## THERMAL AND KINETIC PHYSICS (PHY 214)

# Coursework 8: Week 9

**ISSUE: Tuesday 23 November, 2010 HAND-IN: Tuesday 30 November, 2010**

**Students name (top left corner), course title & exercise number and exercise group (top right corner)** should appear on every sheet of the submitted coursework and sheets should be firmly held together. A stapler is available if needed from the secretaries office.

Hand-in of worked exercises must take place by 4:00 p.m. on the above date at the labeled box provided outside the Teaching Administrators office on the first floor.

This time will be strictly adhered to and no late working will be accepted without written explanation to the course organiser. The solutions will appear shortly after this time on the TKP website.

Each coursework is worth 40 marks and the aggregate coursework mark will count 10% towards the final mark. An indicative mark scheme is given with each question. **Note**: ***I want to see the method of solution. No credit will be given for simply writing down the answer.***

Students should collect new exercise sheets in the Tuesday lecture or download them from the Web. Marked exercises will be returned in exercise classes or via the box outside the Teaching Administrators office on the first floor.

**QUESTION 1: (20 marks)**

For a paramagnetic substance in which volume changes can be ignored the thermodynamic identity can be written in the form

$$dU = TdS + B_0 d\mathcal{M}$$

where $B_0$ is the externally applied magnetic induction and $\mathcal{M}$ is the total magnetic moment of the paramagnetic material. In general we would expect $U$ to depend on two independent variables (its natural variables) which we might take to be the temperature T and the total dipole moment $\mathcal{M}$, $U = U(\mathcal{M}, T)$. We can find out about the functional form of $U$ if we can get information about the two first derivatives `

$$\left(\frac{\partial U}{\partial T}\right)_{\mathcal{M}} \qquad\qquad \left(\frac{\partial U}{\partial \mathcal{M}}\right)_{\mathrm{T}}$$

The heat capacity at constant dipole moment, $C_{\mathcal{M}}$, is defined by

$$C_{\mathcal{M}} = T\left(\frac{\partial S}{\partial T}\right)_{\mathcal{M}}$$

**(a)** Use the thermodynamic identity to express the two first derivatives of $U$ as `

$$\left(\frac{\partial U}{\partial T}\right)_{\mathcal{M}} = C_{\mathcal{M}} \qquad \left(\frac{\partial U}{\partial \mathcal{M}}\right)_{\mathrm{T}} = T\left(\frac{\partial S}{\partial T}\right)_{T} + B_0$$

**[5 mks]**

**(b)** The Helmholtz free energy $F$ for the paramagnet is defined by $F = U - TS$

**i)** Using the thermodynamic identity show that for infinitesimal changes in $F$

$$dF = -SdT + B_0 d\mathcal{M}$$

**[3 mks]**

**ii)** Obtain the Maxwell relation arising from the equation in i) and hence show that we may write `

$$\left(\frac{\partial U}{\partial \mathcal{M}}\right)_{\mathrm{T}} = -T\left(\frac{\partial B_0}{\partial T}\right)_{M} + B_0$$

**[7 mks]**

**(c)** The Curie Law for an ideal paramagnet implies that

$$\mathcal{M} = \frac{CV}{\mu_0 T} B_0$$

where $\mu_0$ and $C$ are constants and $V$ is the (constant) volume of the system.

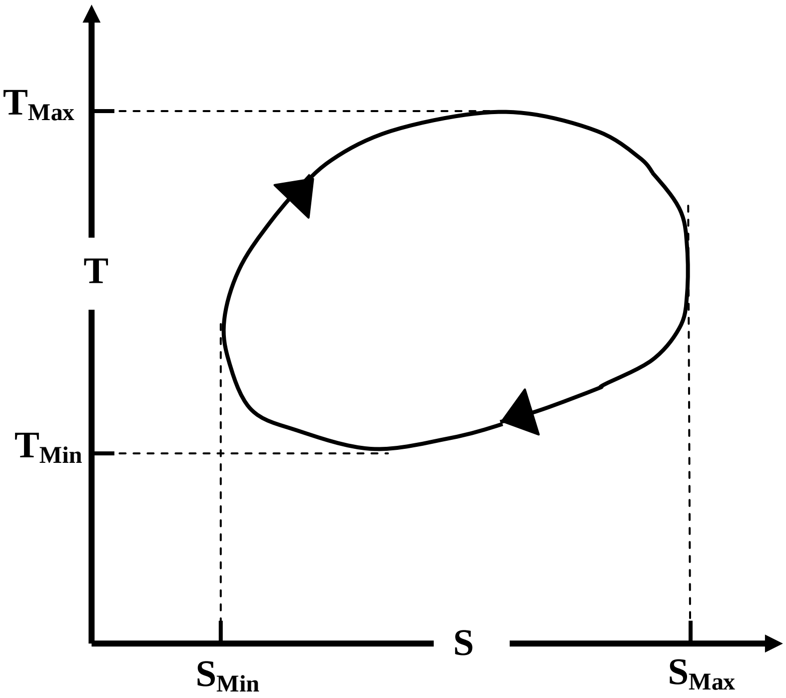
Use this equation of state and the result **ii)** to show that for an ideal paramagnet the internal energy $U$ is a function only of the temperature $T$ and does not depend upon the total magnetisation $\mathcal{M}$.

**[5 mks]**

**QUESTION 2: (20 marks)** (It may pay to read the textbook!)

**(a)** On a *P* - *V* diagram sketch the Carnot cycle which is undergone by a Carnot engine. **[3 mks]**

**(b)** The following diagram represents a general reversible engine cycle, but plotted in a temperature-entropy ($T - S$) plot.




We can think of the cycle as being composed of two parts, an upper curve from Smin to $S_{max}$ and a lower curve back from $S_{max}$ to $S_{min}$.

**i)** What does the area $A_{UP}$ under the upper curve between $S_{min}$ and $S_{max}$ represent? **[4 mks]**

**ii)** What does the area $A_{LOW}$ under the lower curve between these two points represent? **[4 mks]**

**iii) (1 mark)** Express the efficiency of this engine in terms of these two areas.

NOTE:The areas $A_{UP}$ and $A_{LOW}$ are understood as positive magnitudes.

**(c)** Draw the Carnot cycle on a *T* - *S* plot. **[3 mks]**

**(d)** Show that the efficiency of a Carnot engine operating between $T_{\max}$ , $T_{\min}$ , $S_{\max}$ and $S_{\min}$ is greater than that of the engine represented by the general cycle in (b).

**[6 mks]**