THERMAL AND KINETIC PHYSICS (PHY 214)

Coursework 7: Week 8

ISSUE: Tuesday 16 November, 2010 HAND-IN: Tuesday 23 November, 2010

Students name (top left corner), course title & exercise number and exercise group (top right corner) should appear on every sheet of the submitted coursework and sheets should be firmly held together. A stapler is available if needed from the secretaries office.

Hand-in of worked exercises must take place by 4:00 p.m. on the above date at the labeled box provided outside the Teaching Administrators office on the first floor.

This time will be strictly adhered to and no late working will be accepted without written explanation to the course organiser. The solutions will appear shortly after this time on the TKP website.

Each coursework is worth 40 marks and the aggregate coursework mark will count 10% towards the final mark. An indicative mark scheme is given with each question. **Note**: ***I want to see the method of solution. No credit will be given for simply writing down the answer.***

Students should collect new exercise sheets in the Tuesday lecture or download them from the Web. Marked exercises will be returned in exercise classes or via the box outside the Teaching Administrators office on the first floor.

QUESTION 1: (14 marks) (Question 3 on Chapter 8 in Finn)

(a) Calculate the temperature of the sun, assuming it to be a black body, if the solar energy flux falling on the surface of the earth is 1.4×10^3 J m^{-2} s^{-1}. The radius of the sun is 6.96×10^8m and the mean distance from the sun to the earth is 14.9×10^{10}m. **[4mks]**

(b) (From Question 5 of the 2004 Examination) Treating the Earth as a black body of uniform temperature heated solely by radiation absorbed from the Sun (also treated as a black body), derive an expression for the Earth's temperature T_E in terms of the surface temperature of the Sun, the radius of the Sun and the distance L_E from the Earth to the Sun. Show that $T_E \approx 280$ K. **[4mks]**

(c) The asteroid Icarus has an eccentric orbit with minimum distance from the Sun given approximately by $L_{\min} \approx \frac{1}{3} L_E$ and maximum distance $L_{\max} \approx \frac{3}{2} L_E$. Treating Icarus is also a black body heated only by solar radiation, estimate the minimum and maximum temperatures of Icarus in its orbit, given from (b) above that the Earth's mean temperature is about 280 K. **[6mks]**

QUESTION 2: (12 marks)

(a) A deep space probe has the form of a blackened sphere of radius 1.1 m containing a radioactive power source in the form of a few grams of Americium which gives off copious alpha particles which are stopped in a surrounding lead blanket which heats up to a steady temperature which you may take to be 500 K. This source supplies heat energy to the satellite at a rate of 10 kW. The satellite is powered by a Carnot engine which operates between the radioactive source as a high temperature reservoir and the surface of the satellite which can be regarded as the low temperature reservoir. In steady state, calculate the surface temperature of the satellite and the useful power supplied by the Carnot engine. **[6mks]**

(b) The satellite is provided with a blackened panel of area A which it can unfold to increase its total surface area. If the extra area A is equal to the surface area of the original sphere, what is the new steady surface temperature and the power supplied when the panel is deployed?

HINT: ***Questions 1 and 2 are energy balance questions in which a little bit of the geometry of spheres is needed. Think carefully about what geometric properties determine the energy absorbed and the energy radiated by a spherical black body.***

[6mks]

QUESTION 3: (14 marks)

(a) From kinetic theory arguments we derive that, for a photon gas, the internal energy U is given in terms of pressure and volume by U = 3PV. By using Boltzmann's argument we derived an expression for the energy density of a photon gas in the form $u = \frac{U}{V} = \left(\frac{4\sigma}{c}\right) T^4$ where $\sigma = 5.67 \times 10^{-8} W m^{-2} K^{-4}$ is the Stefan-Boltzmann constant and c is the velocity of light.

i) Calculate the energy density, $u = \frac{U}{V}$, of a sample of Argon gas at a pressure of 1 atm. **[2mks]**

ii) At what temperature would a photon gas have the same energy density? **[2mks]**

iii) Part of the unpleasant nature of nuclear weapons is the different way that they give out their energy as compared with conventional explosives. Nuclear weapons are not just bigger bangs, but should be regarded as qualitatively different in kind from chemical explosives. To understand how different they are, consider the fact that in a Hiroshima type explosion immediately after detonation the fireball is about 10 cm in diameter and at a temperature of about 10^6 K. What is the rate (in Watts) at which the fireball is losing energy from its surface (treat it as a black body)? What is the energy flux (energy per second per square metre) at a distance of one kilometer from the fireball? What is the pressure of the photon gas inside the fireball at this temperature?

[4 mks]

(b) The only macroscopic properties of a black hole are its mass M , its electric charge q and its angular momentum J. The change in the internal energy U is given by

$$dU = \frac{\chi c^2}{8\pi G} dA + \Omega dJ + \phi dq \qquad (1)$$

where χ is the so-called surface gravity of the black hole, i.e. the acceleration due to gravity at the horizon,

$$\chi = \frac{GM}{R_g^2} \qquad R_g = \frac{2GM}{c^2}$$

Ω is the angular velocity of the horizon and ϕ is the electrostatic potential at the horizon. The terms in dJ and dq represent work done on the black hole so we can interpret equation (1) as a statement of the First Law if we identify the term $\frac{\chi c^2}{8\pi G} dA$ as the entropy term TdS. It is thought that the black hole entropy is given by the expression

$$S_{BH} = \frac{1}{4}\frac{k_B A}{L_P^2} \qquad (2)$$

Where A is the area of the event horizon, L_P is the Planck length defined by

$$L_P = \sqrt{\frac{hG}{2\pi c^3}}$$

with c the velocity of light, and G is Newton's gravitational constant.

i) Use equations (1) and (2) to identify an expression for the temperature of the black hole. **[3 mks]**

ii) If the sun were compressed to form a black hole its horizon would have an area A of about 100 km^2 . Calculate the entropy and the temperature of this black hole. **[3 mks]**