (and $P_i V = n R T_{Atm}$)

in **stage 2**, $P_{Atm}V = n_1RT$, where T is some intermediate temperature immediately following the expansion of the gas.

and in **stage 3** finally $P_fV = n_1RT_{Atm}$.

Stage 1 to stage 2 is an adiabatic process, $PV^\gamma = const$ or equivalently

$P^{1/\gamma}V = const$ by taking root γ on both sides of the equation, but only if we compare systems with the same number of moles n (or molecules) ie for our initial system only

the sub-volume V_1 containing n_1 moles so we can write $P_i^{1/\gamma}V_1 = P_{atm}^{1/\gamma}V$ and use the equation of state to change the variables

$$P^{1/\gamma}V = P^{1/\gamma-1}PV = P^{1/\gamma-1}n_1RT = const$$

Thus the adiabatic law in terms of pressure and temperature is

$$P^{1/\gamma-1}T = const \quad [1mk]$$

Remember the constant is generally not the same constant!

As we go from stage 1 to stage 2 adiabatically we can write

$$P_i^{1/\gamma-1}T_{Atm} = P_{Atm}^{1/\gamma-1}T \quad [1mk]$$

whence

$$\frac{T}{T_{Atm}} = \left(\frac{P_i}{P_{Atm}}\right)^{1/\gamma-1} = \left(\frac{P_i}{P_{Atm}}\right)^{-2/7}$$

If $P_i = 4$ atm, then $T = T_{Atm} \times 4^{-2/7} = 0.673T_{Atm}$ [2mks]

ii) Now we compare **stage 1** with **stage 3**,

NB. In comparing stages 1 and 3 we need to take care in using equations of state and adiabatic rule as there is a different number of molecules present in stage 1 and 3 (some were lost when the valve was opened). To apply the equations equally to stages 1 and 2 (or 3) we need only consider in stage 1 the volume V_1 containing the molecules n_1

From the equation of state we will have

$$P_iV_1 = P_fV$$

Or equivalently $\frac{P_i}{P_f} = \frac{V}{V_1}$

However, from stage 1 to stage 2 we have the adiabatic relation $P_i V_1^\gamma = P_{Atm} V^\gamma$

Thus

$$\frac{P_i}{P_{Atm}} = \left(\frac{V}{V_1} \right)^\gamma = \left(\frac{P_i}{P_f} \right)^\gamma \quad \text{[1mk]}$$

Taking logs

$$\ln \left(\frac{P_i}{P_{Atm}} \right) = \gamma \ln \left(\frac{P_i}{P_f} \right)$$

$$\gamma = \frac{\ln \left(\frac{P_i}{P_{Atm}} \right)}{\ln \left(\frac{P_i}{P_f} \right)} \quad \text{[3mks]}$$