THERMAL AND KINETIC PHYSICS 2010, (PHY 214)

Outline Solutions to Coursework 7 : Week 8

QUESTION 1: **(total 14 marks)**

(a) This is an energy conservation problem. The total energy per second emitted at the sun's surface must be equal to the total energy per second passing through a spherical surface located at the earth's distance (L_E) from the sun. Total energy per second is calculated from the product of the energy flux times the total area emitting or through which the energy is passing. Thus we have the balance equation

$$\left(4\pi R_S^2\right)\sigma T_S^4 = \left(4\pi L_E^2\right)\times 1.4\times 10^3\,\mathrm{Jm^{-2}s^{-1}}$$

Here R_S is the sun's radius.

Re-arranging;

$$T_S^4 = \left(\frac{L_E}{R_S}\right)^2 \frac{1.4\times 10^3\,\mathrm{Jm^{-2}s^{-1}}}{\sigma} = \left[\frac{14.9\times 10^{10}}{6.96\times 10^8}\right]^2 \frac{1.4\times 10^3\,\mathrm{Jm^{-2}s^{-1}}}{5.7\times 10^{-8}\,\mathrm{Wm^{-2}K^{-4}}}$$

$T_S \approx 5790$ K **[4 mks]**

(b) The sun radiates the same total energy per second as above, $4\pi R_S^2 \sigma T_S^4$. TheEarth catches all the energy that falls on its cross-sectional area, πR_E^2, and so Earth absorbs a fraction

$$\frac{\pi R_E^2}{4\pi L_E^2}\left(4\pi R_S^2 \sigma T_S^4\right)$$

of the sun's total output.

In steady state this must just balance what Earth radiates away (from ALL points on its surface).The balance equation now looks like

$$4\pi R_E^2 \sigma T_E^4 = \frac{\pi R_E^2}{4\pi L_E^2}\left(4\pi R_S^2\right)\sigma T_S^4$$

thus

$$\left(\frac{T_E}{T_S}\right)^4 = \frac{1}{4}\left(\frac{R_S}{L_E}\right)^2 \quad \Rightarrow \quad T_E = T_S\sqrt{\frac{R_S}{2L_E}}$$ **[3 mks]**

giving numerically

$$T_E \approx 280 \text{ K} :$$ **[1 mk]**

(c) We assume that Icarus is heated by the sun in exactly the same manner as is Earth in part (b), therefore, we have also that

$$T_I = T_S\sqrt{\frac{R_S}{2L_I}} = T_S\sqrt{\frac{R_S}{2L_E}}\sqrt{\frac{L_E}{L_I}} = T_E\sqrt{\frac{L_E}{L_I}}$$

From this we deduce that T_I^{Max} corresponds to L_I^{Min} and vice versa. Thus

$$T_I^{Max} = T_E\sqrt{\frac{L_E}{\frac{1}{3}L_E}} = \sqrt{3}T_E \approx 1.732 \times 280\text{K} = 485\text{K}$$ **[3 mks]**

$$T_I^{Min} = T_E\sqrt{\frac{L_E}{\frac{3}{2}L_E}} = \sqrt{\frac{2}{3}}T_E \approx 0.816 \times 280\text{K} = 228.6\text{K}$$ [3 mks]

QUESTION 2: (total 12 marks)

(a) In steady state at maximum power there is an energy balance,

Heat in = Heat radiated away :

$$10\text{kW} = 4\pi R^2 \sigma T_{\text{surface}}^4$$

giving $T_{surface} \approx 328{:}2$ K. **[3 mks]**

For a Carnot engine we have the efficiency as

$$\eta_C = 1 - \frac{T_2}{T_1} = 1 - \frac{328.2}{500} = 1 - 0.66 = 0.34$$

Therefore useful power is given by

$$\dot{W} = \eta_C \dot{Q}_{\text{input}} = 10\text{kW} \times 0.34 = 3.4\text{kW}$$ **[3 mks]**

(b) After the panel is deployed the surface area is effectively doubled so that the steady state equation now reads

$$10\text{kW} = 8\pi R^2 \sigma T_{\text{new}}^4$$

so that

$$\frac{T_{\text{new}}^4}{T_{\text{surface}}^4} = \frac{1}{2}$$

$$T_{new}^4 = \frac{1}{2} T_{surface}^4 = \frac{1}{2}(328.2)^4$$

$$T_{new} = 275.8K$$ **[3 mks]**

We find for the new surface temperature $T_{new} \approx 275{:}8$ K. The new efficiency is then;

$$\eta_C = 1 - \frac{T_2}{T_1} = 1 - \frac{275.8}{500} = 1 - 0.55 = 0.45$$

giving a useful power of .

$$\dot{W} = \eta_C \dot{Q}_{\text{input}} = 10\text{kW} \times 0.45 = 4.5\text{kW}$$ **[3 mks]**

QUESTION 3: (total 14 marks)

(a)

(i) For a monatomic gas like Argon we have $U = \frac{3}{2}PV$ and so $u = \frac{U}{V} = \frac{3}{2}P$ so

$$u_{Ar} = \frac{3}{2} \times 1.01 \times 10^5 \,Pa = 1.52 \times 10^5 \,Jm^{-3}$$

For the photon gas then

$$u_{photon} = \frac{4\sigma}{c} T^4 = 1.52 \times 10^5 \,Jm^{-2}$$ **[2 mks]**

(ii) $$T^4 = \frac{c \times 1.52 \times 10^5 \,Jm^{-2}}{4\sigma} = \frac{3 \times 10^8 \,ms^{-1} \times 1.52 \times 10^5 \,Jm^{-2}}{4 \times 5.67 \times 10^{-8} \,Wm^{-2}K^{-4}} = 2.01 \times 10^{20} \,K^4$$

$$T = 1.19 \times 10^5 \,K$$ **[2 mks]**

(iii) The first part is like Question 1 (a),

Rate of energy loss of the fireball is $-\frac{dE}{dt} \approx 4\pi R^2 \sigma T^4$

$$-\frac{dE}{dt} \approx 4\pi (5 \times 10^{-2} \,m)^2 \times 5.67 \times 10^{-8} \,Jm^{-2}K^{-4}s^{-1} \times (10^6 \,K)^4$$

$$-\frac{dE}{dt} \approx 4 \times 3.14 \times 2.5 \times 5.67 \times 10^{13} \,Js^{-1} = 1.78 \times 10^{15} \,W$$

Energy flux at 1 km = $\frac{\text{Energy loss rate}}{\text{Area at 1km}} = \frac{1.78 \times 10^{15} \,W}{4 \times 3.14 \times 10^6 \,m^2} = 1.42 \times 10^8 \,Wm^{-2}$

$$P = \frac{1}{3}\frac{U}{V} = \frac{1}{3}\frac{4\sigma}{c} T^4 = \frac{4 \times 5.67 \times 10^{-8} \,Wm^{-2}K^{-4}}{3 \times 3 \times 10^8 \,ms^{-1}} \times (10^6)^4$$

$$P = \frac{4 \times 5.67}{9} \times 10^8\, Jm^3 = 2.53 \times 10^8\, Nm^{-2} = 2.53 \times 10^8\, Pa = \frac{2.53 \times 10^8}{1.007 \times 10^5}\, atm = 2.51 \times 10^3\, atm$$

[4 mks]

(b)

(i) We have from $dU = \frac{\chi c^2}{8\pi G} dA + \Omega dJ + \phi dq$ identified the first term on the RHS as the heat or

$$\frac{\chi c^2}{8\pi G} dA = T dS_{BH} = T \frac{k_B}{4L_P^2} dA$$

From Eq. (2) , $S_{BH} = \frac{1}{4} \frac{k_B A}{L_P^2}$,we have that a change of horizon area is related to the corresponding change of entropy by

$$\frac{4L_P^2}{k_B} S_{BH} = A \qquad\qquad dA = \frac{4L_P^2}{k_B} dS_{BH}$$

Using this relation between dS and dA and solving for T

$$T = \frac{4L_P^2}{k_B} \frac{\chi c^2}{8\pi G} = \frac{\chi c^2}{2\pi k_B G} \frac{hG}{2\pi c^3} = \frac{\chi h}{4\pi^2 k_B c} = \frac{c^4}{4GM} \frac{h}{4\pi^2 k_B c}$$

$$T = \frac{hc^3}{16\pi^2 G k_B M} = \frac{\hbar c^3}{8\pi G k_B M}$$ **[3 mks]**

(ii) The entropy is found from $\frac{4L_P^2}{k_B} S_{BH} = A$

$$S_{BH} = \frac{k_B}{4L_P^2} A$$

The Planck length is $L_P = \sqrt{\frac{hG}{2\pi c^3}} = 1.61 \times 10^{-35} \mathrm{m}$

therefore

$$S_{BH} = \frac{k_B}{4L_P^2} A = \frac{1.38 \times 10^{-23} \mathrm{JK}^{-1}}{4 \times \left(1.61 \times 10^{-35}\right)^2 \mathrm{m}^2} \times 100 \times 10^6 \mathrm{m}^2$$

$$S_{BH} = 1.33 \times 10^{54} \mathrm{JK}^{-1}$$

We find the temperature using our derived equation and the mass of the sun, $M = 1.99 \times 10^{30}$kg

$$T = \frac{1.05 \times 10^{-34} \mathrm{Js} \times (3 \times 10^8 \mathrm{ms}^{-1})^3}{8 \times 3{,}14 \times 6.67 \times 10^{-11} \mathrm{m}^3 \mathrm{kg}^{-1} \mathrm{s}^{-2} \times 1.38 \times 10^{-23} \times 1.99 \times 10^{30} \mathrm{kg}}$$

$T \approx 6 \times 10^{-8}$K **[3 mks]**